

Incidence and Environmental Effects of Distortionary Subsidies

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Abstract

We present an analytical general equilibrium model to study the effect of distortionary subsidies on factor input prices and on environmental outcomes. Subsidies modeled include an output subsidy in one sector, capital and emissions input subsidies in one sector, and a direct cash subsidy to prevent one firm from exiting the market. We calibrate and simulate the model to gauge the magnitude of the effects.

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I. Introduction

Nearly all governments support particular firms or sectors by granting low interest financing, reduced regulation, tax relief, price supports, monopoly rights, and a variety of other subsidies. Subsidies are not lump sum, but instead introduce distortions by favoring particular firms, sectors, and/or inputs. This paper determines the incidence of distortions induced by subsidies. Subsidies are common in environmentally sensitive industries. This paper also determines the effect of subsidies on the quality of the environment.

Our subsidies differ from typical analysis of the incidence of subsidies or taxes in the literature in that only some firms receive subsidies. In our equilibrium, subsidized and non-subsidized (hereafter private) firms coexist only if subsidized firms are less productive. This matches the empirical observation that subsidies tend to go to firms to prevent bankruptcy or to support distressed firms facing foreign or domestic competition. We find that subsidies tied to the use of a particular input (e.g. low interest loans, relief from environmental regulation) have two effects. First, subsidized firms tend to increase demand for the subsidized input (capital, emissions) at the expense of substitute inputs. Second, as the input price increases, the private firms reduce usage of the subsidized input in favor of substitute inputs. Overall, the price of the subsidized input and complementary inputs increase at the expense of substitute inputs, and the subsidized sector grows in size. We show that output subsidies can be viewed as a subsidy to all inputs which therefore tend to increase the price of all inputs, unless an increase in the price of complementary inputs depresses demand for another input enough.

In our model, firms must use a minimum quantity of labor in exchange for receiving subsidies. If the minimum labor constraint binds, the marginal product of labor in the subsidized sector falls below the wage and subsidized firms earn negative profits. The government then provides a cash subsidy (hereafter a direct subsidy) to prevent subsidized firms from exiting the market. Increasing direct subsidies and the minimum labor constraint moves labor from the private to the subsidized sector. The demand for capital therefore increases in the subsidized sector but decreases in the private sector. The interest rate rises if the subsidized sector is more capital intensive than the private sector.

To gauge the magnitude of the effects, we calibrate our model to the Chinese economy, where the distinction between subsidizes and private firms is clear in our data. We simulate for base-case parameter values and we conduct sensitivity analysis over the parameters describing

each sector's substitution elasticities in production. The key determinant of the incidence of the policies (the changes in factor prices) is the factor intensities; in our simulation where the subsidized sector is relatively capital-intensive, subsidies hurt labor more than they hurt capital. This pattern is unaffected by the substitution elasticity values. In contrast, the effects on emissions depend crucially on the substitution elasticity values. The greater the ability of one sector to substitute into emissions away from an alternate input whose price increases, the larger that sector's emissions increase.

The subsidies we study are not designed to correct an externality (e.g. R&D subsidies). Instead, the subsidies we study are designed to protect or favor one firm or industry, which are known as "perverse subsidies" (Barde and Honkatukia 2004). Fullerton and Heutel (2010), (2007) use similar analytical methods to study the incidence of taxes and environmental mandates but not perverse subsidies. While Fullerton and Heutel (2007) study an emissions tax and Fullerton and Heutel (2010) study a variety of emissions policies, neither consider non-environmental policies like subsidies.

In many developing countries, subsidies are a significant fraction of GDP. For example, Brandt and Zhu (2000) report that subsidies in China amount to 6.8% of GDP in 1993. Further, van Beers and van den Bergh (2001) estimate worldwide subsidies are 3.6% of world GDP in the mid-1990s. Barde and Honkatukia (2004), report agriculture, fishing, energy (especially coal), manufacturing, transport, and water are all heavily subsidized, environmentally sensitive industries.

Despite the prevalence of subsidies, little is known about their the general equilibrium effect on factor prices and the environment. Barde and Honkatukia (2004) discuss a few channels by which subsidies may affect the quality of the environment. Input and output subsidies, especially in environmentally sensitive industries, encourage the over use of dirty inputs. Bailouts, tax relief, and other cash subsidies prevent the exit from the market of the least efficient producers, which are likely to be the most emissions intensive, which they call a technology lock-in effect. Subsidies in the form of regulatory relief include exemptions from environmental regulation, which directly increase the incentive to emit. Still, their analysis is largely informal. Indeed, they note that "a thorough assessment would require a complex set of general equilibrium analysis (to evaluate the rebound effect on the economy)." (Barde and

Honkatukia 2004, 268) This paper provides such a general equilibrium analysis, including all of the above channels.

Subsidies can also be used to protect favored industries against foreign competition. Indeed many trade agreements explicitly call for a reduction in subsidies. For example, Bajona and Kelly (2006) examine the effect on the environment of eliminating the subsidies required for China to enter the WTO and find that elimination of subsidies reduces steady state emissions of three of four pollutants studied. van Beers and van den Bergh (2001) show in a static, partial equilibrium setting how subsidies increase output and therefore emissions in a small open economy. For example, if subsidies are sufficiently large, a country may move from importing to exporting an environmentally sensitive good. The increase in output in turn increases emissions.

A number of authors study the effect of agricultural subsidies on the environment (Antle, Lekakis and Zanias 1998), (Pasour and Rucker 2005). Price supports and output and input subsidies encourage the use of dirty inputs such as fertilizer and pesticides, and encourage marginal land to be converted from conservation to farming. On the other hand, the USDA in 2003 had over 17 agricultural subsidy programs (\$1.9 billion) designed in part to improve environmental quality, primarily by paying farmers to remove environmentally sensitive land from production (Pasour and Rucker 2005). However, such restrictions have an ambiguous effect on erosion and fertilizer and pesticide use, since such restrictions encourage farmers to farmer the remaining land more intensively (Pasour and Rucker 2005, 110). Other subsidies, such as output subsidies, magnify this effect. It is therefore important to analyze all subsidies together in general equilibrium, as they can have offsetting or magnifying effects.

Previous work, then, has provided an important first step in identifying the extent of subsidies and likely channels by which they effect the environment. Still, the previous literature, with the exception of Bajona and Kelly (2006), does not examine the general equilibrium effects, or the effect of multiple subsidies. Further, Bajona and Kelly (2006) consider only two kinds of subsidies, direct subsidies and interest subsidies.

To understand emissions in such a setting requires a theory of firms and industry structure with subsidies. Bajona and Kelly (2006) provide a model where private and subsidized firms coexist. Subsidized firms have restrictions on the number of people they can lay off (Yin 2001), which they model as a minimum labor requirement. In exchange, subsidized firms receive

low interest loans from the government or state owned banks (modeled as an interest rate subsidy) and receive direct subsidies to cover the negative profits that result from the excess use of labor. Finally, subsidized firms have lower TFP relative to private sector firms. They prove the existence of an equilibrium in which subsidized firms and private firms co-exist with the share of production of subsidized firms determined endogenously by the subsidies, labor requirement, and technology difference. We extend their framework by considering as well output subsidies and regulatory relief and by making emissions endogenous.

