Modeling Peak Oil

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"Peak oil" refers to the future decline in world production of crude oil and to the accompanying potentially calamitous effects. The majority of the literature on peak oil is non-economic and ignores price effects even when analyzing policies. Unfortunately, most economic models of depletable resources do not generate production peaks. I present four models which generate production peaks in equilibrium. Production increases in the models are driven by: demand increases, cost reductions through advancing technology, cost reductions through reserve additions, and production capacity increases through site development. Production decreases are driven by scarcity. The models do not rely on market failures and indicate that a peak in production may arise from efficient intertemporal optimization. The models shows that prices are a better indicator of impending scarcity than peaking is and that peak production can occur when any percentage from 0-100% of the original deposit remains.

JEL codes: Q3 and Q4.
Keywords: Depletable resources, Hotelling, peak oil.

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1 Introduction

The term "peak oil" has come to be synonymous with a host of concerns about future energy supplies.[1] A vast non-economic literature addresses whether global oil production has peaked or will soon peak; what consequences that could have for fossil fuel dependent societies; and what should be done about it.[2] Unfortunately, most of this literature fails to recognize the role that prices could play in allocating scarce oil. For example, Hirsch *et al.* (2005) notes a wedge between projections of oil production, which peaks, and oil consumption, which does not.[3] The authors analyze a variety of mitigation policies and conclude that prompt action is required to prevent future shortfalls and economic disruptions. However, they do not mention the effects of these policies on prices or the effects of prices on these policies. Similarly, Lovins *et al.* (2005) proposes a variety of demand reduction policies for "getting the United States completely, attractively, and profitably off oil." However, the analysis ignores the fact that these policies would decrease demand for oil, presumably decreasing the price of oil and the profitability of the policies.

Most economic models of depletable resources do not seem to offer additional insight because they do not explicitly generate a peak in production.[4] For example, the seminal model of Hotelling (1931) predicts that (net) prices should grow at the rate of interest and that production should steadily decline over time.[5] Extensions of this model for uncertainty, limited capacity, set-up costs, different grades of ore, and increasing costs with cumulative extraction, also do not generate peaks in production.[6] This raises the question as to whether the observed production peak could have arisen from an economic model. Is the production peak itself evidence of some market failure or disequilibrium? Is oil peaking just a series of happy (or unhappy) accidents that

[1] "Peak oil" refers to the peak in U.S. crude oil production in 1970. This peak, also known as Hubbert's peak, was correctly predicted by M.K. Hubbert (1956).

[2] Recent books include Abdullah (2005), Cooke (2004), Deffeyes (2005), Deffeyes (2001), Kunstler (2005), and Simmons (2005). Numerous websites and on-line discussions (often bordering on hysteria) are devoted to peak oil. In Aug. 2005, the Wall Street Journal hosted an online forum on peak oil in which the invited economists agreed that peak oil is a "greater challenge than the 'looming crisis' in Social Security" and "one of the most important economic transitions that many of us [...will...] witness." Econoblog (2005). See Lynch (2003) for an opposing view.

[3] This report, funded by the U.S. Dept. of Energy, is not published and does not appear on the DoE website.

[4] Notable exceptions, discussed below, are Pindyck (1978), Slade (1982) and Livernois and Uhler (1987).

[5] In his review of the peak oil literature, Porter (2006) states that "the standard Hotelling model offers little insight into the oil market."

[6] See Krautkraemer (1998) for a survey of this vast literature.

is not amenable to economic analysis?[7]

This paper answers these questions by presenting four economic models that generate peaks in production without resorting to market failures. The models are solidly based on the classic Hotelling theoretical framework of optimizing producers and consumers.[8] Thus the models show that peaking is consistent with dynamic efficiency and is not evidence of some market failure.

The peak in each model is generated by opposing forces tending to increase or decrease equilibrium production. In the models, increasing demand, improvements in technology, additional reserves, and new site development tend to increase production while scarcity tends to decrease production. Given the fundamental nature of these forces, it would be more surprising if production did not peak than if it did peak!

Indeed, oil production in many regions has peaked. After increasing for over 100 years, U.S. annual crude production peaked in 1970 at 3.5 billion barrels of oil and has generally declined since. Brandt (2006) analyzes 139 (potentially overlapping) oil producing regions throughout the world and argues that production in 123 regions can be reasonably modeled as single peaked and that production in 74 of these regions has already peaked.[9] Furthermore, production of other resources has also peaked.[10] This widespread empirical evidence of production peaking highlights the importance of understanding why production of an exhaustible resource might peak.

