

# Modeling Peak Oil

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## Abstract

“Peak oil” refers to the future decline in world production of crude oil and to the accompanying potentially calamitous effects. The vast majority of the literature on peak oil is non-economic and ignores price effects even when analyzing policies. Unfortunately, most economic models of depletable resources do not generate production peaks and thus are not applicable. I present four models which generate production peaks in equilibrium. Production increases in the models are driven by: demand increases, cost reductions through advancing technology, cost reductions through reserve additions, and production capacity increases through site development. Production decreases are driven by scarcity. The models do not rely on market failures and thus indicate that a peak in production may well arise from efficient intertemporal optimization. A simulation using realistic parameters indicates that a carbon tax can delay the peak, delay exhaustion, and reduce long-term production volatility. However, the carbon tax can have unintended consequences: hastening the peak and increasing short-term production volatility. These short-term jumps in the price path are exacerbated by delaying implementation of the tax and are reduced, but not eliminated, by phasing in the tax.

**Keywords:** Depletable resources, Hotelling, peak oil.

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# Modeling Peak Oil

## Abstract

“Peak oil” refers to the future decline in world production of crude oil and to the accompanying potentially calamitous effects. The vast majority of the literature on peak oil is non-economic and ignores price effects even when analyzing policies. Unfortunately, most economic models of depletable resources do not generate production peaks and thus are not applicable. I present four models which generate production peaks in equilibrium. Production increases in the models are driven by: demand increases, cost reductions through advancing technology, cost reductions through reserve additions, and production capacity increases through site development. Production decreases are driven by scarcity. The models do not rely on market failures and thus indicate that a peak in production may well arise from efficient intertemporal optimization. A simulation using realistic parameters indicates that a carbon tax can delay the peak, delay exhaustion, and reduce long-term production volatility. However, the carbon tax can have unintended consequences: hastening the peak and increasing short-term production volatility. These short-term jumps in the price path are exacerbated by delaying implementation of the tax and are reduced, but not eliminated, by phasing in the tax.

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# 1 Introduction

The term “peak oil” has come to be synonymous with a host of concerns about future energy supplies.<sup>1</sup> A vast non-economic literature addresses whether global oil production has peaked or will soon peak; what consequences that could have for fossil fuel dependent societies; and what should be done about it.<sup>2</sup> Unfortunately, most of this literature fails to recognize the role that prices could play in allocating scarce oil. For example, Hirsch *et al.* (2005) notes a wedge between projections of oil production, which peaks, and oil consumption, which does not.<sup>3</sup> The authors analyze a variety of mitigation policies and conclude that prompt action is required to prevent future shortfalls and economic disruptions. However, they do not mention the effects of these policies on prices or the effects of prices on these policies. Similarly, Lovins *et al.* (2005) proposes a variety of demand reduction policies for “getting the United States completely, attractively, and profitably off oil.” However, the analysis ignores the fact that these policies would decrease demand for oil, presumably decreasing the price of oil and the profitability of the policies.

Most economic models of depletable resources do not seem to offer additional insight because they do not explicitly generate a peak in production.<sup>4</sup> For example, the seminal model of Hotelling (1931) predicts that (net) prices should grow at the rate of interest and that production should steadily decline over time.<sup>5</sup> Extensions of this model for uncertainty, limited capacity, set-up costs, different grades of ore, and increasing costs with cumulative extraction, also do not generate peaks in production.<sup>6</sup> This raises the question as to whether the observed production peak could have arisen from an economic model. Is the production peak itself evidence of some market failure or disequilibrium? Is oil peaking just a series of happy (or unhappy) accidents that

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<sup>1</sup>“Peak oil” refers to the peak in U.S. crude oil production in 1970. This peak, also known as Hubbert’s peak, was correctly predicted by M.K. Hubbert (1956).

<sup>2</sup>Recent books include Abdullah (2005), Cooke (2004), Deffeyes (2005), Deffeyes (2001), Kunstler (2005), and Simmons (2005). Numerous websites and on-line discussions (often bordering on hysteria) are devoted to peak oil. In Aug. 2005, the Wall Street Journal hosted an online forum on peak oil in which the invited economists agreed that peak oil is a “greater challenge than the ‘looming crisis’ in Social Security” and “one of the most important economic transitions that many of us [...will...] witness.” Econoblog (2005). See Lynch (2003) for an opposing view.

<sup>3</sup>This report, funded by the U.S. Dept. of Energy, is not published and does not appear on the DoE website.

<sup>4</sup>Notable exceptions, discussed below, are Pindyck (1978), Slade (1982) and Livernois and Uhler (1987).

<sup>5</sup>In his review of the peak oil literature, Porter (2006) states that “the standard Hotelling model offers little insight into the oil market.”

<sup>6</sup>See Krautkraemer (1998) for a survey of this vast literature.

is not amenable to economic analysis?<sup>7</sup>

This paper answers these questions by presenting four economic models that generate peaks in production without resorting to market failures. The models are solidly based on the classic Hotelling theoretical framework of optimizing producers and consumers.<sup>8</sup> Thus the models show that peaking is consistent with dynamic efficiency and is not evidence of some market failure.

The peak in each model is generated by opposing forces tending to increase or decrease equilibrium production. In the models, increasing demand, improvements in technology, additional reserves, and new site development tend to increase production while scarcity tends to decrease production. Given the fundamental nature of these forces, it would be more surprising if production did not peak than if it did peak!

Indeed, oil production in many regions has peaked. After increasing for over 100 years, U.S. annual crude production peaked in 1970 at 3.5 billion barrels of oil and has generally declined since. Brandt (2006) analyzes 139 (potentially overlapping) oil producing regions throughout the world and argues that production in 123 regions can be reasonably modeled as single peaked and that production in 74 of these regions has already peaked.<sup>9</sup> Furthermore, production of other resources has also peaked.<sup>10</sup> This widespread empirical evidence of production peaking highlights the importance of understanding why production of an exhaustible resource might peak.

Although, the peak-oil literature focuses attention on peak production, the underlying concern seems to be that the transition from cheap oil to expensive substitutes will be sudden, chaotic, and costly. This attention to peak production is misplaced for several reasons. First, peak production is irrelevant to concerns about the transition to substitute resources. The transition to renewables should occur when oil resources are depleted such that their price rises to the production cost of renewables. The models show that the transition should occur after production peaks since oil should be used to smooth the transition to the renewable resource. Second, the price path is a

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<sup>7</sup>Kaufmann and Cleveland (2001) argue that the observed asymmetric relationship between prices and production indicates a failure of the basic Hotelling model and that this failure warrants a greater degree of government intervention in the transition from oil.

<sup>8</sup>The focus is on competitive models. Modeling a monopoly or oligopoly would likely change the shape of the production peak, but not eliminate it.

<sup>9</sup>Non-conforming regions either have multiple peaks, *e.g.*, Ohio, or have chaotic production, *e.g.*, Iraq. Regions have peaked if they have sufficient production data to fit a curve after the peak.

<sup>10</sup>Bardi (2004) describes the production peak in whale oil in the mid-19th century.

better indicator of impending resource scarcity than the production path is. As the models show, prices will begin to increase before production peaks and thus are an earlier indicator of future scarcity.<sup>11</sup>

While the transition to renewables will surely have some surprises for everyone, energy use is likely to be smooth across the transition. There are two reasons the equilibrium price path (of a barrel of oil equivalent) cannot jump during the transition. First, a forward-looking firm, or government, could profitably save some (or all) of its oil for production after any price jump. Since oil is virtually costless to store in its natural reservoir, such intertemporal arbitrage would eliminate any jumps in the price path. Second, even if there were *no* forward-looking firms or governments with secure property rights to oil, the increasing cost of oil extraction from different deposits would prevent jumps in the price path. For example, if the marginal extraction cost of the highest cost deposit were \$200 per barrel and the cost of the renewable substitute were \$260 (the cost of solar in the following simulation), then completely myopic firms would exhaust the oil at a price of \$200, and the price would then jump to \$260.<sup>12</sup> However, since oil production costs are smoothly increasing, there are oil deposits with production costs between \$200 and \$260. Even completely myopic firms without secure property rights would wait to produce from these deposits until the price were high enough to cover the extraction costs. Production from these high-cost deposits will ensure a smooth (although possibly inefficient) transition from oil to renewable resources.