II. Model

Consider a closed economy consisting of two representative firms producing an identical output good. Each firm has access to a production technology utilizing three inputs: capital, labor, and pollution. Production here is net of abatement costs, so higher pollution input means lower abatement costs, and therefore higher net production.¹ One firm is subsidized by the government, as described later; call its output level G . The other firm is non-subsidized or private; call its output level P . The subsidized firm's production function is

$$G = A_G F(K_G, L_G, E_G),$$

where K_G , L_G , and E_G are the quantities of capital, labor, and pollution used by the subsidized firm, and A_G is total factor productivity. We assume F is a constant returns to scale function.

Similarly,

$$P = A_P F(K_P, L_P, E_P),$$

with A_P , K_P , L_P , and E_P defined analogously. Henceforth, we normalize A_P to one.

Households supply \bar{K} units of capital and \bar{L} units of labor inelastically. Thus,

$$K_P + K_G = \bar{K},$$

$$L_P + L_G = \bar{L}.$$

Totally differentiating each of these equations, noting that both \bar{K} and \bar{L} are constant, yields

$$\hat{K}_P \lambda_{KP} + \hat{K}_G \lambda_{KG} = 0, \tag{1}$$

$$\hat{L}_P \lambda_{LP} + \hat{L}_G \lambda_{LG} = 0. \tag{2}$$

¹ See, for example, Bartz and Kelly (2008) for a derivation of a production function with pollution as an input from abatement cost functions.

Here λ_{ij} is the fraction of the total supply of factor i that is employed by firm j (e.g. $\lambda_{KP} = K_P/\bar{K}$). A variable with a caret represents a proportional change in that variable (e.g. $\hat{K}_P \equiv dK_P/K_P$).

The private firm faces three prices, r , w , and τ , for inputs K_P , L_P , and E_P , respectively. The price of pollution τ is a government policy variable, while the prices of capital and labor are endogenous. We follow Mieszkowski (1972) in modeling the private firm's choices over its inputs. Totally differentiating the private firm's three input demand equations, yields two independent equations:²

$$\hat{K}_P - \hat{L}_P = (e_{P,KK} - e_{P,KL})\theta_{PK}\hat{r} + (e_{P,KE} - e_{P,EL})\theta_{PE}\hat{\tau} + (e_{P,KL} - e_{P,LL})\theta_{PL}\hat{w}, \quad (3)$$

$$\hat{E}_P - \hat{L}_P = (e_{P,KE} - e_{P,KL})\theta_{PK}\hat{r} + (e_{P,EE} - e_{P,EL})\theta_{PE}\hat{\tau} + (e_{P,EL} - e_{P,LL})\theta_{PL}\hat{w}. \quad (4)$$

The parameters $e_{P,ij}$ are the Allen elasticities of substitution (Allen 1938). The Allen elasticity $e_{P,ij}$ is positive when inputs i and j are substitutes and negative when they are complements. The own price Allen elasticity $e_{P,ii}$ is always negative. The parameters θ_{Pi} represent the share of total revenue spent on input i , e.g. $\theta_{PK} \equiv rK_P/q_P P$, where q_P is the output price the good. Constant returns to scale implies zero profits, so $\theta_{PK} + \theta_{PL} + \theta_{PE} = 1$.

Totally differentiating the private firm's zero-profit condition yields

$$\hat{q}_P + \hat{P} = \theta_{PK}(\hat{r} + \hat{K}_P) + \theta_{PL}(\hat{w} + \hat{L}_P) + \theta_{PE}(\hat{\tau} + \hat{E}_P). \quad (5)$$

Totally differentiating the private firm's production function, and substitute in the first-order conditions from the firm's profit maximization problem (marginal revenue product equals marginal cost, for each input), gives

$$\hat{P} = \theta_{PK}\hat{K}_P + \theta_{PL}\hat{L}_P + \theta_{PE}\hat{E}_P. \quad (6)$$

The subsidized firm faces a different problem than the private firm, in four aspects. First, subsidized firms receive a discount on their capital costs. The discount may arise from the government guaranteeing repayment of funds borrowed by subsidized firms, direct loans from the government at reduce interest rates, state owned enterprises (SOEs) borrowing at the government's rate of interest, or as the government steering deposits at state owned banks to subsidized firms (common in developing countries) at reduced interest rates. Any of these

² The analytical general equilibrium modeling and solution strategy that we employ is in the style of Harberger (1962). Other papers using similar methods include Mieszkowski (1972), Fullerton and Metcalf (2002), Fullerton and Heutel (2007), and Fullerton and Heutel (2010). See, for example, Fullerton and Heutel (2007) for a derivation of equations (3)-(4).

implicit or explicit subsidies imply a lower effective price of capital for the subsidized firm, $r_G = (1 - \gamma)r$, where γ is the interest subsidy rate.

Second, subsidized firms receive a subsidy of ε per unit of output. The output subsidy may also be interpreted as a price support which applies only to the subsidized firm (for example, the government buys excess demand at an above market price from the subsidized firm and distributes the goods to households). If the consumer price of its output is q_G , then the effective output price that the subsidized firm faces is $q_{GN} = (1 + \varepsilon)q_G$.

Third, the government reduces the environmental regulatory burden the subsidized firm faces. In particular, the subsidized firm pays a pollution tax rate of $\tau_G = \tau(1 - \phi)$. Here $1 - \phi$ may also represent the fraction of emissions reported by the subsidized firm if for example the government monitors subsidized firms less often (Gupta and Saksena, 2002 find subsidized SOEs are monitored less often and Wang, 2002 finds SOEs enjoy bargaining power over environmental compliance).

Fourth, if subsidized and non-subsidized firms co-exist, some cost to receiving subsidies must exist. Following Bajona and Kelly (2006), we model this cost in a simple way. In particular, we assume that in order to receive subsidies, the government requires employment at subsidized firms to be greater than or equal to \bar{L}_G . The minimum labor constraint means the cost of receiving subsidies is a labor cost, which may include hiring lobbyists and/or hiring labor in key districts to increase bargaining power. In exchange for using \bar{L}_G fraction of the total labor supply, the government covers any losses through a direct subsidy (cash payment), S . The labor constraint binds if and only if the marginal product of labor in subsidized firms is below the wage rate, which causes subsidized firms to earn negative profits. Subsidized and private firms then co-exist if subsidized firms receive a direct subsidy large enough for the subsidized firm's profits to be non-negative, including the direct subsidy. If the constraint does not bind, subsidized firms have a competitive advantage and will drive the private firms from the market. Since this case is less interesting, we consider only the case where the constraint binds.

The subsidized firm's cost-minimization problem is analogous to the private firm's cost-minimization problem, except the subsidized firm faces different input costs (for example, replacing r with r_G , the subsidized capital price) and the binding constraint on labor input, \bar{L}_G . Because of this constraint, the subsidized firm no longer is able to set its marginal revenue product of labor equal to the wage. Rather, the constraint creates a shadow price of labor for the

subsidized firm, which we call w_G . This shadow price equals the difference between the market wage w and the multiplier on the labor constraint. Since we always assume that the labor constraint is binding, the market price w is strictly greater than the shadow price w_G .