Although, the peak-oil literature focuses attention on peak production, the underlying concern seems to be that the transition from cheap oil to expensive substitutes will be sudden, chaotic, and costly. This attention to peak production is misplaced for several reasons. First, peak production is irrelevant to concerns about the transition to substitute resources. The transition to renewables should occur when oil resources are depleted such that their price rises to the production cost of renewables. The models show that the transition should occur after production peaks since oil should be used to smooth the transition to the renewable resource. Second, the price path is a

[7] Kaufmann and Cleveland (2001) argue that the observed asymmetric relationship between prices and production indicates a failure of the basic Hotelling model and that this failure warrants a greater degree of government intervention in the transition from oil.

[8] The focus is on competitive models. Modeling a monopoly or oligopoly would likely change the shape of the production peak, but not eliminate it.

[9] Non-conforming regions either have multiple peaks, *e.g.,* Ohio, or have chaotic production, *e.g.,* Iraq. Regions have peaked if they have sufficient production data to fit a curve after the peak.

[10] Bardi (2004) describes the production peak in whale oil in the mid-19th century.

better indicator of impending resource scarcity than the production path is. As the models show, prices will begin to increase before production peaks and thus are an earlier indicator of future scarcity.[11]

While the transition to renewables will surely have some surprises for everyone, energy use is likely to be smooth across the transition. There are two reasons the equilibrium price path (of a barrel of oil equivalent) cannot jump during the transition. First, a forward-looking firm, or government, could profitably save some (or all) of its oil for production after any price jump. Since oil is virtually costless to store in its natural reservoir, such intertemporal arbitrage would eliminate any jumps in the price path. Second, even if there were *no* forward-looking firms or governments with secure property rights to oil, the increasing cost of oil extraction from different deposits would prevent jumps in the price path. For example, if the marginal extraction cost of the highest cost deposit were $200 per barrel and the cost of the renewable substitute were $260, then completely myopic firms would exhaust the oil at a price of $200, and the price would then jump to $260.[12] However, since oil production costs are smoothly increasing, there are oil deposits with production costs between $200 and $260. Even completely myopic firms without secure property rights would wait to produce from these deposits until the price were high enough to cover the extraction costs. Production from these high-cost deposits will ensure a smooth (although possibly inefficient) transition from oil to renewable resources.

The four models are presented and discussed in Section 2. Each model incorporates fundamental economic forces into the basic Hotelling model and generates an endogenous peak in equilibrium production.

Although the first three models have clear antecedents in the literature, the fourth model is novel. The main motivation for this model comes from casual observation of the oil industry. The original oilfields in the U.S. were centered around Pennsylvania. Over time, these resources were depleted, and the center of U.S. oil production moved to Texas. As these reserves have been depleted, production has moved to Alaska and abroad. The fourth model explicitly allows for

[11] Bardi (2004) argues that prices for whale oil began to increase well before the production peak in approximately 1850.

[12] Gaudet *et al.* (2002) describe a model without secure property rights in which the transition from low-cost oil (with constant marginal extraction cost) to the high-cost substitute is indeed abrupt.

sites to be developed at different times and acknowledges that much of the cost of oil production comes from exploration, development, and installation of production capacity. All these features are excluded from other models in the literature. Interestingly, these features alone can cause a peak in production even without any cost reductions as illustrated by the model.

Section 3 derives insights common to all the models which would hold in a combined models. The first result addresses whether prices or peaking is a better indicator of impending resource scarcity. The second result describes what can (or cannot) be learned from production peaking. Section 4 concludes.

2 Models of Peak Oil

This section presents four Hotelling-style models with peaks in production. Unlike the curve fitting model of Hubbert (1956) or the noneconomic literature on peak oil, these models recognize the incentives of both producers and consumers and that these competing effects must be balanced to determine equilibrium prices and consumption. The models are presented in order of increasing complexity and evaluated based on the empirical evidence.