The four models are presented and discussed in Section 2. Although the first three models have clear antecedents in the literature, the fourth model is novel. Each model generates an endogenous peak in equilibrium production, and combining the models shows that production will peak after the price path reaches its minimum. In Section 3, a model is calibrated with realistic parameters from the literature and used to analyze a variety of policy proposals. The simulations show that a carbon tax can be effective in delaying the peak and delaying exhaustion of oil. The tax also can reduce the long-term volatility of production, *i.e.*, reduce peak production, but only at the cost of introducing short-term volatility in production. This short-term volatility is surprisingly

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<sup>11</sup>Bardi (2004) argues that prices for whale oil began to increase well before the production peak in approximately 1850.

<sup>12</sup>Gaudet *et al.* (2002) describe a model without secure property rights in which the transition from low-cost oil (with constant marginal extraction cost) to the high-cost substitute is indeed abrupt.

hard to eliminate, persisting even when the carbon tax is phased in. Section 4 concludes.

## 2 Models of Peak Oil

This section presents four Hotelling-style models with peaks in production. Unlike the curve fitting model of Hubbert (1956) or the noneconomic literature on peak oil, these models recognize the incentives of both producers and consumers and that these competing effects must be balanced to determine equilibrium prices and consumption. The models are presented in order of increasing complexity and evaluated based on the empirical evidence.

### 2.1 Model 1: Demand shift

In Model 1, the increase in production is caused by increasing demand. Let  $D_t(p)$  be aggregate demand derived from the consumer's optimization. Let demand,  $D_t$ , be downward sloping, *i.e.*,  $D'_t < 0$ , and growing over time, *i.e.*,  $D_t(p) \leq D_{t+1}(p)$  for every  $p$ . To simplify the transversality conditions, assume demand has a finite choke price, possibly at the cost of a substitute resource.<sup>13</sup>

Now consider supply of a depletable resource. Assume the finite resource stock,  $S$ , can be extracted at cost  $C(q)$  where  $C' > 0$  and  $C'' \geq 0$ . Thus profit maximization,

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t)],$$

is subject to the stock constraint

$$\sum_{t=1}^{\infty} q_t \leq S \tag{1}$$

where  $\beta$  is the discount factor, and  $q_t$  and  $p_t$  are production and price in period  $t$ . The well-known Hotelling supply relation is seen from the first order condition:

$$p_t = C_q + (1+r)^t \lambda$$

where  $\lambda$  is the shadow value of the stock and the rate of interest,  $r$ , is defined by  $\beta = 1/(1+r)$ . Thus equilibrium in the model is characterized by a net price that grows at the rate of interest.

Figure 1 illustrates how production can increase in this model. The *full marginal cost*—defined as the marginal extraction plus marginal scarcity costs, here  $C_q + (1+r)^t \lambda$ —is increasing

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<sup>13</sup>Specifically, assume there exists some  $\bar{p}$  such that  $D_t(\bar{p}) = 0$  for every  $t$ .

over time due to the increasing scarcity of the depletable resource. This tends to decrease production. However, the demand shift tends to increase consumption over time. In this figure, the demand increase effect is larger, and equilibrium production increases. With a finite choke price, the demand increase will eventually be less than the full marginal cost increase, and equilibrium production will decrease. Thus the model predicts a peak in equilibrium production.

Model 1 has a strong empirical implication: namely, prices should rise over time, even while production is increasing. Econometric evidence on the crude oil price series has tested for deterministic trends.<sup>14</sup> Slade (1982), using data from 1870-1978, regresses the price series for several commodities, including petroleum, on linear and quadratic trends and finds evidence for quadratic trends. However, this result may be spurious since she did not test for a unit root.<sup>15</sup> Lee, List, & Strazicich (2006) reject the unit root hypothesis for petroleum and find evidence of a quadratic trend when allowing for structural breaks in the time series. The plot from their linear trend model shows a declining trend through 1896 which suggests that prices have not been monotonically non-decreasing.<sup>16</sup>

Model 1 shows that demand growth can cause a peak in production. However, since this model may not be consistent with the empirical evidence on prices, the model may be missing some important economic forces. Models 2-4 focus on supply side mechanisms and yield peaks in production together with U-shaped equilibrium price paths.

## 2.2 Model 2: Technological change

This model, based on Slade (1982), simply assumes that costs decrease exogenously over time due to technological change. Let the cost function be  $C(q, t)$  where  $C_q > 0$ ,  $C_{qq} > 0$ ,  $C_t < 0$  and  $C_{qt} < 0$ . The producer's optimization is given by:

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, t)]$$

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<sup>14</sup>Several well-publicized bets over natural resource prices have been resolved in favor of the "optimists."

<sup>15</sup>Berck and Roberts (1996) extend Slade's data and cannot reject a unit root leading them to conclude that the commodity prices do not have a deterministic trend. Ahrens & Sharma (1997) reject the unit root hypothesis for some commodities, including petroleum.

<sup>16</sup>Their analysis finds structural breaks for petroleum in 1896 and 1971 when allowing for a linear trend and breaks in 1914 and 1926 when allowing for a quadratic trend.

subject to the stock constraint in equation 1. The first order condition is

$$p_t = C_q(q_t, t) + (1 + r)^t \lambda.$$

This first order condition implies that the net price again grows at the rate of interest.

Figure 2 illustrates a production increase in Model 2 where price is constant, *i.e.*,  $p_t = p_{t+1} = \bar{p}$ . The marginal cost curve,  $C_q$ , shifts down over time due to technological progress. However, the scarcity cost,  $(1 + r)^t \lambda$ , is increasing over time due to scarcity. Thus production can increase if the decrease in marginal extraction costs outweighs the increase in scarcity costs, as illustrated in Figure 2. Since the growth rate of the scarcity cost is  $r$ , the scarcity-cost increase will eventually outweigh the decrease in marginal extraction costs (and production will fall) as long as the growth rate of the decline in marginal costs is less than  $r$ .

Thus far, only the supply side of the model has been analyzed. Because of increasing supply, equilibrium in this model can lead to a U-shaped price path even with a stationary demand. With a stationary demand, the peak in production would occur simultaneously with the lowest price. If demand increases over time, the peak in production occurs after the low point on the price path.

The relationship between technological change and costs is intuitively appealing. Clearly advances in drilling and exploration technology have made additional oilfields accessible. An optimist might assume that such technological advances will continue indefinitely. However, existing empirical evidence does not strongly support a negative relationship between technological change and costs. Cuddington and Moss (2001) construct a measure of technological diffusions but did not find that it significantly reduced exploration and development costs for crude oil reserves—although they find that it reduced finding costs for nonassociated natural gas.

This technological change model is somewhat unsatisfactory from a theoretical perspective since the primary driver of interest, technological change, is unexplained in the model. The next model does not rely on an exogenous mechanism but has reductions in extraction costs derived endogenously through discovery of additional reserves.

### 2.3 Model 3: Reserves growth

This model is based on Pindyck (1978) who argued that there is an inverse relationship between marginal extraction costs and reserves. In this model, a firm has an incentive to explore and develop new reserves in order to drive down marginal extraction costs.

Let  $R_t$  be reserves in period  $t$ . Additions to reserves,  $f(w, S_t)$ , depend on effort,  $w$ , and cumulative discoveries,  $S_t$ , where  $f_w > 0$ ,  $f_S < 0$ ,  $S_{t+1} - S_t = f(w, S_t)$  and  $S_1 = 0$ . Changes in reserves are then  $R_{t+1} - R_t = f(w_t, S_t) - q_t$  where  $R_1 = 0$ . Let the cost of effort be  $c(w)$ , where  $c' > 0$  and  $c'' \geq 0$ , and costs of extraction be  $C(q, R)$  where  $C_q > 0$ ,  $C_{qq} \geq 0$ ,  $C_R < 0$ , and  $C_{qR} < 0$ . Thus costs and marginal costs are decreasing in reserves. Profit maximization is

$$\max_{q_t, w_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, R_t) - c(w_t)]$$

subject to the equations of motion for reserves and cumulative reserve additions. The first order conditions can be written

$$p_t = C_q(q_t, R_t) + (1+r)^t \lambda_t \quad (2)$$

$$\beta^t c'(w_t) - \gamma_t f_w(w_t, S_t) = \lambda_t f_w(w_t, S_t) \quad (3)$$

$$\lambda_{t+1} - \lambda_t = \beta^t C_R(q_t, R_t) \quad (4)$$

and

$$\gamma_{t+1} - \gamma_t = -(\lambda_t + \gamma_t) f_S(w_t, S_t) \quad (5)$$

where  $\lambda_t > 0$  and  $\gamma_t < 0$  are the shadow values of reserves and of cumulative additions to reserves at time  $t$ . Equation 2 sets the marginal benefit of oil equal to the full marginal cost. Equation 3 sets the marginal cost of effort plus the scarcity cost of effort equal to the marginal benefit of effort in terms of increased reserves. Equations 4 & 5 are the equations of motion for the shadow values.