The solution to the firm's cost-minimization problem can thus be written in terms of the shadow price of the constraint and the other subsidized prices in a manner similar to equations (3) and (4).³

$$\widehat{K}_G - \widehat{L}_G = (e_{G,KK} - e_{G,KL})\theta_{GK}\hat{r}_G + (e_{G,KE} - e_{G,EL})\theta_{GE}\hat{t}_G + (e_{G,KL} - e_{G,LL})\theta_{G\bar{L}}\widehat{w}_G \quad (7)$$

$$\widehat{E}_G - \widehat{L}_G = (e_{G,KE} - e_{G,KL})\theta_{GK}\hat{r}_G + (e_{G,EE} - e_{G,EL})\theta_{GE}\hat{t}_G + (e_{G,EL} - e_{G,LL})\theta_{G\bar{L}}\widehat{w}_G \quad (8)$$

In equations (7) and (8), the parameters θ_{GK} and θ_{GE} are the share of total subsidized firm revenues paid to capital and pollution, respectively, less government subsidies (e.g. $\theta_{GK} \equiv r_G K_G / q_{GN} G$). However, the parameter $\theta_{G\bar{L}}$ is not equal to $w L_G / p_{GN} G$. Rather, $\theta_{G\bar{L}} \equiv w_G L_G / p_{GN} G$, where w_G is the shadow price of labor. In equations (7) and (8), the firm's demand for labor is a function of prices. Although the constraint implies $L_G = \bar{L}_G$, the constraint creates the shadow price w_G so that labor demand under that shadow price, according to equations (7) and (8), is just equal to the minimum labor. That is, the subsidized firm faces a shadow price of labor which is less than the wage, which encourages the subsidized firm to use too much labor.

The subsidized firm's profits are

$$\pi_G = q_{GN} G - r_G K_G - w L_G - \tau_G E_G + S.$$

We first model the case where equilibrium profits, net of subsidies, must equal zero. Such an equilibrium condition might arise under free entry, for example.⁴ In this case, one of the subsidy values is determined in equilibrium; we assume that the zero-profit condition determines S once the regulator chooses all of the other subsidies. Substituting the firm's first-order conditions from the cost-minimization problem into the zero-profit condition implies:

$$\pi_G = q_{GN} G - q_{GN} G_K K_G - q_{GN} G_E E_G - w L_G + S = 0,$$

where G_K and G_E are the derivatives of the production function with respect to inputs capital and pollution. Then, since production is constant returns to scale, using Euler's theorem for homogeneous functions yields

³ An alternate method of modeling the binding labor constraint does not use the shadow price of labor, and instead derives equations similar to (7) and (8) but that are in terms of the value of the labor constraint itself rather than the shadow price. This is similar to the method in Fullerton and Heutel (2010). This solution method is available upon request from the authors.

⁴ Alternatively, if one interprets S as a "bailout," the government may be motivated only prevent bankruptcy, not to give positive profits to the subsidized firm.

$$\pi_G = w_G L_G - w L_G + S = 0.$$

Thus equilibrium subsidies are $S = (w - w_G)L_G > 0$, since the minimum labor constraint binds. Substituting equilibrium direct subsidies into the zero profit condition and totally differentiating yields

$$\hat{q}_{GN} + \hat{G} = \theta_{GK}(\hat{r}_G + \hat{K}_G) + \theta_{GL}(\hat{w}_G + \hat{L}_G) + \theta_{GE}(\hat{\tau}_G + \hat{E}_G). \quad (9)$$

Similarly, totally differentiating the production function and substituting in the first-order conditions from the profit-maximization problem yields

$$\hat{G} = \theta_{GK}\hat{K}_G + \theta_{GL}\hat{L}_G + \theta_{GE}\hat{E}_G. \quad (10)$$

The direct subsidy S drops out of equations (7)-(10). One could solve for the change in the direct subsidy by totally differentiating the above expression linking S to w , w_G , and L_G , but it is not necessary to include to solve the system (that would add one equation and one variable that does not show up in any other equation). Intuitively, the lump-sum subsidy does not affect the firm's decisions and therefore has no effect on any general equilibrium outcomes.⁵

It follows that the equations describing the subsidized firm's decisions (equations (7) through (10)) are independent of the assumption that the firm earns zero profits. Indeed, assume instead that the firm's profits π_G are allowed to be positive, and that the level of firm profits is an exogenous policy parameter that can be controlled by the government. That is, the profits π_G represent rents that the regulator allows the firm to capture, perhaps through barriers to entry.

To see this, assume that $\pi_G > 0$. Then the equation relating firm profits to the shadow price of the constraint still holds, but firm profits no longer must equal zero:

$$\pi_G = w_G L_G - w L_G + S.$$

The endogenous direct subsidy S is now larger, given the positive exogenous level of profit π_G . Totally differentiating this equation yields

$$\hat{\pi}_G = \beta_{w_G}(\hat{w}_G + \hat{L}_G) - \beta_w(\hat{w} + \hat{L}_G) + \beta_S \hat{S},$$

where $\beta_{w_G} \equiv \frac{w_G L_G}{\pi_G}$, $\beta_w \equiv \frac{w L_G}{\pi_G}$, and $\beta_S \equiv \frac{S}{\pi_G}$. Similarly, totally differentiating the definition of the firm's profits yields

$$\hat{\pi}_G = \beta_{p_G}(\hat{p}_{GN} + \hat{G}) - \beta_r(\hat{r}_G + \hat{K}_G) - \beta_\tau(\hat{\tau}_G + \hat{E}_G) - \beta_w(\hat{w} + \hat{L}_G) + \beta_S \hat{S},$$

⁵ The subsidized firm can only increase direct subsidies by increasing losses, so the direct subsidy is lump sum in the sense that profits net of subsidies are unchanged regardless of the subsidized firm's decisions.

where $\beta_{p_G} \equiv \frac{Gp_{GN}}{\pi_G}$, $\beta_\tau \equiv \frac{\tau_G E_G}{\pi_G}$, and $\beta_r \equiv \frac{r_G K_G}{\pi_G}$. Combining the above two equations, canceling terms, and multiplying everything by $\frac{\pi_G}{q_{GN}G}$ yields equation (9). The derivation of equation (10) does not depend on the assumption of zero profits. Thus, equations (7) through (10) hold as long as the direct subsidy is large enough so that subsidized profits are non-negative. We are interested in the case where both firms choose to operate, where neither firm is at a corner solution for any of its input demands and our differential analysis is applicable. The condition $S \geq (w - w_G)L_G$ ensures the subsidized firm operates, and the private firm operates if and only if the cost of receiving subsidies is positive, which occurs when the minimum labor constraint binds, $w - w_G > 0$.⁶

Finally, since the minimum labor constraint binds:

$$\hat{L}_G = \hat{L}_G. \quad (11)$$

Thus equations (7)-(11) characterize the subsidized firm's decisions. An alternative method to incorporating the binding labor constraint would be to set the input demand equation for labor equal to the labor constraint, then totally differentiate:

$$\hat{L}_G = e_{G,KL}\theta_{GK}\hat{r}_G + e_{G,EL}\theta_{GE}\hat{\tau}_G + e_{G,LL}\theta_{GL}\hat{w}_G + \hat{G}. \quad (11')$$

Equation (11') demonstrates that the input demand equation, in terms of the Allen elasticities and the input prices, characterizes the subsidized firm's demand for labor, which must equal the required minimum labor. It can be shown that this equation above can be derived from equations (7), (8) and (11) given known restrictions on the Allen elasticities. Thus, replacing equation (11) with (11') yields identical solutions.

The two firms produce identical goods, so perfect competition implies that consumer prices are equal:

$$\hat{q}_P = \hat{q}_G. \quad (12)$$

The consumer price for the subsidized firm's output is not equal to the price net of subsidies that the firm faces q_{GN} .