2.1 Model 1: Demand shift

In Model 1, the increase in production is caused by increasing demand. Let $D_t(p)$ be aggregate demand derived from the consumer's optimization. Let demand, D_t, be downward sloping, *i.e.,* $D_t' < 0$, and growing over time, *i.e.,* $D_t(p) \leq D_{t+1}(p)$ for every p. To simplify the transversality conditions, assume demand has a finite choke price, possibly at the cost of a substitute resource.[13]

Now consider supply of a depletable resource. Assume the finite resource stock, S, can be extracted at cost $C(q)$ where $C' > 0$ and $C'' \geq 0$. Thus profit maximization,

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t)], \tag{1}$$

is subject to the stock constraint

$$\sum_{t=1}^{\infty} q_t \leq S \tag{2}$$

[13] Specifically, assume there exists some $\bar{p}$ such that $D_t(\bar{p}) = 0$ for every t.

where β is the discount factor, and q_t and p_t are production and price in period t. The well-known Hotelling supply relation is seen from the first order condition:

$$p_t = C_q + (1+r)^t \lambda \tag{3}$$

where λ is the shadow value of the stock and the rate of interest, r, is defined by $\beta = 1/(1+r)$. Thus equilibrium in the model is characterized by a net price that grows at the rate of interest.

Figure 1 illustrates how production can increase in this model. The *full marginal cost*—defined as the marginal extraction plus marginal scarcity costs, here $C_q + (1+r)^t\lambda$—is increasing over time due to the increasing scarcity of the depletable resource. This tends to decrease production. However, the demand shift tends to increase consumption over time. In this figure, the demand increase effect is larger, and equilibrium production increases. With a finite choke price, the demand increase will eventually be less than the full marginal cost increase, and equilibrium production will decrease. Thus the model predicts a peak in equilibrium production.

Model 1 has a strong empirical implication: namely, prices should rise over time, even while production is increasing. Econometric evidence on the crude oil price series has tested for deterministic trends.[14] Slade (1982), using data from 1870-1978, regresses the price series for several commodities, including petroleum, on linear and quadratic trends and finds evidence for quadratic trends. However, this result may be spurious since she did not test for a unit root.[15] Lee, List, & Strazicich (2006) reject the unit root hypothesis for petroleum and find evidence of a quadratic trend when allowing for structural breaks in the time series. The plot from their linear trend model shows a declining trend through 1896 which suggests that prices have not been monotonically non-decreasing.[16]

Model 1 shows that demand growth can cause a peak in production. However, since this model may not be consistent with the empirical evidence on prices, the model may be missing some important economic forces. Models 2-4 focus on supply side mechanisms and yield peaks in production together with U-shaped equilibrium price paths.

[14] Several well-publicized bets over natural resource prices have been resolved in favor of the "optimists."

[15] Berck and Roberts (1996) extend Slade's data and cannot reject a unit root leading them to conclude that the commodity prices do not have a deterministic trend. Ahrens & Sharma (1997) reject the unit root hypothesis for some commodities, including petroleum.

[16] Their analysis finds structural breaks for petroleum in 1896 and 1971 when allowing for a linear trend and breaks in 1914 and 1926 when allowing for a quadratic trend.

2.2 Model 2: Technological change

This model, based on Slade (1982), simply assumes that costs decrease exogenously over time due to technological change. Let the cost function be $C(q,t)$ where $C_q > 0$, $C_{qq} > 0$, $C_t < 0$ and $C_{qt} < 0$. The producer's optimization is given by:

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, t)] \tag{4}$$

subject to the stock constraint in [2]. The first order condition is

$$p_t = C_q(q_t, t) + (1+r)^t \lambda. \tag{5}$$

This first order condition implies that the net price again grows at the rate of interest.

Figure 2 illustrates a production increase in Model 2 where price is constant, *i.e.*, $p_t = p_{t+1} = \bar{p}$. The marginal cost curve, C_q, shifts down over time due to technological progress. However, the scarcity cost, $(1+r)^t\lambda$, is increasing over time due to scarcity. Thus production can increase if the decrease in marginal extraction costs outweighs the increase in scarcity costs, as illustrated in Figure 2. Since the growth rate of the scarcity cost is r, the scarcity-cost increase will eventually outweigh the decrease in marginal extraction costs (and production will fall) as long as the growth rate of the decline in marginal costs is less than r.

Thus far, only the supply side of the model has been analyzed. Because of increasing supply, equilibrium in this model can lead to a U-shaped price path even with a stationary demand. With a stationary demand, the peak in production would occur simultaneously with the lowest price. If demand increases over time, the peak in production occurs after the low point on the price path.