For a constant price,  $\bar{p}$ , substituting Equation 2 into Equation 4 shows that

$$r[\bar{p} - C_q(q_t, S_t)] + C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t) + (1+r)C_R(q_t, S_t) = 0. \quad (6)$$

Since the first term in Equation 6 is positive and the last term is negative, the change in marginal extraction costs,  $C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t)$ , can be either positive or negative. In fact, Pindyck

shows that if initial reserves are low, it is optimal for the firm to exert effort to find reserves to drive down the marginal extraction cost. This leads to a peak in production and, in equilibrium, a U-shaped price path.

Model 3 is theoretically appealing since the peak is clearly endogenously determined and is not driven by exogenous shifts. However, the empirical evidence of a negative relationship between costs and reserves is not conclusive. Livernois & Uhler (1987) present empirical evidence arguing that aggregate reserves and cost are not negatively correlated and in fact are positively correlated. They claim that this may be due to the fact that lower cost reserves tend to be found first. Thus, even if new discoveries are sufficient to increase reserves, they may not lower costs. They argue for a disaggregated analysis and find evidence for the assumed negative correlation between costs and reserves at a disaggregate level.

Pesaran (1990) uses a similar model for an integrated econometric estimation using data on North Sea production.<sup>17</sup> The author argues that the estimates give a “reasonable degree of support to the theory.”

## 2.4 Model 4: Site development

Models 2 & 3 relied on decreases in marginal extraction costs to drive the peak and U-shaped price path. The peak in Model 4 is driven by increases in aggregate production capacity due to production at newly developed sites, despite the fact that costs are unchanged at all previously developed sites.

Each period in this model, firms choose how large a site to explore and develop and the production capacity to install in that site. Once capacity is installed, production continues from that site until all oil is exhausted. Assume the density of oil is  $X$  so the stock of oil in a site of size  $s$  is  $X \times s$ . Costs of exploring a site of size  $s$ , are given by  $G(s)$  where  $G' > 0$  and  $G'' \geq 0$ . Convex exploration costs could arise from the fixed number of trained geologists or exploration crews in the short run. Before production can take place at a given site, firms must install production capacity,  $K$ , in the site. Costs of installing capacity,  $F$ , are assumed to be

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<sup>17</sup>Pickering (2002) applies a similar model to find evidence of the “discovery decline phenomenon.”

increasing in the amount of capacity,  $F' > 0$ , at a nondecreasing rate,  $F'' \geq 0$ .<sup>18</sup> Convex costs in capacity installation could arise due to a fixed number of experienced drilling crews and trained engineers or due to the declining natural pressure of the reservoir.<sup>19</sup> Finally, extraction costs for any site,  $C(q, K)$ , depend on the amount extracted as well as the installed capacity with  $C_q > 0$ ,  $C_{qq} > 0$ ,  $C_K < 0$ ,  $C_{KK} > 0$ , and  $C_{qK} < 0$ . Thus, pumping costs and marginal pumping costs are increasing in output but decreasing in capacity. The remaining cost assumption,  $C_{KK} > 0$ , implies that the decrease in costs from adding capacity is smaller at higher levels of capacity, *i.e.*,  $d(-C_K)/dK < 0$ .<sup>20</sup>

Profit maximization is

$$\max_{q_{it}, K_t, s_t} \sum_{t=1}^{\infty} \beta^t [p_t Q_t - \sum_{i=1}^t C(q_{it}, K_i)] - \beta^t F(K_t) - \beta^t G(s_t)$$

subject to

$$q_{it} = 0 \quad \text{if } i > t$$

$$Q_t = \sum_{i=1}^t q_{it} \quad \forall t$$

$$\sum_{t=1}^{\infty} q_{it} \leq X \times s_i \quad \forall i$$

$$\sum_{i=1}^{\infty} s_i \leq S \quad \forall i$$

where  $q_{it}$  is production from site  $i$  at time  $t$ ,  $K_t$  is capacity installed in site  $t$  at time  $t$ , and  $s_t$  is the size of the site explored at time  $t$ , and  $S$  is the total area available for exploration. Production profits are revenue minus production costs from all sites that have previously been explored and developed. In each period, an additional site is explored and developed (*i.e.*, production capacity is installed in the site) and the final two terms of the objective function capture the costs of development and exploration of a new site.<sup>21</sup> The first constraint prevents extraction from a site

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<sup>18</sup>For simplicity, assume that capacity costs are independent of the size of the site.

<sup>19</sup>If  $K$  is interpreted as effective capacity, then increasing effective capacity by 100 bpd would require a smaller capital investment when capacity is smaller, due to the natural pressure within the reservoir.

<sup>20</sup>This specification of the cost function does not allow an inverse L-shaped marginal cost curve. The allowable function  $C(q, K) = cq + (K - q)^{-\alpha}$  defined for  $q < K$  approximates an inverse L-shaped marginal cost curve as  $\alpha \rightarrow \infty$ .

<sup>21</sup>Decreasing returns in exploration ensure the entire available area is not explored and developed in the first period.

before it is explored and developed, and the second constraint defines aggregate supply as the sum of production from all developed sites. The third constraint is the stock constraint for site  $i$ , and the final constraint ensures that the size of all the developed area is less than the total area available for development.

The first order conditions can be written:

$$p_t \leq C_q(q_{it}, K_i) + (1+r)^t \lambda_i \quad \forall t, \quad \forall i < t \quad (7)$$

$$\sum_{t=i}^{\infty} \beta^t [-C_K(q_{it}, K_i)] = \beta^i F'(K_i) \quad \forall i \quad (8)$$

$$G'(s_i) + (1+r)^i \gamma = (1+r)^i \lambda_i \times X \quad \forall i \quad (9)$$

where  $\lambda_i$  is the shadow value of oil at site  $i$ , and  $\gamma$  is the shadow value of area for development. The first condition says that the price equals the full marginal cost at each site with positive production. The second condition implies that the present value of the sum of cost reductions from an additional unit of capital must equal the present value of the cost of the additional unit of capital. The third condition says that the marginal cost of exploration plus the scarcity cost of the additional explored area must equal the benefit from an additional barrel of oil times the density of oil.

To illustrate the solution to the model, first let price be constant, *i.e.*,  $p_t = \bar{p}$  for every  $t$ , and let  $S = \infty$ . With unlimited area to develop, the shadow value,  $\gamma$ , would be zero, and the exploration and development problems would be stationary. This stationarity implies that it is optimal to explore the same size site in every period and to install the same capacity in every period, *i.e.*,  $s_t = s_1$  and  $K_t = K_1$  for every  $t$ . Equation 9 then implies that  $\lambda_t = (1+r)\lambda_{t+1}$  so that the shadow values of oil are declining at sites that are developed later.<sup>22</sup> Now turn to production from site 1. Since capacity is fixed after installation, the marginal cost curve is fixed over time. However since the scarcity cost of oil at site  $i$ ,  $(1+r)^t \lambda_i$ , is growing over time, production from site  $i$  is decreasing, and oil at the site is eventually exhausted. Figure 3 illustrates this declining production over time from site  $i$  as the full marginal cost curve shifts upward. Now consider aggregate production over time. In period 1, only one site has been explored and developed, so

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<sup>22</sup>Intuitively, a marginal increment of oil to a site that is developed later is worth less than the marginal increment to a site that is developed sooner.

aggregate production is just  $q_{11}$ . In period 2, two sites have been explored and developed, so production is  $q_{12} + q_{22}$ . By stationarity,  $q_{11} = q_{22}$  so aggregate production increases in the second period even though production at site 1 has decreased. In the third period, production begins at site 3 so aggregate production increases despite decreasing production at the two sites that were previously explored and developed. As more sites are developed, aggregate production increases further until site 1 is exhausted. After this, new production in each period exactly offsets the declines in production at all the other sites, and aggregate production would be at the steady-state level,  $X \times s_1$ .