Finally, government policy determines the relative prices faced by the subsidized firm. Totally differentiating the definition $r_G \equiv r(1 - \gamma)$ yields

⁶ In the case of Cobb-Douglas production functions, we can write the condition $w - w_G > 0$ in terms of the parameters. In the more general formulation here where we impose no functional forms, we cannot specify $w - w_G > 0$ in terms of the parameters. However, since in practice one can check $w - w_G > 0$ directly, this is not a great concern.

$$\hat{r}_G = \hat{r} - \hat{\gamma}, \quad (13)$$

where \hat{r}_G and \hat{r} again are proportional changes (e.g. $\hat{r} = dr/r$), but $\hat{\gamma} \equiv d\gamma/(1 - \gamma)$.

Similarly,

$$\hat{t}_G = \hat{t} - \hat{\phi} \quad (14)$$

$$\hat{q}_{GN} = \hat{q}_G + \hat{\varepsilon}, \quad (15)$$

where $\hat{\phi} \equiv d\phi/(1 - \phi)$ and $\hat{\varepsilon} = d\varepsilon/(1 + \varepsilon)$.

The model consists of equations (1) through (15). The five exogenous policy variables are $\hat{\varepsilon}, \hat{t}, \hat{\phi}, \hat{\gamma}, \hat{L}_G$. The sixteen endogenous variables are:

$\hat{K}_P, \hat{L}_P, \hat{K}_G, \hat{L}_G, \hat{r}, \hat{w}, \hat{q}_P, \hat{P}, \hat{q}_G, \hat{G}, \hat{r}_G, \hat{w}_G, \hat{t}_G, \hat{q}_{GN}, \hat{E}_G, \hat{E}_P$. The model does not determine the price level, so the solution requires a normalization. We normalize to output by setting $\hat{q}_P = 0$. Now, the remaining fifteen endogenous variables are the solution to the linear system of equations (1)-(15).

III. Solution

We solve the model through successive substitution. The steps of the solution method are available from the authors. We present the results in two parts. First, we present the incidence results, that is, the effect of policy changes on the returns to capital and to labor. Second, we present the emissions results.

III.A Incidence

We derive a closed-form solution for \hat{r} and \hat{w} . However, here we present only the expression for \hat{r} . By subtracting equation (6) from equation (5) and invoking the normalization $\hat{q}_P = 0$, it can be shown that $\hat{w} = -\frac{\theta_{PK}}{\theta_{PL}}\hat{r} - \frac{\theta_{PE}}{\theta_{PL}}\hat{t}$. Thus, if the policy variable τ remains unchanged, then the sign of the change in the wage resulting from any other policy change is the opposite of the sign of the change in the rental rate. This does not mean that one factor gains and one loses, since both of these prices are relative to an arbitrary numeraire. Rather, if $\hat{r} > 0$ and $\hat{w} < 0$, then labor bears a disproportionately high share of the burden of the policy change relative to capital.

The solution for \hat{r} is

$$\hat{r} = \frac{1}{D} \left\{ \begin{array}{l} -\theta_{GK}\lambda_{KG}(e_{G,KK} - 2e_{G,KL} + e_{G,LL})\hat{\gamma} + \lambda_{KG}(e_{G,KL} - e_{G,LL})\hat{\varepsilon} + \\ -\theta_{GE}\lambda_{KG}B_G\hat{\phi} + \lambda_{KP}\left(\frac{\lambda_{KG}}{\lambda_{KP}} - \frac{\lambda_{LG}}{\lambda_{LP}}\right)\hat{L}_G + (\theta_{GE}\lambda_{KG}B_G + \theta_{PE}\lambda_{KP}B_P)\hat{t} \end{array} \right\} \quad (16)$$

Here $B_P \equiv e_{P,KE} - e_{P,EL} - e_{P,KL} + e_{P,LL}$, $B_G \equiv e_{G,KE} - e_{G,EL} - e_{G,KL} + e_{G,LL}$, and

$D \equiv -\lambda_{KG}\theta_{GK}(e_{G,KK} - 2e_{G,KL} + e_{G,LL}) - \theta_{PK}\lambda_{KP}(e_{P,KK} - 2e_{P,KL} + e_{P,LL})$. In the appendix we show that both terms in D are positive given constant returns to scale.

Given both terms in D are positive, the coefficient on $\hat{\gamma}$ in the expression for \hat{r} must be positive. An increase in γ , the subsidy to capital, increases the demand for capital by the subsidized firm, which pushes up the price of capital.⁷

The sign of the coefficient on $\hat{\phi}$ is opposite of the sign of B_G . We show in the appendix that $B_G > 0$ if and only if an increase in emissions raises the marginal product of capital ($f_{KE} > 0$). Therefore, the sign of the coefficient on $\hat{\phi}$ is positive if and only if capital and emissions are substitutes in production. A subsidy to emissions decreases the price of emissions for the subsidized firm. If emissions and capital are complements, demand for capital, and therefore the interest rate, rises with a decrease in the price of emissions. The coefficient on \hat{t} is exactly the opposite sign, since an increase in \hat{t} increases the price of emissions.⁸

An increase in the output subsidy can be viewed as equivalently a decrease in the price of all three inputs. In the appendix, we show the coefficient on $\hat{\varepsilon}$ in equation (16) consists of two terms. The first term represents the increase in demand for capital caused by the decrease in the price of capital. The second term is positive if the cross price elasticity between emissions and capital is positive, since in this case the decrease in the price of emissions increases demand for capital. No term exists for the decrease in the price of labor since labor is fixed by the constraint. Therefore, capital demand and therefore the interest rate increase unless capital and emissions are sufficiently strong substitutes.

The coefficient on the labor constraint, \hat{L}_G , has the same sign as the expression $\lambda_{KG}/\lambda_{KP} - \lambda_{LG}/\lambda_{LP}$. This expression is positive when the subsidized firm is capital-intensive relative to the private firm; that is, when firm G has a higher capital to labor ratio than P . One

⁷ Since the supply of capital is fixed, the increase in capital in the subsidized firm replaces demand in the private firm through the higher interest rate.

⁸ Technically, we assume that emissions and capital do not switch from being substitutes to complements or the reverse. If so, then it is possible that emissions and capital are substitutes for one firm, and complements for the other. In this case, an decrease in \hat{t} may have a different effect than an increase in $\hat{\phi}$ as \hat{t} directly affects emissions of both firms.

might think that a tightening of the labor constraint must hurt labor, since it is forcing its quantity employed to be lower. However, there is a fixed labor stock, and any reduction in labor used in one firm equals the increased use of labor in the other firm. Rather, a tightening of the labor constraint is a burden that falls only on the subsidized firm. If the subsidized firm is capital-intensive, then that burden will fall harder on capital than on labor, and the price of capital falls.

III.B. Emissions

Consider first the effect on emissions in the private sector E_P . Instead of presenting a closed-form solution, it is more helpful to present an intermediate equation in the solution steps that expresses the change in emissions as a function of just the labor constraint, the emissions price, and the endogenous capital price:

$$\hat{E}_P = -\frac{\lambda_{LG}}{\lambda_{LP}}\hat{L}_G + \theta_{PE}(e_{P,EE} - 2e_{P,EL} + e_{P,LL})\hat{\tau} + \theta_{PK}B_P\hat{r}. \quad (17)$$

If \bar{L}_G decreases, then more labor must be used in the private firm, and holding all else constant this increases the amount of emissions used in the private firm (from equation (4)). If the emissions price τ increases, then holding all else constant the emissions used in the private firm decreases. Lastly, if the policy change is such that the price of capital increases, then the quantity of emissions used in the private sector decreases as long as $B_P < 0$. When capital and emissions are very substitutable, $B_P > 0$, the substitution effect from the capital price increase dominates, and emissions increases.