The relationship between technological change and costs is intuitively appealing. Clearly advances in drilling and exploration technology have made additional oilfields accessible. An optimist might assume that such technological advances will continue indefinitely. However, existing empirical evidence does not strongly support a negative relationship between technological change and costs. Cuddington and Moss (2001) construct a measure of technological diffusions but did not find that it significantly reduced exploration and development costs for crude oil reserves—although they find that it reduced finding costs for nonassociated natural gas.

This technological change model is somewhat unsatisfactory from a theoretical perspective since the primary driver of interest, technological change, is unexplained in the model. The next model does not rely on an exogenous mechanism but has reductions in extraction costs derived endogenously through discovery of additional reserves.

2.3 Model 3: Reserves growth

This model is based on Pindyck (1978) who argued that there is an inverse relationship between marginal extraction costs and reserves. In this model, a firm has an incentive to explore and develop new reserves in order to drive down marginal extraction costs.

Let R_t be reserves in period t. Additions to reserves, $f(w, S_t)$, depend on effort, w, and cumulative discoveries, S_t, where $f_w > 0$, $f_S < 0$, $S_{t+1} - S_t = f(w, S_t)$ and $S_1 = 0$. Changes in reserves are then $R_{t+1} - R_t = f(w_t, S_t) - q_t$ where $R_1 = 0$. Let the cost of effort be $c(w)$, where $c' > 0$ and $c'' \geq 0$, and costs of extraction be $C(q, R)$ where $C_q > 0$, $C_{qq} \geq 0$, $C_R < 0$, and $C_{qR} < 0$. Thus costs and marginal costs are decreasing in reserves. Profit maximization is

$$\max_{q_t, w_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, R_t) - c(w_t)] \tag{6}$$

subject to the equations of motion for reserves and cumulative reserve additions. The first order conditions can be written

$$p_t = C_q(q_t, R_t) + (1 + r)^t \lambda_t \tag{7}$$

$$\beta^t c'(w_t) - \gamma_t f_w(w_t, S_t) = \lambda_t f_w(w_t, S_t) \tag{8}$$

$$\lambda_{t+1} - \lambda_t = \beta^t C_R(q_t, R_t) \tag{9}$$

and

$$\gamma_{t+1} - \gamma_t = -(\lambda_t + \gamma_t) f_S(w_t, S_t) \tag{10}$$

where $\lambda_t > 0$ and $\gamma_t < 0$ are the shadow values of reserves and of cumulative additions to reserves at time t. [7] sets the marginal benefit of oil equal to the full marginal cost. [8] sets the marginal cost of effort plus the scarcity cost of effort equal to the marginal benefit of effort in terms of increased reserves and [9 & [10] are the equations of motion for the shadow values.

For a constant price, $\bar{p}$, substituting [7] into [9] shows that

$$r[\bar{p} - C_q(q_t, S_t)] + C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t) + (1+r)C_R(q_t, S_t) = 0. \tag{11}$$

Since the first term in [11] is positive and the last term is negative, the change in marginal extraction costs, $C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t)$, can be either positive or negative. In fact, Pindyck shows that if initial reserves are low, it is optimal for the firm to exert effort to find reserves to drive down the marginal extraction cost. This leads to a peak in production and, in equilibrium, a U-shaped price path.

Model 3 is theoretically appealing since the peak is clearly endogenously determined and is not driven by exogenous shifts. However, the empirical evidence of a negative relationship between costs and reserves is not conclusive. Livernois & Uhler (1987) present empirical evidence arguing that aggregate reserves and cost are not negatively correlated and in fact are positively correlated. They claim that this may be due to the fact that lower cost reserves tend to be found first. Thus, even if new discoveries are sufficient to increase reserves, they may not lower costs. They argue for a disaggregated analysis and find evidence for the assumed negative correlation between costs and reserves at a disaggregate level.

Pesaran (1990) uses a similar model for an integrated econometric estimation using data on North Sea production.[17] The author argues that the estimates give a "reasonable degree of support to the theory."

2.4 Model 4: Site development

Models 2 & 3 relied on decreases in marginal extraction costs to drive the peak and U-shaped price path. The peak in Model 4 is driven by increases in aggregate production capacity due to production at newly developed sites, despite the fact that costs are unchanged at all previously developed sites.