Since there is limited area to develop, *i.e.*,  $S$  is finite, the exploration and development problems are not stationary. Consider  $s_i^*$ , the optimal site size in period  $i$ . In period  $i + 1$ , suppose the firm were to explore the same size site. As argued above, the shadow value of oil,  $\lambda_i(s_i^*)$ , which is decreasing in  $s_i^*$ , must be smaller at the later site such that  $\lambda_i(s_i^*) = (1 + r)\lambda_{i+1}(s_i^*)$ . But this implies that  $G'(s_i^*) + (1 + r)^{i+1}\gamma > G'(s_i^*) + (1 + r)^i\gamma = (1 + r)^i\lambda_i(s_i^*) \times X = (1 + r)^{i+1}\lambda_{i+1}(s_i^*) \times X$ . Comparing this with equation 9 shows that in period  $i + 1$  the marginal cost of exploration would be greater than the marginal benefit of exploration if the firm explored the same size site. Thus the optimal site size is declining over time, *i.e.*,  $s_i > s_{i+1}$ . This argument is illustrated graphically in Figure 4 which shows the marginal benefit of exploration  $(1 + r)^i\lambda_i(s)X$  and the marginal cost of exploration  $G'(s) + (1 + r)^i\gamma$  both conditional on site size  $s$  for periods  $i$  and  $i + 1$ . If  $s$  is the same in two periods, then  $\lambda_i(s) = (1 + r)\lambda_{i+1}(s)$ , and the marginal benefit of exploration is equal in periods  $i$  and  $i + 1$ . However, the marginal cost of exploration increases due to the scarcity of sites as illustrated.

Now turn to capacity installation. Let  $K_i^*$  be the optimal capacity to be installed at site  $i$ . The firm could install the same capacity in the next site in the following period. Let  $q_{(i+1)t}^*$  be the optimal production path at site  $i + 1$  conditional on installing capacity  $K_i^*$ . This implies that:

$$\begin{aligned} \sum_{t=i+1}^{\infty} \beta^{t-i-1} [-C_K(q_{(i+1)t}^*, K_i^*)] &< \sum_{t=i+1}^{\infty} \beta^{t-i-1} [-C_K(q_{i(t-1)}, K_i^*)] \\ &= \sum_{t=i}^{\infty} \beta^{t-i} [-C_K(q_{it}, K_i^*)] = F'(K_i^*). \end{aligned} \quad (10)$$

The first inequality holds because site  $i + 1$  is smaller than site  $i$ . This implies that optimal

extraction from site  $i + 1$  is smaller than extraction from site  $i$  in the preceding period, *i.e.*,  $q_{(i+1)t}^* \leq q_{i(t-1)}$  for all  $t$  with strict inequality when  $q_{i(t-1)} > 0$ . Since  $C_{qK} < 0$ , it follows that  $-C_K(q_{(i+1)t}^*, K_i^*) < -C_K(q_{i(t-1)}, K_i^*)$  for  $q_{i(t-1)} > 0$ . The second equality in Equation 10 follows by rearranging the indices, and the final equality follows from equation 8. But the inequality in Equation 10 implies that the marginal benefit of installing capacity  $K_i^*$  at site  $i + 1$  (the reduction in pumping costs) is less than the marginal capacity cost of  $K_i^*$ . Thus, optimal capacity is smaller at the smaller site, *i.e.*,  $K_i > K_{i+1}$  for every  $i$ .

**Characterization of the Optimal Supply:** With a stationary output price, limited total area for exploration, and stationary costs of production, exploration and development, the following hold:

- (i) **Declining production at each site:**  $q_{it} > q_{i(t+1)}$  if  $q_{it} > 0$ .
- (ii) **Exhaustion at each site:**  $\exists T(i) > i$  such that  $q_{it} = 0$  for every  $t > T(i)$ .
- (iii) **Decreasing capacity:**  $K_i > K_{i+1}$ .
- (iv) **Decreasing site size:**  $s_i > s_{i+1}$ .
- (v) **Increasing and decreasing aggregate production:** Production increases if additional production at the newly developed site offsets production declines at all the previously developed sites. Eventually production declines as newly developed sites become smaller.

Figure 5 simulates optimal production in Model 4 for the first seven sites and for the aggregate.<sup>23</sup> In the first year, production is only from the first site so aggregate production is  $q_{11}$ . In the second year, production starts from the second site while production only declines slightly from the first site. Thus aggregate production increases in the second year. Aggregate production continues to increase until year 13 as new production offsets declines in production at existing sites. After the peak, new production cannot offset declines in production at existing sites. The first site is exhausted by year 17 and thereafter at least one site is exhausted every year. The last site is developed in year 51, and all sites are exhausted by year 53.

Figure 6 shows a schematic of Hubbert's curve taken from a primer on peak oil (Energy Bulletin 2006). The schematic, not based on an underlying model, illustrates the observed

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<sup>23</sup>Simulation details are in the appendix. Parameters of the simulation are  $\bar{p} = \$50$ ,  $C(q, K) = q^2/K$ ,  $G(s) = s^2/2$ ,  $F(K) = K^2/2$ ,  $S = 20,000$ ,  $X = 100$  and  $r = 0.05$ .

(hypothesized?) relationship between aggregate production and production at individual wells. Model 4 could be parameterized to match this schematic, thus showing economic assumptions which generate such a schematic in equilibrium.

## 2.5 Discussion

Since each of Models 1-4 incorporate important economic forces into the standard model of exhaustible resources, a model combining each of these elements might be even more realistic. Combining demand growth, technological change, reserve additions, and site development into a single model would be easy but tedious. However, the analysis of the models suggests an interesting result for the combined model.

Recall that in Model 1, the oil price increased monotonically. The peak in production arose because initially the demand growth was stronger than the price increase. Eventually, the increase in price became stronger than the increase in demand, and production decreased. In Models 2-4 with stationary demand, production increased only when the economic forces caused the price to fall and production to increase. Thus in Models 2-4, the production peak was coincident with the minimum (trough or low point) in the price path.

What happens when all four models are combined? Clearly, while the equilibrium price is falling, all forces align so that production increases. However, when the price path reaches its minimum what happens to production? For a fine enough division of time, the price path is essentially flat at the minimum. If demand is still growing, production must increase for some time after the price has reached its minimum. Eventually, the increasing price will outstrip the demand growth (as in Model 1) and production will peak. The preceding argument is summarized in the following proposition:

**Proposition 1** *In a model incorporating the economic forces of Models 1-4, the peak in production cannot occur before the minimum price is reached and occurs strictly after the minimum price is reached if demand growth is sufficiently strong.*

This result has an important implication for those concerned about peaking production in the oil market. Namely, prices are a better indicator of impending oil scarcity than are peaks in production. Therefore focusing attention on peak production is misguided since focusing attention

on price would given an earlier predictor of impending resources scarcity.

### 3 Policy analysis

These models can be used to analyze a variety of policy proposals, the simplest of which is a carbon tax.<sup>24</sup> Such a tax could improve efficiency if it corrected some market failure such as: external environmental costs, external national security costs, incomplete futures markets, macroeconomic adjustment costs, and/or insufficient patience.<sup>25</sup> Normative analysis would require more complete modeling of these market failures. Here I simply assess the positive effects of a carbon tax with special attention to the effects on the peak. In particular, the simulation analyzes whether the tax: (i) delays the peak, (ii) delays exhaustion of the resource, and/or (iii) reduces volatility in oil production.

Any of Models 1-4 could be used for policy analysis. Since Model 1 is the simplest, it requires the fewest parameters and is the basis for the following simulation. The parameterization follows Chakravorty *et al.* (1997), which follows Nordhaus (1973), but departs from the former analyses in two ways.<sup>26</sup> First, since oil is our primary interest, coal and natural gas production are not modeled and only transportation demand is modeled. Since transportation accounted for 60% of oil use in 1990, the simulation estimates total oil consumption as this constant multiple of oil for transportation. Second, the costs of oil extraction are a continuous, increasing function of cumulative extraction rather than a step function.<sup>27</sup> This simplifies the computation since only one shadow value must be calculated and the step function need not be approximated. Moreover, the total amount of oil extracted in equilibrium is continuous in the policy variables. The simulation estimates the equilibrium price and production paths through a simple looping procedure.<sup>28</sup>

The result of the baseline simulation is presented in Figure 7. World oil production in 1990

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<sup>24</sup>Kaufmann (Econoblog 2005) proposes a “large” energy tax phased in over 20 years. Stiglitz (2006) argues for a global environmental tax on greenhouse gas emissions.

<sup>25</sup>Caplin & Leahy (2004) point out that the competitive equilibrium is the most impatient Pareto optimal intertemporal allocation.

<sup>26</sup>Following Chakravorty *et al.*, GDP grows at 3% annually, declining by 1% per year; the baseline year is 1990; the discount rate is 2%; price elasticity is 1.28 and income elasticity is 0.81 for transportation; and solar (available at constant marginal cost) is the backstop resource.