All of the effects on emissions in the private firm from any of the policy changes occur via their effect on r , as seen in equation (17), except for the additional effects from \hat{L}_G and $\hat{\tau}$ (which also affect \hat{r}). The closed-form solution for \hat{E}_P can be found by substituting equation (16) into equation (17). The results provide the same intuition as does equation (17). For instance, the coefficient on $\hat{\gamma}$ in the closed-form expression for \hat{E}_P is

$$-\frac{1}{D}\theta_{PK}B_P\theta_{GK}\lambda_{KG}(e_{G,KK} - 2e_{G,KL} + e_{G,LL}).$$

When $B_P < 0$, this coefficient is negative. An increase in γ increases r , which decreases E_P .

The coefficients on \hat{L}_G and $\hat{\tau}$ include both the direct effect in the equation (17) and the indirect effects via the effects on \hat{r} from equation (16).

An analogous expression for \hat{E}_G is

$$\hat{E}_G = \hat{L}_G + \theta_{GE}(e_{G,EE} + e_{G,LL})(\hat{\tau} - \hat{\phi}) + (e_{G,EL} - e_{G,LL})\hat{\varepsilon} + \theta_{GK}B_G(\hat{r} - \hat{y}). \quad (18)$$

The first term comes directly from equation (8), where if nothing else changes, then the change in emissions equals the change in labor. The second term shows that the change in the net emissions price to the subsidized firm, $\hat{\tau} - \hat{\phi}$, negatively affects the emissions used by the subsidized firm. The third term demonstrates that an increase in the subsidized firm's output subsidy, ε , increases its use of the emissions input. Lastly, the final term is the effect of the change in the net price to the subsidized firm of capital, $\hat{r} - \hat{y}$. It includes the endogenous \hat{r} . When the net price increases, the quantity of emissions used decreases, unless the substitution effect between capital and emissions dominates and $B_G > 0$.

As with equation (17), equation (18) is not a closed-form expression, but the closed-form solution can be found by substituting in equation (16). Most of the resulting closed-form coefficients conform to the intuition presented by merely examining equation (18). For instance, the coefficient on $\hat{\varepsilon}$ in the closed-form expression for \hat{E}_G is

$$(e_{G,EL} - e_{G,LL}) + \frac{1}{D}(\theta_{GK}B_G\lambda_{KG}(e_{G,KL} - e_{G,LL})).$$

The first two terms in the first set of parentheses are positive and represent the direct effect that can be seen in equation (18), and the rest of the terms are from the effect of $\hat{\varepsilon}$ on \hat{r} . Equation (16) shows that an increase in ε will increase r . Thus, combined with equation (18), as long as $B_G < 0$, this second term will be negative. In this case the direct effect from $\hat{\varepsilon}$ shown in equation (18) and the indirect effect via its effect on \hat{r} move in opposite directions.

One closed-form coefficient is worth showing on its own. The coefficient on \hat{L}_G in the expression for \hat{E}_G is

$$\begin{aligned} \frac{1}{D\lambda_{LP}} \{ & -\lambda_{LP}\lambda_{KP}\theta_{PK}(e_{P,KK} - 2e_{P,KL} + e_{P,LL}) \\ & + \theta_{GK}[\lambda_{KG}\lambda_{LP}(e_{G,KL} - e_{G,KK}) + \lambda_{KP}\lambda_{LG}(e_{G,KL} - e_{G,LL}) \\ & + (\lambda_{KG}\lambda_{LP} - \lambda_{LG}\lambda_{KP})(e_{G,KE} - e_{G,EL})] \} \end{aligned}$$

All of the terms in this expression are positive, with the exception of the last line of the expression. The first part of the last line, $\lambda_{KG}\lambda_{LP} - \lambda_{LG}\lambda_{KP}$, is positive whenever the subsidized firm is capital-intensive. The second part of the last line, $e_{G,KE} - e_{G,EL}$, is positive whenever capital is a better substitute for emissions than is labor, in the subsidized firm.

Most of the coefficient is positive and picks up the fact that a reduction in \bar{L}_G is a burden on the subsidized firm and causes it to contract, reducing its demand of input E_G . However, the firm can also substitute among its inputs. The subsidized firm could respond to its requirement to decrease labor demand (i.e. its increase in the shadow price of labor) by demanding more emissions or more capital. If it is capital-intensive and if labor is a better substitute for emissions than is capital, then the increase in the shadow price will lead to a substitution effect that works to increase emissions in that sector. Even in this case, this effect will only dominate if it is larger than all of the other positive terms in the above coefficient.

IV. Calibration and Simulation

We choose parameter values to calibrate the model to investigate the magnitude of the effects found from the analytical solutions, many of which were difficult to sign unambiguously. The 2006 China Statistical Yearbook provides data on capital and labor inputs, profits, and emissions. Because the Chinese data are separated into state-owned enterprises (SOEs) and private firms, these data provide a good source for the calibration. We consider the SOEs to be the subsidized sector and the private firms to be the private sector.

The data gives the value of capital and labor inputs in both sectors, and thus we can directly calculate the input shares λ_{KP} , etc. Calibration of the other parameters, including the expenditure share parameters θ_{ij} , is described in the Appendix. Lastly, the Allen elasticities of substitution in each sector are not provided in the China Statistical Yearbook. Instead, we use the same values as in Fullerton and Heutel (2010). Those values, based on earlier estimates, suggest that capital is a slightly better substitute for pollution than is labor. We assume that the cross-price Allen elasticities are identical across the two sectors. But, because of the different expenditure shares across sectors, the own-price elasticities are not identical across sectors.

Table 1 summarizes the calibrated parameter values (the own-price elasticities are not presented but are derivable from the rest of the parameter values). The subsidized sector is relatively capital-intensive ($\frac{\lambda_{KG}}{\lambda_{LG}} > \frac{\lambda_{KP}}{\lambda_{LP}}$). The share of expenditures that goes towards pollution taxes in both industries (θ_{GE} and θ_{PE}) is very small.

We simulate the effects of four different exogenous policy changes. First, we set $\hat{\gamma} = 10\%$, simulating an increase in the capital subsidy rate to the subsidized firm. Second, $\hat{\varepsilon} = 10\%$, simulating an increase in the output subsidy rate to the subsidized firm. Third,

$\hat{\phi} = 10\%$, simulating an increase in the pollution tax subsidy rate to the subsidized firm. Last, $\hat{L}_G = -10\%$, simulating an decrease in the minimum labor constraint faced by the subsidized firm (this is negative so that all four policy simulations are a benefit to the subsidized firm, although the direct subsidy will in equilibrium decrease to leave the subsidized firm's profits unchanged). The results from these four policy changes are presented in panels B and C of Table 2. Row 1 in each of those panels represents the simulations using the base case parameter values; rows 2 through 5 represent sensitivity analysis about the parameters (see panel A).