Each period in this model, firms choose how large a site to explore and develop and the production capacity to install in that site. Once capacity is installed, production continues from that site until all oil is exhausted. Assume the density of oil is X so the stock of oil in a site

[17] Pickering (2002) applies a similar model to find evidence of the "discovery decline phenomenon."

of size s is $X \times s$. Costs of exploring a site of size s, are given by $G(s)$ where $G' > 0$ and $G'' \geq 0$. Convex exploration costs could arise from the fixed number of trained geologists or exploration crews in the short run. Before production can take place at a given site, firms must install production capacity, K, in the site. Costs of installing capacity, F, are assumed to be increasing in the amount of capacity, $F' > 0$, at a nondecreasing rate, $F'' \geq 0$.[18] Convex costs in capacity installation could arise due to a fixed number of experienced drilling crews and trained engineers or due to the declining natural pressure of the reservoir.[19] Finally, extraction costs for any site, $C(q, K)$, depend on the amount extracted as well as the installed capacity with $C_q > 0$, $C_{qq} > 0$, $C_K < 0$, $C_{KK} > 0$, and $C_{qK} < 0$. Thus, pumping costs and marginal pumping costs are increasing in output but decreasing in capacity.[20] The remaining cost assumption, $C_{KK} > 0$, implies that the decrease in costs from adding capacity is smaller at higher levels of capacity, *i.e.*, $d(-C_K)/dK < 0$.[21]

Profit maximization is

$$\max_{q_{it}, K_t, s_t} \sum_{t=1}^{\infty} \beta^t [p_t Q_t - \sum_{i=1}^{t} C(q_{it}, K_i)] - \beta^t F(K_t) - \beta^t G(s_t) \tag{12}$$

subject to

$$q_{it} = 0 \qquad \text{if} \;\; i > t \tag{13}$$

$$Q_t = \sum_{i=1}^{t} q_{it} \qquad \forall t \tag{14}$$

$$\sum_{t=i}^{\infty} q_{it} \leq X s_i \quad \forall i \tag{15}$$

$$\sum_{i=1}^{\infty} s_i \leq S \tag{16}$$

where q_{it} is production from site i at time t, K_t is capacity installed in site t at time t, and s_t is the size of the site explored at time t, and S is the total area available for exploration. Production

[18]For simplicity, assume that capacity costs are independent of the size of the site.

[19]If K is interpreted as effective capacity, then increasing effective capacity by 100 bpd would require a smaller capital investment when capacity is smaller, due to the natural pressure within the reservoir.

[20]A referee points out that due to declining natural field pressure, too much capacity could actually increase costs. Since here capacity is endogenous and the firm has secure property rights, too much capacity will not be installed.

[21]This specification of the cost function does not allow an inverse L-shaped marginal cost curve. The allowable function $C(q, K) = cq + (K - q)^{-\alpha}$ defined for $q < K$ approximates an inverse L-shaped marginal cost curve as $\alpha \to \infty$.

profits are revenue minus production costs from all sites that have previously been explored and developed. In each period, an additional site is explored and developed (*i.e.,* production capacity is installed in the site) and the final two terms of the objective function capture the costs of development and exploration of a new site.[22] The first constraint prevents extraction from a site before it is explored and developed, and the second constraint defines aggregate supply as the sum of production from all developed sites. The third constraint is the stock constraint for site i, and the final constraint ensures that the size of all the developed area is less than the total area available for development.

The Lagrangian for this constrained optimization is[23]

$$L = \sum_{t=1}^{\infty}\left(\beta^t[p_tQ_t - \sum_{i=1}^{t} C(q_{it}, K_i)] - \beta^t F(K_t) - \beta^t G(s_t)\right) + \sum_{i=1}^{\infty}\lambda_i[Xs_i - \sum_{t=i}^{\infty} q_{it}] + \gamma[S - \sum_{i=1}^{\infty} s_i] \tag{17}$$

where λ_i is the shadow value of oil at site i, and γ is the shadow value of area for development. The first order conditions can be written:

$$p_t \leq C_q(q_{it}, K_i) + (1+r)^t\lambda_i \qquad \forall t, \forall i, \text{ where } i < t \tag{18}$$

$$\sum_{t=i}^{\infty}\beta^t[-C_K(q_{it}, K_i)] = \beta^i F'(K_i) \qquad \forall i \tag{19}$$

$$G'(s_i) + (1+r)^i\gamma = (1+r)^i\lambda_i X \qquad \forall i \tag{20}$$

The first condition says that the price equals the full marginal cost—here $C_q(q_{it}, K_i) + (1+r)^t\lambda_i$—at each site with positive production. The second condition implies that the present value of the sum of cost reductions from an additional unit of capital must equal the present value of the cost of the additional unit of capital. The third condition says that the marginal cost of exploration plus the scarcity cost of the additional explored area must equal the benefit from an additional barrel of oil times the density of oil.