<sup>27</sup>The precise equation is (15) on p. 1215 of Chakravorty *et al.*

<sup>28</sup>For an arbitrary shadow value, the model calculates the price and consumption paths based on the equations of motion for the stock of oil and the shadow value. The shadow value is then adjusted based on whether it becomes negative or there is excess supply. The approximation converges quite quickly.

is approximately 19 billion barrels. Production rises to a peak of 35 billion barrels in 2064 and then falls until oil stocks are exhausted in 2130 and energy for transportation is produced by the solar backstop. The “backstop barrels” path, reflecting demand growth, illustrates the growth in energy use at a constant price of \$260, which is the cost of producing a barrel of oil equivalent for transportation with solar power. Notice that energy use continues to grow after oil stocks are exhausted in 2130 due to the continued growth in demand. The equilibrium price path begins at \$30 per barrel in 1990 and grows smoothly to the solar backstop cost of \$260 per barrel of oil equivalent.<sup>29</sup>

Figure 7 also presents the simulation of a \$10 per barrel tax on oil introduced in 2010. This carbon tax (about 20% of the 2010 price) causes production to drop by about 5% (1.5 billion barrels) in 2010 but then to continue on much the same trajectory. Production peaks 5 years later than in the baseline, and the transition to the solar backstop occurs one year later. Note that less total oil is extracted with a tax since deposits of oil, which were (marginally) economic without a tax, are not economic with the \$10 tax. This reduced stock effect is offset by the reductions in production in most years so the tax delays the transition by a year. In general these offsetting effects will imply that the policies have little effect on the timing of the transition from oil to the solar backstop.<sup>30</sup>

Panel A of Table 1 presents the results of simulating different taxes announced and implemented in 2010. With a larger tax, the peak production year is delayed more, the transition year is delayed more, and peak production is smaller. Note that these effects are relatively small. Even with a large tax of \$30, approximately 65% (!) of the 2010 price, the peak is only delayed by 15 years, and peak production is only decreased by 4%. However, these policies have substantial effects in the short run. In particular, the tax, when announced in 2010, induces an immediate drop in equilibrium production of 2.86 billion barrels (an 11% drop) with a \$20 tax and of 15% with a \$30 tax. Thus, although these policies reduce the long-term volatility of oil production, *i.e.*, reduce the peak production, they do so at the cost of substantial short-term production volatility

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<sup>29</sup>The simulated price in 2006 is approximately \$40 in 1990 dollars.

<sup>30</sup>If the carbon tax were phased out near exhaustion, the deposits would become economic, and the policy would delay the transition more.

in the adoption year.

These policy simulations show that in addition to delaying oil exhaustion, the carbon tax also delays the production peak. This result turns out to be sensitive to the adoption date of the carbon tax. Panel B of Table 1 presents the results of simulating a \$10 carbon tax implemented (and announced simultaneously) at different dates. Implementing the tax at a slightly later date has little effect on the peak year, peak production, or the transition year. Implementing the tax slightly later leads to a slightly larger absolute decrease in production when the tax is implemented since demand is higher. However, since the price is rising over time, the \$10 tax is a smaller percentage of the oil price and thus can lead to a smaller short-run decrease in production. Note, however, that when the tax is implemented in 2060 (near the baseline peak year) the tax actually shifts peak production to 2060, the year the tax is implemented. Thus, the carbon tax may not necessarily delay the peak production year.

To avoid the short-term volatility from implementing an unannounced policy, policy makers sometimes announce policy changes in advance so that firms can prepare for the new policy environment. However, announcing a carbon tax before it is implemented would be a particularly bad idea. Figure 8 shows the results of simulating a \$10 tax implemented in 2020 that is announced in 2010. Although the tax does delay peak production by five years, it also has the unintended consequence of increasing production over the ten years between the announcement and the time the tax is implemented. This unfortunate policy causes production to jump twice: production increases when the tax is announced and then decreases when the tax is implemented. Note that the decrease in production in 2020 (2.09 billion barrels) is substantially larger than the decrease in production would be with a tax in 2010 (1.45 billion barrels). Panel C of Table 1 shows simulations of a variety of implementation dates of a \$10 carbon tax announced in 2010. Production is always above the baseline before the tax is implemented and then drops below the baseline after the tax takes effect. These drops in production are larger than would result from simply announcing the tax and implementing it simultaneously.

The economic intuition for this result follows from an analysis of the tax burden. When the tax is announced, the shadow value of the oil decreases immediately since firms optimally begin

to bear their portion of the burden immediately, *i.e.*, spread their portion of the burden across all remaining oil. Since consumers do not store oil, consumption is not linked intertemporally. Thus consumers do not begin to bear their portion of the burden until the tax actually comes into effect. Thus the producer's (and consumer's) price of oil jumps down immediately when the tax is announced causing production to increase. When the tax is implemented, the consumer's price jumps up by the amount of the tax, and consumption and production decrease. Note that the producer's price does not jump at this time, since it has already incorporated the entire effect of the carbon tax.

While announcing a tax before its implementation leads to two jumps in production, phasing in the tax can only eliminate one of the jumps. This follows because producers again spread their share of the entire burden across all production. Thus the shadow price of oil immediately jumps to reflected the diminished value of the oil in the ground. Phasing in the tax implies that consumers begin bearing their portion of the burden immediately so the wedge between producer's and consumer's prices increases as the tax is phased in. Note that production only jumps at the announcement date when the tax is phased in.

Figure 9 illustrates production when large carbon tax (\$30) is phased in over twenty years.<sup>31</sup> When the tax is announced, the shadow value and the (producer's and consumer's) price immediately fall and production increases discontinuously. As the tax is phased in, production falls relative to the baseline (in this example production is relatively flat) until the tax is completely phased-in in 2030. Panel D of Table 1 shows carbon taxes of \$10, \$20, and \$30 phased in over twenty years. In each case, production jumps up when the tax is announced and remains above baseline production for approximately five years. Note that phasing in the tax reduces the jump in production by about one third relative to simply implementing the tax when it is announced (compare panels A and D).

To summarize, the simulations show that a carbon tax can delay the peak year and transition year if the tax is implemented soon enough. While the tax can reduce the long-term volatility of production (reduce the peak production), the reduction comes at the cost of an increase in

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<sup>31</sup>This tax is still 45% of the baseline price in 2030 when the tax is fully phased in.

short-term production volatility. This short-term production volatility is surprisingly hard to eliminate. Delaying implementation of the tax after the announcement date only increases short-term volatility by inducing two jumps in the production path. Phasing in the carbon tax, while reducing the magnitude of the short-term jump and eliminating one production jump, cannot eliminate both jumps since firms optimally begin to bear their portion of the entire tax burden at the announcement date.

In addition to carbon taxes, other policies, many of which serve to reduce future demand for oil, have been proposed. For example, extensive investment in carbon fiber cars would reduce future demand for oil. Are such demand reduction policies amenable to this analysis?

In short, yes. Suppose investing in carbon fiber cars would reduce demand in ten years. This policy would affect the production path exactly like an announced tax increase in ten years. The investment would cause firms to produce more oil now at higher relative prices and less in the future at lower relative prices. As with the announced tax increase, this policy could have unintended consequences.

## 4 Conclusion

The four models isolate four possible causes of increases in production: increasing demand, cost reductions through technological change, cost reductions through exploration, and increasing production from additional site exploration and development. In each model, the underlying scarcity of the resource ultimately leads to a decline in production. This shows that a peak in production is not evidence of market failure but rather that a peak in production could well arise from efficient intertemporal optimization.

Models 2-4 generate U-shaped price paths in equilibrium. While there is some econometric evidence for a U-shaped price path, the evidence is inconclusive. However, these models can generate production peaks even if price is constant. Thus the conditions under which the models generate production peaks are quite general. In fact, the economic forces underlying the production peaks are so fundamental that it would only be surprising if production did not peak.

Combining the four models shows that the price path will reach its minimum before pro-

duction peaks. This insight suggests that prices, rather than production, are a better indicator of impending resource scarcity. Thus research on determining production peaks may be better focused on determining the minimum of the price path if the goal is early detection of coming resource scarcity. However, given substantial short-run volatility in oil prices, it may be difficult to identify the underlying, long-run price trend from short-run changes in prices.