Focusing first on the base case simulations, the effect of either the capital subsidy or the output subsidy on incidence (\hat{r} and \hat{w}) is identical: a 10% increase increases the rental rate by 5.45% and decreases the wage rate by 2.32% (like the analytical results, these price changes are normalized relative to the price of the output good). This change is driven by the factor share parameters. Since the subsidized sector is capital-intensive, a subsidy to it is likely to benefit capital relatively more than labor. The pollution tax subsidy in panel C does not affect the rental rate or the wage to two decimal places. This is because pollution expenditure shares are so small that a change in that input price has minimal general equilibrium effects. The change in the labor constraint reduces the capital price by 6.56% and increases the labor price by 2.79%. This may seem counterintuitive, since a decrease in the required use of labor seems like it should decrease demand for labor and therefore its price. But, the overall labor resource constraint always binds, and thus the reduction of the labor constraint in the subsidized sector is equivalent to forcing more labor into the private sector. Because the private sector must set the wage equal to its marginal value product of labor, the policy actually drives up the wage.

Next, consider the pollution effects of all four policy simulations under the base case parameter values. The capital subsidy and output subsidy both have identical effects on pollution in the private sector (a 0.83% reduction), since the private sector's demand responds only to the input prices, which are identical under the two policy changes. By contrast, the 10% capital subsidy causes only a 0.33% increase in the subsidized sector's emissions, while the 10% output subsidy causes a 3.69% increase in the subsidized sector's emissions. The output subsidy substantially alters the quantity of output produced by the subsidized sector, not just its relative input prices. This increase in output of the subsidized sector is the primary driver of its increased emissions. The change in the pollution tax subsidy has no substantial effect (to two digits) on the private sector's emissions, but it increases the subsidized sector's emissions by 3.36%. Lastly,

the change in the labor constraint increases the private sector's emissions and decreases the subsidized sector's emissions. The decrease of 10% in the subsidized firm's labor requirement reduces its emissions by just under 10%. This is reflective of the expression at the end of section III for the coefficient on \hat{L}_G in the expression for \hat{E}_G . Although that expression could not be signed, most of its terms are positive, indicating an output effect. The negative terms indicate the substitution effects from the switching of inputs. Since the overall sign ends up positive in this simulation, the output effect dominates.

Table 2 also considers alternate parameter values. We consider alternate values for the substitution elasticities in production, since these are the parameters about which we know the least (but of course we could vary other parameters in other sensitivity analyses). We investigate changes in two of the cross-price Allen elasticities for each of the two sectors. In each of the rows 2 through 5, we change one of those Allen elasticities to 1 and leave all other parameter values unchanged in the base case. Row 2 in Table 2 sets $e_{P,KE} = 1$; this represents a case where emissions and capital are strong substitutes in the private sector. In row 3, emissions and labor are strong substitutes in the private sector ($e_{P,EL} = 1$). Rows 4 and 5 make the same assumptions, respectively, in the subsidized sector ($e_{G,KE} = 1$ and $e_{G,EL} = 1$). The results of the simulations for each of these new parameter values are presented in the remaining panels of Table 2.

Consider first how these different elasticity values affect the incidence results (\hat{r} and \hat{w}). Rounded to the second decimal place, there is no difference in these values between rows 1 through 5 for any of the four policy variables. They are slightly different from each other, but only beyond the second decimal point. The takeaway is that the substitution elasticities in production do not substantively affect general equilibrium factor prices.

Next, consider how the different elasticity values affect the emissions results (\hat{E}_P and \hat{E}_G). For the capital subsidy $\hat{\gamma}$, the largest decrease in private sector emissions occurs in row 3, where labor and emissions are strong substitutes in the private sector. Similarly, the only decrease in subsidized sector emissions occurs in row 4, where capital and emissions are strong substitutes in the subsidized sector. In each of these rows, the changes in the relative input prices (the same across all rows) have different effects on the demand for emissions based on each sector's substitution elasticity.

For the output subsidy $\hat{\varepsilon}$, the emissions from the private sector mimics its emissions under the capital subsidy $\hat{\gamma}$ since the factor prices are the same. But the subsidized sector expands relative to the private sector under this policy, and so emissions increases in each row. When the subsidized sector is more able to substitute emissions for either capital or for labor, as it is in rows 4 and 5, its resultant increase in emissions is even greater than under its base case substitution elasticity values.

The emissions tax subsidy $\hat{\phi}$ has no effect on emissions in the private sector since it does not change any prices that sector faces. It changes the relative price of emissions for the subsidized sector, and thus the increased subsidy (lower net emissions price) increases the emissions of that sector. When the substitutability between emissions and either input is large in the subsidized sector (rows 4 and 5), the increase in its emissions is larger.

The loosening of the labor constraint ($\hat{L}_G = -10\%$) always increases emissions in the private sector and decreases emissions in the subsidized sector. This is due to an output effect (the private sector expands and the subsidized sector contracts). But the magnitude of the change in emissions is due to substitution effects. When the private sector has more ability to substitute between emissions and labor (row 3), its emissions increase is larger. When the subsidized sector has more ability to substitute between emissions and capital (row 4), its emissions decrease is larger.

V. Conclusion

We present an analytical general equilibrium model to study the effects on incidence and on pollution of distortionary subsidies. Policies intended to support one firm or one sector, like input or output price subsidies, can also have unintended general equilibrium price effects and effects on environmental quality. We calibrate the model to gauge the numerical magnitude of these effects, and in particular to examine how they depend on the substitution elasticities in production. The incidence effects (factor prices) are relatively unaffected by substitution elasticities, but emissions are substantitally affected by the substitution elasticities. The better substitute pollution is for an input whose price increases, the more emissions will increase.

Our model is simple and omits many salient features of an economy that might also affect emissions and incidence. However, its simplicity is a virtue in that it allows us to isolate individual effects without confounding complications. Our calibration is based on data from

China simply because of access to a data source that clearly delineates between subsidized and non-subsidized firms, though the model can be applied to other economies and other industries. Further work could consider, for example, how subsidies to domestic auto manufacturing firms affected prices and emissions from that sector in the United States.

References

- Allen, R.G.D. *Mathematical Analysis for Economists*. New York: St. Martin's, 1938.
- Antle, John, Joseph Lekakis, and George Zanas. *Agriculture, trade and the environment: The impact of liberalization on sustainable development*. Northampton, MA: Edward Elgar, 1998.
- Bajona, Claustre, and David Kelly. "Trade and the Environment With Pre-Existing Subsidies: A Dynamic General Equilibrium Analysis." *University of Miami Working Paper 2006-3*, 2006.
- Barde, Jean-Philippe, and Outi Honkatukia. "Environmentally Harmful Subsidies." In *The International Yearbook of Environmental and Resource Economics 2004/2005*, by Tom Tietenberg and Henk Folmer, 254-288. Northampton, MA: Edward Elgar, 2004.
- Brandt, L., and Z. Zhu. "Redistribution of a Decentralized Economy: Growth and Inflation in China Under Reform." *Journal of Political Economy* 108 (2000): 422-439.
- Fullerton, Don, and Garth Heutel. "The General Equilibrium Incidence of Environmental Mandates." *American Economic Journal: Economic Policy* 2, no. 3 (August 2010): 64-89.
- Fullerton, Don, and Garth Heutel. "The General Equilibrium Incidence of Environmental Taxes." *Journal of Public Economics* 91, no. 3-4 (April 2007): 571-591.
- Fullerton, Don, and Gilbert Metcalf. "Tax Incidence." In *Handbook of Public Economics*, by A. Auerbach and M. Feldstein. Amsterdam: North Holland, 2002.
- Harberger, Arnold. "The Incidence of the Corporation Income Tax." *Journal of Political Economy* 70, no. 3 (June 1962): 215-240.
- Mieszkowski, Peter. "On the Theory of Tax Incidence." *Journal of Political Economy* 75, no. 3 (June 1967): 250-262.