Proposition 1 Characterization of the Optimal Supply in Model 4: *With a stationary output price, limited total area for exploration, and stationary costs of production, exploration and development, the following hold:*

(i) Declining production at each site: $q^*_{it} > q^*_{i(t+1)}$ *if* $q^*_{it} > 0$.

[22]Decreasing returns in exploration ensure the entire available area is not explored and developed in the first period.

[23]Since the shadow values, λ_i, are constant, the Hamiltonian shortcuts are not necessary here.

(ii) Exhaustion at each site: *$\exists\, T_i > i$ such that $q^*_{it} = 0$ for every $t > T_i$.*

(iii) Decreasing site size: *$s^*_i > s^*_{i+1}$.*

(iv) Decreasing capacity: *$K^*_i > K^*_{i+1}$.*

(v) Increasing and decreasing aggregate production: *Production increases if additional production at the newly developed site offsets production declines at all the previously developed sites. Eventually production declines as newly developed sites become smaller.*

Declining production at each site follows from [18]. Once capital in site i is sunk, the marginal extraction cost $C_q(q_{it}, K_i)$ does not change. However, the full marginal cost increases since $(1+r)^t\lambda_i$ grows at the rate of interest. Figure 3 illustrates declining production from site i for a stationary output price $\bar{p}$.

Exhaustion at each site also follows from [18]. Once capital is sunk, the full marginal cost, which includes one term growing at the rate of interest, will eventually exceed $\bar{p}$ at which time extraction from site i ceases and the deposit is exhausted. Exhaustion would be illustrated in Figure 3 when the full marginal cost rises above $\bar{p}$, *i.e.,* when $C_q(0, K_i) + (1+r)^t\lambda_i > \bar{p}$.

Decreasing site size is demonstrated in the appendix. The argument is illustrated graphically in Figure 4. If the same size site is developed in period i and period $i+1$, then the appendix shows that the marginal benefit of development, which is declining in s, is the same in present value for i and $i+1$, *i.e.,* $(1+r)^i\lambda_i(s)X = (1+r)^{i+1}\lambda_{i+1}(s)X$. Since the marginal cost of site development $G'(s) + (1+r)^i\gamma$ includes a scarcity cost, which grows at the rate of interest, the marginal benefit would be less than the marginal cost in period $i+1$ as illustrated. Thus $s^*_i > s^*_{i+1}$.

Decreasing capacity is also demonstrated in the appendix by a similar argument comparing the marginal benefit and marginal cost of capacity installation. It is not surprising that smaller capacity is installed in a smaller site.

The production peak (increasing and decreasing aggregate production) follows since aggregate production initially increases as more sites are developed for production. In the first period, there is only production from one site so aggregate production is simply q_{11}. In the second period, production occurs from two sites so aggregate production is $q_{12} + q_{22}$. If new production from site 2 is sufficient to offset the decline in production at site 1, then aggregate production increases. In each subsequent period, new production becomes available, but production from all previously developed sites declines. Since the size of new sites is optimally declining, eventually the additional production from a new site will not be sufficient to offset the decline in production from all existing sites, and aggregate production falls.

Figure 5 simulates optimal production in Model 4 for the first seven sites and for the aggregate.[24] In the first year, production is only from the first site so aggregate production is q_{11}. In the second year, production starts from the second site while production only declines slightly from the first site. Thus aggregate production increases in the second year. Aggregate production continues to increase until year 13 as new production offsets declines in production at existing sites. After the peak, new production cannot offset declines in production at existing sites. The first site is exhausted by year 17 and thereafter at least one site is exhausted every year. The last site is developed in year 51, and all sites are exhausted by year 53.

Figure 6 shows a schematic of Hubbert's curve taken from a primer on peak oil (Energy Bulletin 2006). The schematic, not based on an underlying model, illustrates the observed (hypothesized?) relationship between aggregate production and production at individual wells. Model 4 could be parameterized to match this schematic, thus showing economic assumptions which generate such a schematic in equilibrium.

3 Results

Since each of Models 1-4 incorporate important economic forces into the standard model of exhaustible resources, a model combining each of these elements might be even more realistic. Combining demand growth, technological change, reserve additions, and site development into a single model would be easy but tedious. However, the analysis of the models suggests some interesting results common to all the models, which would hold in a combined model.