The models do not explicitly address possible market failures, *e.g.*, externalities, excessive impatience, or macroeconomic adjustment costs and thus cannot analyze the efficiency of policies. This is an important area for future research. However, the models can illustrate the positive effects of policies. Simulations using realistic parameters show that a carbon tax can delay the peak, depletion, and the transition to a renewable backstop. A carbon tax can also reduce long-term production volatility in the sense of reducing peak production. These effects however come at the cost of increased short-term volatility in production. In fact, the jumps in production from even a modest carbon tax can be significant. For example, a \$10 carbon tax induces a 5% drop in production the year of adoption. The simulations show that these short-term jumps in production are exacerbated by announcing the tax before it takes effect and are reduced, but not eliminated, by phasing in the tax. These unintended consequences illustrate the importance of careful economic modeling in policy evaluation.

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## Appendix: Simulation of Model 4

Optimal production, exploration, and capacity installation in Model 4 can be simulated using an algorithm of nested loops. For any given  $\gamma$ ,  $s_1$ ,  $K_1$ , and  $\lambda_1$ , the extraction path  $q_{1t}$  is determined by equation 7. If total extraction from site 1 is greater (less) than  $s_1 \times X$ , then the marginal user cost,  $\lambda_1$ , must have been too small (large). Using this adjustment rule, the optimal shadow value, conditional on  $\gamma$ ,  $s_1$ , and  $K_1$ , can be calculated by looping. Next equation 8 is used to determine whether too much or too little capacity has been installed in this site. This adjustment rule allows the optimal capacity to be calculated by looping where the optimal shadow value is calculated during each iteration. Once  $K_1$  and  $\lambda_1$  are computed optimally, equation 9 can be used to determine whether the given  $s_1$  is too large or too small. This adjustment rule can be used, with nested loops for  $K_1$  and  $\lambda_1$ , to compute the optimal site size,  $s_1$ , for a given  $\gamma$ . To determine whether  $\gamma$  is too high or too low, the optimal site size must be computed for each site. If the sum of the sites is greater (less) than  $S$ , then  $\gamma$  was too low (high). This adjustment rule, with nested loops for  $s_i$ ,  $K_i$ , and  $\lambda_i$ , allows the optimal  $\gamma$  to be computed.

**Figures**

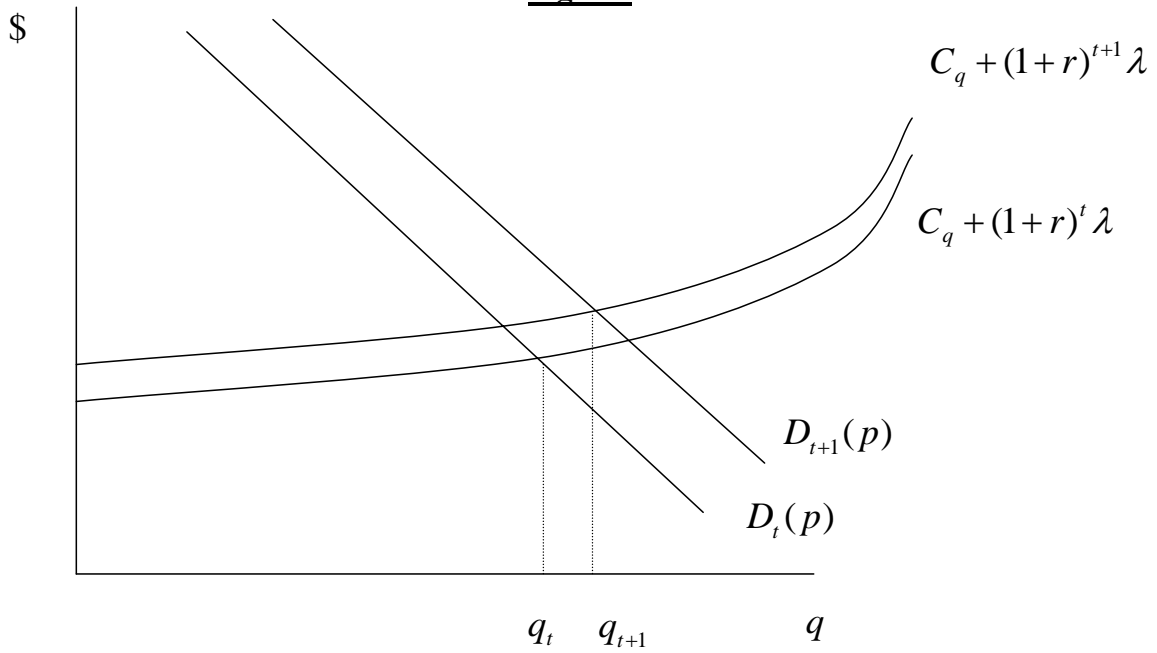


Figure 1: Illustration of a production increase in Model 1, the demand shift model.

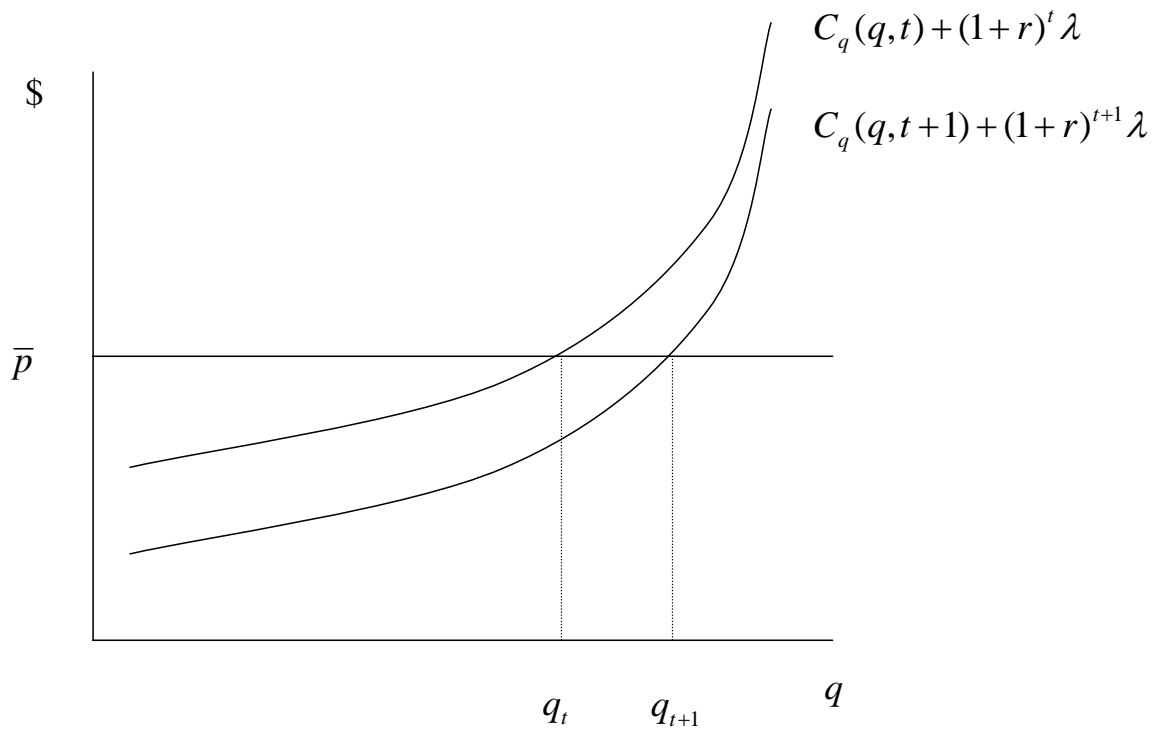


Figure 2: Illustration of increasing production in Model 2, the technological change model.

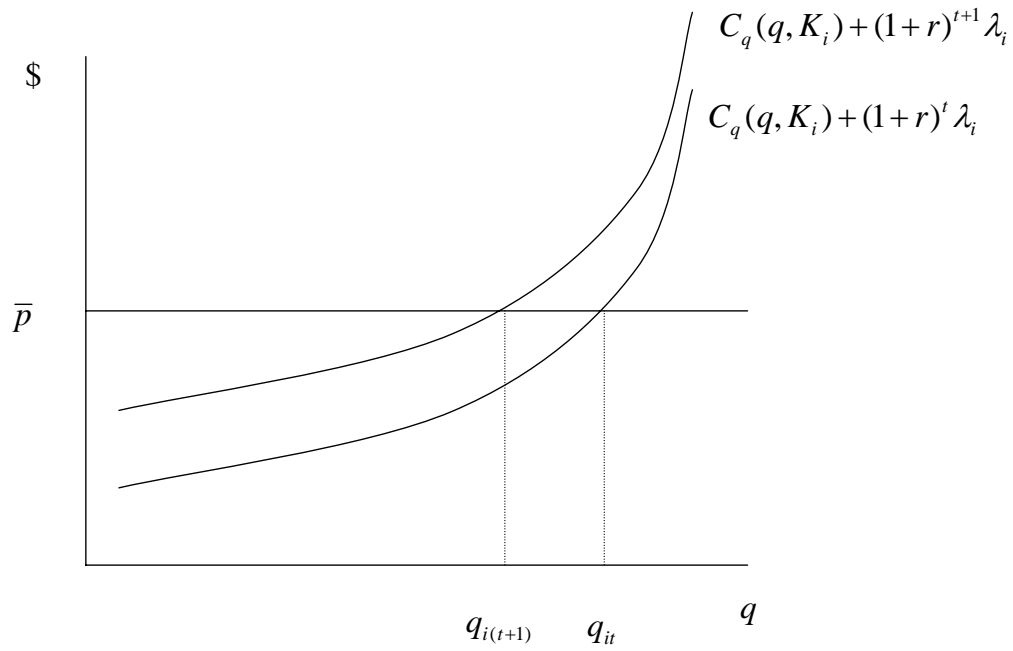


Figure 3: Graphical demonstration of declining production at a site for Model 4, the site development model.