Mieszkowski, Peter. "The Property Tax: An Excise Tax or a Profits Tax?" *Journal of Public Economics* 1, no. 1 (April 1972): 73-96.

Pasour, E.C., and Randal Rucker. *Plowshares and Pork Barrels: The Political Economy of Agriculture*. Oakland, CA: Independent Institute, 2005.

van Beers, C., and J.C. van den Bergh. "Perseverance of Perverse Subsidies and their Impact on Trade and Environment." *Ecological Economics* 36 (2001): 475-486.

Yin, Xiangkang. "A Dynamic Analysis of Overstaff in China's State-Owned Enterprises." *Journal of Development Economics* 66, no. 1 (October 2001): 87-99.

Table 1: Calibrated Parameters

<i>Parameter</i>	<i>Value</i>
λ_{KP}	0.4166
λ_{LP}	0.7281
λ_{KG}	0.5834
λ_{LG}	0.2719
θ_{PK}	0.2986
θ_{PL}	0.7012
θ_{PE}	0.0002
θ_{GK}	0.1795
θ_{GL}	0.8202
θ_{GE}	0.0002
$e_{P,KL}$	0.5
$e_{P,KE}$	0.5
$e_{P,EL}$	0.3
$e_{G,KL}$	0.5
$e_{G,KE}$	0.5
$e_{G,EL}$	0.3

Note: See Appendix for details of calibration.

Table 2: Simulation

<i>Panel A: Parameter Values</i>				
	$e_{P,KE}$	$e_{P,EL}$	$e_{G,KE}$	$e_{G,EL}$
1 (base case)	0.5	0.3	0.5	0.3
2	1	0.3	0.5	0.3
3	0.5	1	0.5	0.3
4	0.5	0.3	1	0.3
5	0.5	0.3	0.5	1

<i>Panel B: Simulation Results</i>								
	$\hat{\gamma} = 10\%$				$\hat{\varepsilon} = 10\%$			
	\hat{r}	\hat{w}	\hat{E}_P	\hat{E}_G	\hat{r}	\hat{w}	\hat{E}_P	\hat{E}_G
1	5.45%	-2.32%	-0.83%	0.33%	5.45%	-2.32%	-0.83%	3.69%
2	5.45%	-2.32%	-0.02%	0.33%	5.45%	-2.32%	-0.02%	3.69%
3	5.45%	-2.32%	-1.97%	0.33%	5.45%	-2.32%	-1.97%	3.69%
4	5.45%	-2.32%	-0.83%	-0.07%	5.45%	-2.32%	-0.83%	4.18%
5	5.45%	-2.32%	-0.83%	0.91%	5.45%	-2.32%	-0.83%	10.01%

<i>Panel C: Simulation Results</i>								
	$\hat{\phi} = 10\%$				$\hat{L}_G = -10\%$			
	\hat{r}	\hat{w}	\hat{E}_P	\hat{E}_G	\hat{r}	\hat{w}	\hat{E}_P	\hat{E}_G
1	0.00%	0.00%	0.00%	3.36%	-6.56%	2.79%	4.74%	-9.52%
2	0.00%	0.00%	0.00%	3.36%	-6.56%	2.79%	3.76%	-9.52%
3	0.00%	0.00%	0.00%	3.36%	-6.56%	2.79%	6.11%	-9.52%
4	0.00%	0.00%	0.00%	4.26%	-6.56%	2.79%	4.74%	-10.11%
5	0.00%	0.00%	0.00%	9.10%	-6.56%	2.79%	4.74%	-8.69%

Note: Panel A shows the parameter values that are varied in simulations 2 through 5. All other parameter values remain at their base case values (Table 1). Panels B and C show simulation results for four endogenous variables (\hat{r} , \hat{w} , \hat{E}_P , and \hat{E}_G) in response to one of four exogenous policy changes.

Appendix

Proof that $\mathbf{D} > \mathbf{0}$.

If $e_{i,KK} - 2e_{i,KL} + e_{i,LL}$ is negative regardless of $i = G, L$, then $D > 0$. Dropping the subscript i for convenience and using the definition of Allen elasticity, we have:

$$\begin{aligned} e_{KK} - 2e_{KL} + e_{LL} &= \frac{|B_{KK}|}{|B|K^2} - 2\frac{|B_{KL}|}{|B|KL} + \frac{|B_{LL}|}{|B|L^2} \\ &= \frac{1}{|B|K^2L^2} (L^2|B_{KK}| - 2|B_{KL}|KL + K^2|B_{LL}|) \end{aligned}$$

Here B is the bordered Hessian from the firm's cost minimization problem, and B_{ij} is the bordered Hessian with column i being all zeros except a one at row j . Since the determinant of the bordered Hessian is negative by concavity, we must show the last term is positive.

Evaluating the determinants in the last term results in:

$$\begin{aligned} &L^2|B_{KK}| - 2|B_{KL}|KL + K^2|B_{LL}| \\ &= 2L^2f_Lf_E - L^2f_L^2f_{EE} - L^2f_E^2f_{LL} + 2KLf_Ef_Kf_{EL} - 2KLf_E^2f_{KL} \\ &\quad - 2KLf_Kf_Lf_{EE} + 2KLf_Ef_Lf_{KE} + 2K^2f_Ef_Kf_{KE} - K^2f_K^2f_{EE} - K^2f_E^2f_{KK} \end{aligned}$$

Since f is constant returns to scale, the marginal products are homogenous degree zero.

Therefore:

$$f_{KK} = -\frac{Lf_{KL} + Kf_{KE}}{K}, \quad (\text{A1})$$

$$f_{LL} = -\frac{Kf_{KL} + Ef_{EL}}{L}, \quad (\text{A2})$$

$$Lf_{EL} + Kf_{KE} = -Ef_{EE}. \quad (\text{A3})$$

Substituting in equations (A1)-(A2), collecting terms, and then substituting in (A3) results in:

$$L^2|B_{KK}| - 2|B_{KL}|KL + K^2|B_{LL}| = -f_{EE}(Lf_L + Kf_K + Ef_E)^2 = -f_{EE}Y^2 > 0.$$

Here the last equality follows from Euler's theorem and the term is positive by concavity. Hence, we have:

$$e_{i,KK} - 2e_{i,KL} + e_{i,LL} = \frac{-f_{i,EE}Y_i^2}{|B_i|K_i^2L_i^2} > 0.$$

The sign of the above equation is independent of i and is negative by concavity which completes the proof.

Condition for $\mathbf{B}_G > \mathbf{0}$.

We have:

$$\begin{aligned} B_G &\equiv e_{G,KE} - e_{G,EL} - e_{G,KL} + e_{G,LL}, \\ &= \frac{1}{|B|KEL^2} (L^2|B_{KE}| - KL|B_{EL}| - EL|B_{KL}| + KE|B_{LL}|). \end{aligned}$$

Evaluating the determinants, substituting in (A1)-(A2), and the (A3) yields:

$$B_G = \frac{1}{|B|KEL^2} (-f_{KE}(Lf_L + Kf_k + Ef_E)^2) = \frac{-f_{KE}Y^2}{|B|KEL^2}.$$

Since the determinant of B is negative by concavity, the above equation is positive if and only if an increase in emissions raises the marginal product of capital ($f_{KE} > 0$).