The first result concerns prices as an indicator of impending scarcity. Recall that in Model 1, the oil price increased monotonically. The peak in production arose because initially the demand growth was stronger than the price increase. Eventually, the increase in price became stronger than the increase in demand, and production decreased. In Models 2-4 with stationary demand, production increased only when the economic forces caused the price to fall and pro-

[24] Simulation details are in the appendix. Parameters of the simulation are $\bar{p} = \$50$, $C(q, K) = q^2/K$, $G(s) = s^2/2$, $F(K) = K^2/2$, $S = 20,000$, $X = 100$ and $r = 0.05$.

duction to increase. Thus in Models 2-4, the production peak was coincident with the minimum (trough or low point) in the price path.

What happens when all four models are combined? Clearly, while the equilibrium price is falling, all forces align so that production increases. However, when the price path reaches its minimum what happens to production? For a fine enough division of time, the price path is essentially flat at the minimum. If demand is still growing, production must increase for some time after the price has reached its minimum. Eventually, the increasing price will outstrip the demand growth (as in Model 1) and production will peak. The preceding argument is summarized in the following proposition:

Proposition 2 *In a model incorporating the economic forces of Models 1-4, the peak in production cannot occur before the minimum price is reached and occurs strictly after the minimum price is reached if demand growth is sufficiently strong.*

This result has an important implication for those concerned about peaking production in the oil market. Namely, prices are a better indicator of impending oil scarcity than are peaks in production. Therefore focusing attention on peak production is misguided since focusing attention on price would given an earlier predictor of impending resources scarcity.

A question remains as to what information is contained by peaking production. The second proposition addresses this question:[25]

Proposition 3 *In a model incorporating the economic forces of Models 1-4, the peak in production can occur when none of the exhaustible resource has been used, or when all of the resource has been used.*

Proposition 2 can be demonstrated by an example. Figure 7 illustrates production from a simple parameterization of Model 1 where the growth rate of demand is α.[26] For $\alpha = 0.01$, production is always falling, so none of the resource has been exhausted before the peak. For $\alpha = 0.05$, the peak production occurs in year 11 when approximately 50% of the resource remains. For $\alpha = 0.2$, the peak occurs in year 10 when 35% of the resource remains. For $\alpha = 3$, the peak occurs when none of the resource remains.[27]

[25] See Smith (2007) for an earlier discussion of this point.

[26] Demand is parameterized by $D(p, t) = (100 - p)(1 + \alpha)^t$, and the 2000 units of the exhaustible resource can be extracted at no cost.

[27] Similar examples could be constructed for each of the four models.

This proposition shows that peaking has little or no relationship with the amount of resource remaining to be extracted. Moreover, the proposition shows that the distribution of production need not be symmetric about the peak but could be skewed either early or late.

4 Conclusion

The four models isolate four possible causes of increases in production: increasing demand, cost reductions through technological change, cost reductions through exploration, and increasing production from additional site exploration and development. In each model, the underlying scarcity of the resource ultimately leads to a decline in production. This shows that a peak in production is not evidence of market failure but rather that a peak in production could well arise from efficient intertemporal optimization.

Models 2-4 generate U-shaped price paths in equilibrium. While there is some econometric evidence for a U-shaped price path, the evidence is inconclusive. However, these models can generate production peaks even if price is constant. Thus the conditions under which the models generate production peaks are quite general. In fact, the economic forces underlying the production peaks are so fundamental that it would be more surprising if production did not peak.

Combining the models shows that the price path will reach its minimum before production peaks and that production can peak when any percentage (from 0-100%) of the original resource remains. This suggests, first, that production peaks are not a reliable indicator of the amount oil remaining, and second, that prices are a better indicator of impending resource scarcity than production. Thus research on determining production peaks may be better focused on determining the minimum of the price path if the goal is early detection of coming resource scarcity. However, given substantial short-run volatility in oil prices, it may be difficult to identify the underlying, long-run price trend from short-run changes in prices.

Although policy makers would find it helpful to have a reliable indicator of resource scarcity, this research suggests that production peaks are not a reliable indicator of resource scarcity.