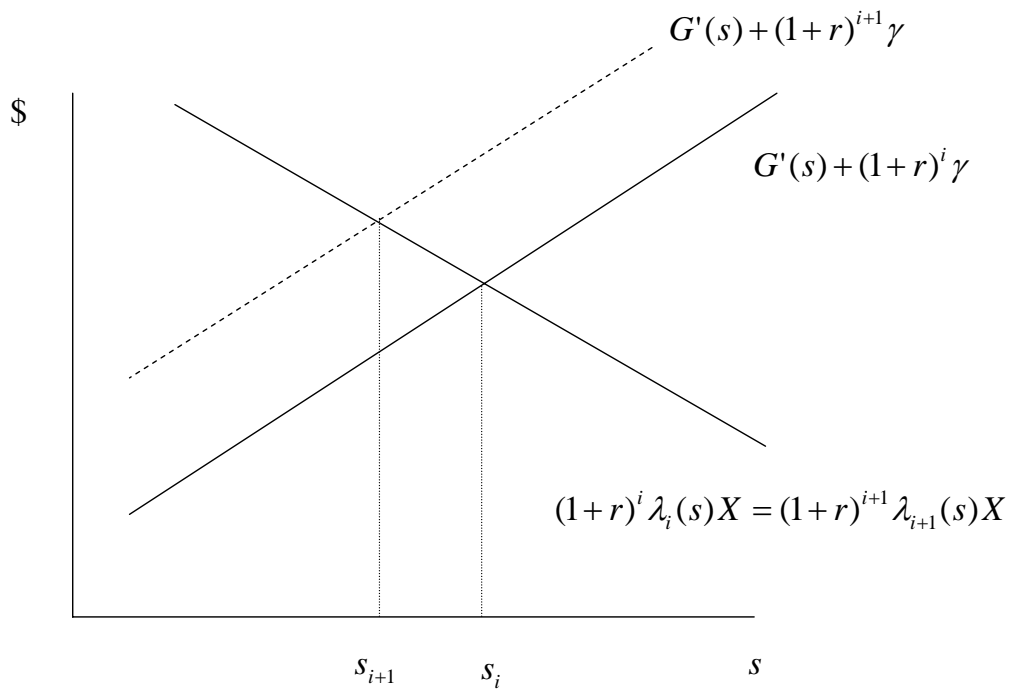


Figure 4: Graphical demonstration of declining site size for Model 4, the site development model.

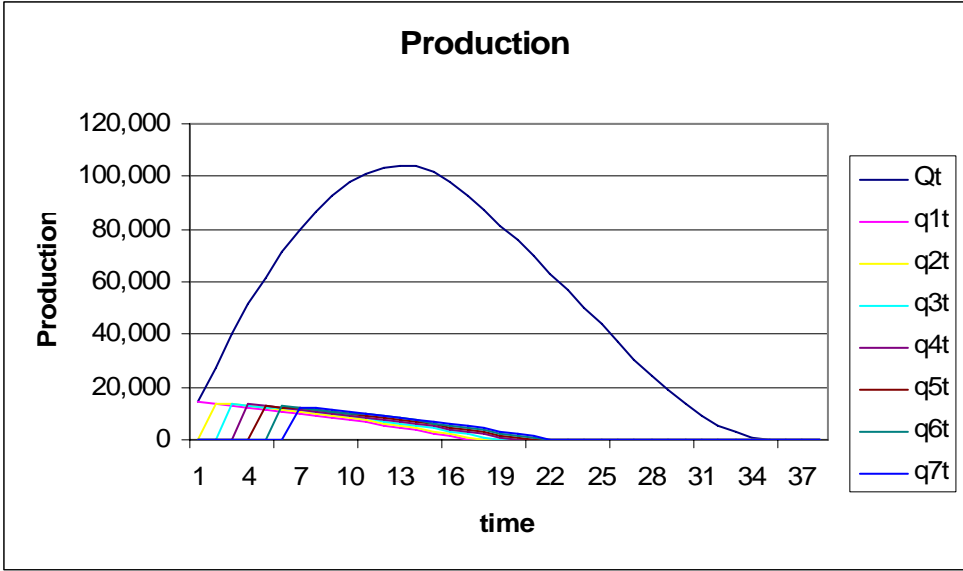


Figure 5: Simulation of aggregate production and production from individual sites in Model 4, the site development model.

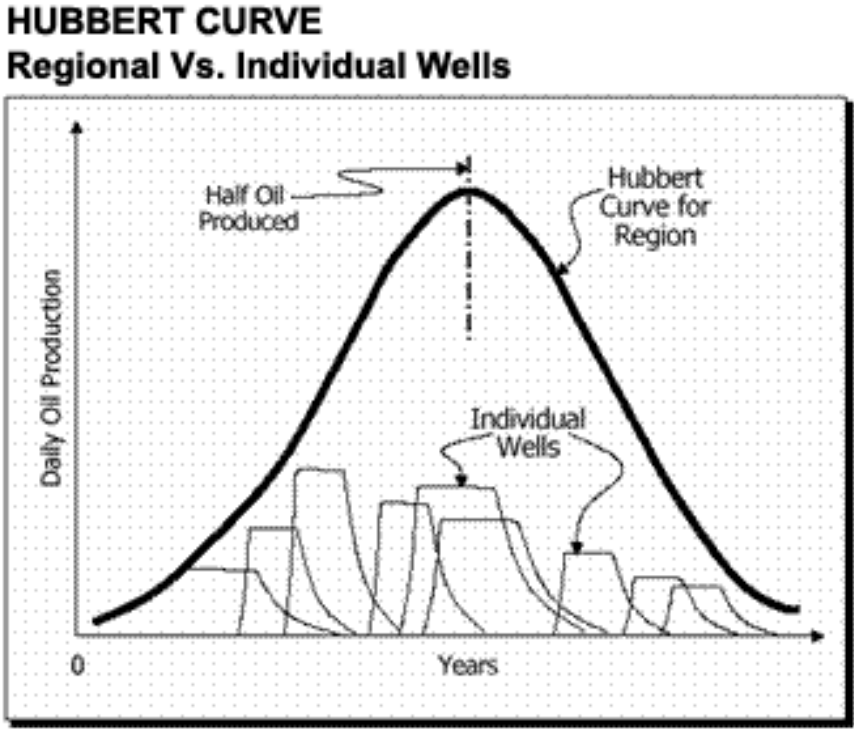


Figure 6: Schematic of Hubbert's curve for a region. Source: Peak oil primer from [www.energybulletin.net/primer.php](http://www.energybulletin.net/primer.php).