Calibration

The data in Appendix Table 1 come from the 2006 China Statistical Yearbook (CSY).

The factor share ratios λ_{ij} are calculated directly from these data, e.g. $\lambda_{KP} = \frac{K_P}{K} = \frac{59,628}{143,144} = 0.42$.

Calibration of policies ϕ, γ and ε

Calibration of ϕ and γ depends on the functional form of the production function. Assuming Cobb-Douglas production, the firm's first order conditions (China has production taxes t , which we add and assume are identical for G and P) imply

$$\begin{aligned} r &= (1-t)F_{K,P} = (1-t)\alpha \left(\frac{Y_P}{K_P} \right), \\ r_G &= r \frac{1-\gamma}{1-\varepsilon} = F_{K,G} = \alpha(1-t) \frac{Y_G}{K_G}, \end{aligned}$$

where α is the capital share parameter. We set $\varepsilon = 0$, since some argue in China that actually SOEs are forced to sell at below market prices. These equations may then be combined to get

$$\gamma = 1 - \frac{K_P Y_G}{Y_P K_G} = 0.57$$

Similarly, for ϕ ,

$$\phi = 1 - \frac{E_P Y_G}{Y_P E_G} = 1 - \frac{\sigma_P}{\sigma_G}$$

Here σ is the emissions intensity of output. Bajona and Kelly (2006) estimates $\frac{\sigma_G}{\sigma_P} = 5$ for SO₂ and Wang and Wheeler (2003) estimate it at 5.7 for chemical oxygen demand (COD).

Construct τ , E_G

We use the emissions ratios to construct emissions by sector.

$$\frac{\sigma_G}{\sigma_P} = \frac{E_G Y_P}{E_P Y_G} = \frac{E_G}{1 - E_G} \frac{Y_P}{Y_G}$$

$$E_G = \frac{\frac{\sigma_G}{\sigma_P} Y_G E}{Y_P + \frac{\sigma_G}{\sigma_P} Y_G}$$

This gives emissions of $E_G = 642$ for SO₂ and $E_G = 382$ for COD.

Wang (2005, footnote 13) reports τ equals 0.5 yuan/kilogram for COD and 0.4 for SO₂. This corresponds to 0.05 100M yuan per 10K metric tons for SO₂ and 0.04 for COD.⁹

Construct r , total subsidies

To get the interest rate, we need to compute depreciation as depreciation is included in value added but not profits. Assuming a depreciation rate of $\delta = 0.06$, we have:

$$DEP_G = \delta K_G = 5,011$$

and the same for private. The interest rate is then

$$r = \frac{rk}{k} = \frac{\pi_G + \pi_P + DEP_G + DEP_P}{K_G + K_P} = 0.16$$

This is a high interest rate, but probably not unreasonable given China's very fast economic growth. We add depreciation to capital income in all the calculations below.

We now construct capital and emissions subsidies:

$$\gamma r K_G = 7,764$$

$$\phi \tau E_G = 21 \text{ for SO}_2, = 16 \text{ for COD.}$$

Calibration of Wages

We have from the profit equations (including a tax t on value added):

$$\pi_G = (1 - t)F(K_G, L_G, E_G) - r_G K_G - w L_G - \tau_G E_G + S.$$

Let us calibrate assuming after-subsidy profits equal zero. This yields

$$w L_G = (1 - t)F(K_G, L_G, E_G) - r_G K_G - \tau_G E_G + S$$

⁹ The emissions tax revenue does not measure all spending on environmental regulations, but total environmental compliance spending is still small relative to GDP.

$$\begin{aligned}
&= Y_G - tY_G - rK_G + \gamma rK_G - \tau E_G + \phi \tau E_G + S \\
&= 27,177 - (6,220 - 0.05 \cdot 642) + 7,764 - 0.05 \cdot 642 + 21 + 193 = 17,404.
\end{aligned}$$

This assumes that the environmental taxes are counted in the data on total taxes paid. The private data wage equation is the same equation with no subsidies, which results in $wL_P = 27,852$. The calibration is slightly different using COD instead of SO_2 .

Given the wage data, we have

$$\begin{aligned}
S &= wL_G - w_G L_G \\
&= 17,211
\end{aligned}$$

Calibration of Shares

We calibrate the shares using the standard assumption that tax income is allocated proportionally to each factor according to their factor shares. For private shares, we have

$$\begin{aligned}
\theta_{PK} Y_P &= rK_P + \theta_{PK} (\text{taxes} - \text{env. taxes}_P) \\
\theta_{PK} &= \frac{rK_P}{Y_P - (\text{taxes} - \text{env. taxes}_P)} \\
\theta_{PK} &= \frac{11,860}{45,010 - 5,298 + \tau E_P} = 0.2986
\end{aligned}$$

Similarly,

$$\begin{aligned}
\theta_{PL} &= \frac{wL_P}{Y_P - (\text{taxes} - \text{env. taxes}_P)} = 0.7012 \\
\theta_{PE} &= \frac{\tau E_P}{Y_P - (\text{taxes} - \text{env. taxes}_P)} = 0.0002
\end{aligned}$$

This is the calibration for SO_2 ; the values for COD are only slightly different.

For the subsidized firm, we must also include the subsidy revenue.

$$\begin{aligned}
\theta_{GK} Y_G &= r_G K_G = rK_G - \gamma rK_G \\
&= \text{profits} + \text{depreciation} + \theta_{GK} (\text{taxes} - \text{env. taxes}_G) - \gamma rK_G \\
\theta_{GK} &= \frac{rK_G - \gamma rK_G}{Y_G - (\text{taxes} - \text{env. taxes}_G)} \\
\theta_{GK} &= \frac{11,531 - 7,764}{27,177 - 6,220 + 21} = 0.1795
\end{aligned}$$

The share for G is lower, but this is a comparison of apples to oranges. $\frac{rK_G}{Y_G}$ is 0.55, which is higher than private as expected since SOEs tend to be more capital-intensive.

Similarly,

$$\theta_{GE} = \frac{\tau E_G - \phi \tau E_G}{Y_G - (\text{taxes} - \text{env. taxes}_G)} = 0.0002$$

$$\theta_{GL} = \frac{w_G L_G}{Y_G - (\text{taxes} - \text{env. taxes}_G)} = 0.8202.$$

We report only four significant digits, so the sum of shares differs slightly from one due to rounding.

Appendix Table 3

Variable	Symbol	Value
SOE value added	Y_G	27,177
Private value added	Y_P	45,010
SOE capital	K_G	83,515
Private capital	K_P	59,628
SOE total employees	L_G	1,875
Private total employees	L_P	5,021
SOE taxes paid	N/A	6,220
Private taxes paid	N/A	5,298
SOE profits	rk_G	6,520
Private profits	rk_P	8,283
SO ₂ Emissions	E	855
COD Emissions	E	493
Subsidies to loss making enterprises	S	193

Notes: Data are from the 2006 China Statistical Yearbook, tables 14.4 and 14.8, except: SO₂ emissions which is from table 12-11, subsidies to loss making enterprises which is table 8.2, and COD which is table 2.11 of the China Environment Yearbook. All numbers are 100 million yuan except total employees which are in 10,000 workers, and emissions which are 10,000 tons.