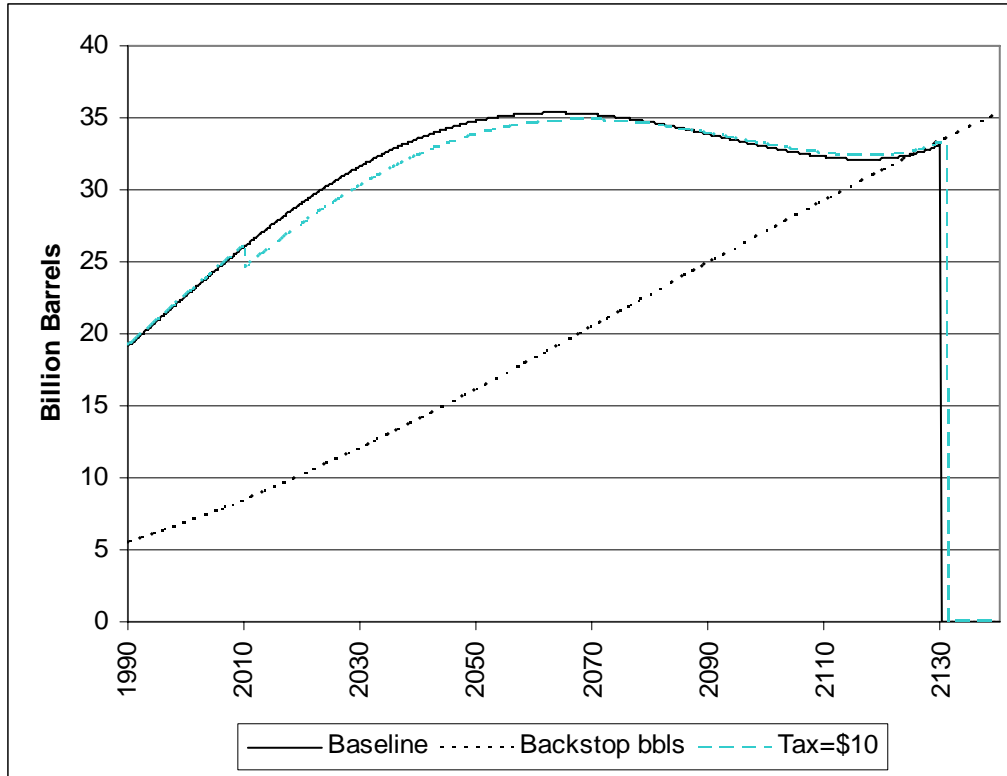


Figure 7. Simulated peak in world oil production with and without \$10 energy tax.

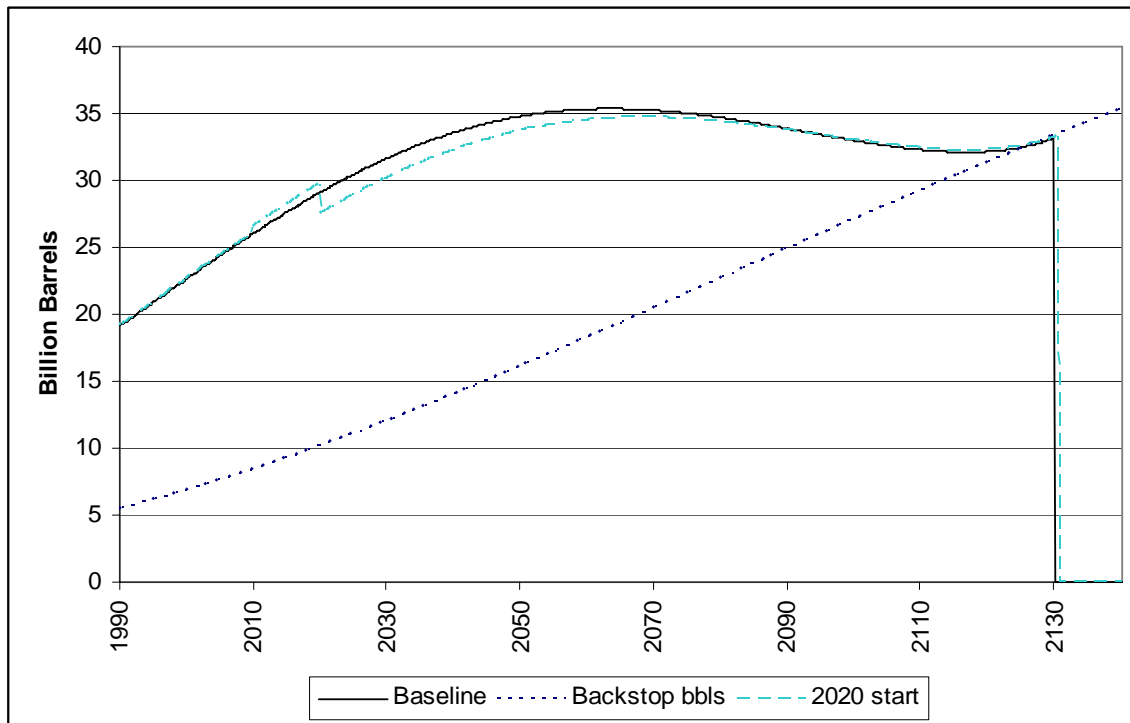


Figure 8. Simulated peak in world oil production with \$10 energy tax announced in 2010 and implemented in 2020.

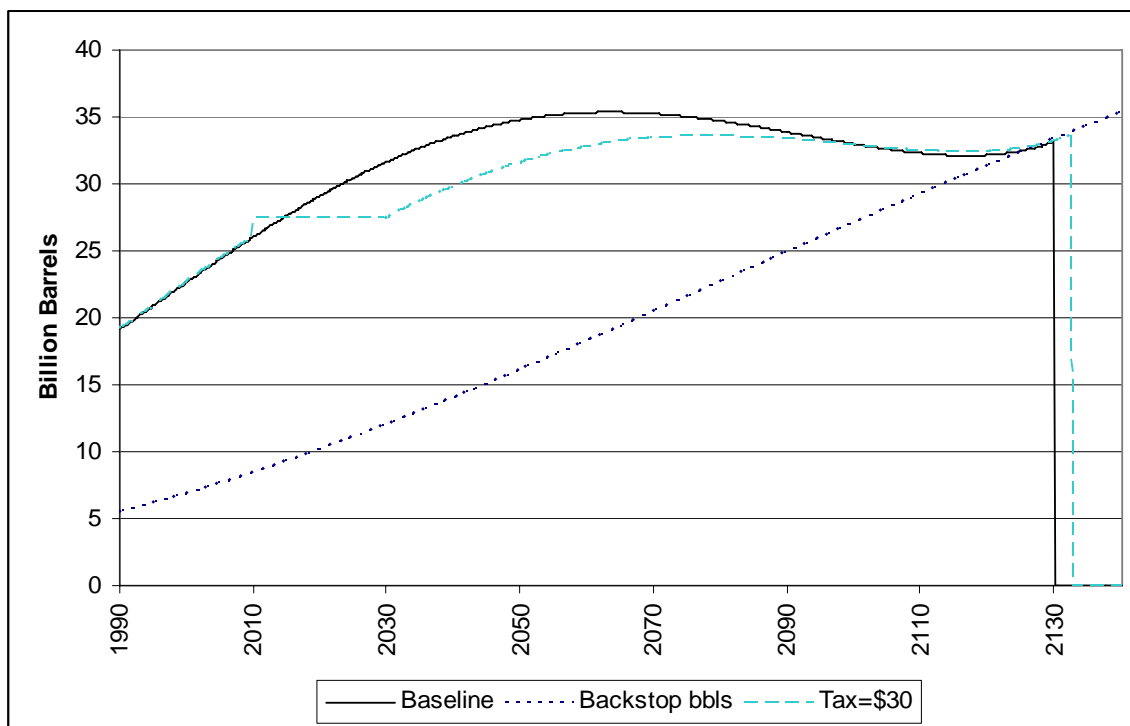


Figure 9. Simulated peak in world oil production with \$30 energy tax phased in over 20 years.

## Tables

Table 1. Energy tax policy simulations.

*Panel A: Tax announced and implemented in 2010*

	Peak Year	Peak Production	Transition Year	Decrease in production
baseline	2064	35.4	2130	n.a.
\$10	2069	34.8	2131	1.45
\$20	2074	34.3	2132	2.86
\$30	2079	33.9	2133	4.14

*Panel B: Tax of \$10*

Announcement Year	Peak Year	Peak Production	Transition Year	Decrease in production
baseline	2064	35.4	2130	n.a.
2010	2069	34.8	2131	1.45
2020	2069	34.7	2131	1.50
2030	2069	34.6	2130	1.46
2060	2060	35.3	2130	1.18

*Panel C: Tax of \$10 announced in 2010*

Implementation Year	Peak Year	Peak Production	Decrease in production	Years above baseline
baseline	2064	35.4	na	na
2010	2069	34.8	1.45	0
2015	2069	34.8	2.09	5
2020	2069	34.7	2.15	10
2030	2069	34.6	2.21	20
2050	2050	35.5	2.12	30

*Panel D: Tax announced in 2010 and phased in over 20 years*

Tax	Peak Year	Peak Production	Increase in 2010 production	Years above baseline
baseline	2064	35.4	n.a.	n.a.
\$10	2069	34.69	0.55	4.5
\$20	2074	34.11	1.02	4.5
\$30	2078	33.59	1.47	4.25

Note: All production in billion barrels of oil.