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Appendix: Proof of Proposition 1 (iii) & (iv)

To demonstrate decreasing site size, *i.e.,* $s_i^* > s_{i+1}^*$, assign site sizes (and their shadow value γ) exogenously, but let capacity and production be determined optimally. The proof uses [20] to compare the marginal cost, $G'(s)+(1+r)^i\gamma$, and marginal benefit, $(1+r)^i\lambda_i X$, of expanding the site size in periods i and $i+1$. The marginal cost is easier. Since $G'(s)+(1+r)^{i+1}\gamma > G'(s)+(1+r)^i\gamma$ for all s, the marginal cost of expanding later sites (conditional on the site sizes being the same) is higher in $i+1$ than in i due to the higher scarcity cost of the exhaustible sites. (See Figure 4.)

The marginal benefits are harder since the shadow value λ_i will depend on the (here exogenous) size of the site s. First, write this explicitly as $\lambda_i(s)$ and note that $\lambda_i' < 0$, *i.e.,* the shadow value is smaller for a larger stock. I will compare $\lambda_i(s)$ and $\lambda_{i+1}(s)$, but first must show that conditional on equal site sizes it is optimal to install equivalent capacity, *i.e.,* $K_i(s) = K_{i+1}(s)$ where $K_i(s)$ is the optimal capacity. To do this I show that if K solves [19] for period i then it also solves [19] for period $i+1$. First note that if capacity, K, and site size, s, are the same in periods i and $i+1$ then optimal production from site $i+1$ would equal lagged production from site i in each period, *i.e.,* $q_{(i+1)t} = q_{i(t-1)}$ for all t. But then

$$\sum_{t=i+1}^{\infty} \beta^{t-(i+1)}[-C_K(q_{(i+1)t}, K)] = \sum_{t=i+1}^{\infty} \beta^{t-(i+1)}[-C_K(q_{i(t-1)}, K)]$$

$$= \sum_{t-1=i}^{\infty} \beta^{(t-1)-i}[-C_K(q_{i(t-1)}, K)] = \sum_{t=i}^{\infty} \beta^{t-i}[-C_K(q_{it}, K)] = F'(K). \tag{21}$$

The first equality follows from the relationship between lagged production, the second equality from algrebra, the third equality by relabeling the index, and the final equality follows since [19] is satisfied in period i. Since [21] implies that [19] is satisfied in period $i+1$, it follows that $K = K_i(s) = K_{i+1}(s)$. This further implies that equating lagged production is optimal, *i.e.,* $q_{(i+1)t} = q_{i(t-1)}$ for all t is optimal conditional on s. Thus:

$$(1+r)^t\lambda_{i+1}(s) = \bar{p} - C_q(q_{(i+1)t}, K) = \bar{p} - C_q(q_{i(t-1)}, K) = (1+r)^{t-1}\lambda_i(s) \tag{22}$$

for all t where the first equality follows from [18], the second equality follows from the lagged production relationship, and the third equality follows again from [18]. But this implies that $(1+r)^{i+1}\lambda_{i+1}(s)X = (1+r)^i\lambda_i(s)X$ which implies that the marginal benefit of expanding the site in period i or period $i+1$ is equal. (See Figure 4.)

The proof of (iii) then follows by noting that $G'(s_i^*) + (1+r)^{i+1}\gamma > G'(s_i^*) + (1+r)^i\gamma = (1+r)^i\lambda_i(s)X = (1+r)^{i+1}\lambda_{i+1}(s)X$ which implies that $s_i^* > s_{i+1}^*$. (See Figure 4.)

To prove (iv), let K_i^* be the optimal capacity at site i. Let $q_{(i+1)t}^{**}$ be the optimal production at site $i+1$ conditional on installing capacity K_i^* at site $i+1$. This implies that:

$$\sum_{t=i+1}^{\infty} \beta^{t-(i+1)}[-C_K(q_{(i+1)t}^{**}, K_i^*)] < \sum_{t=i+1}^{\infty} \beta^{t-(i+1)}[-C_K(q_{i(t-1)}^{*}, K_i^*)]$$

$$= \sum_{t=i}^{\infty} \beta^{t-i}[-C_K(q_{it}^{*}, K_i^*)] = F'(K_i^*). \tag{23}$$

The inequality holds because site $i+1$ is smaller than site i, which implies that optimal extraction from site $i+1$ is smaller than lagged extraction from site i, *i.e.*, $q_{(i+1)t}^{**} \leq q_{i(t-1)}^{*}$, for all t with strict inequality when $q_{i(t-1)}^{*} > 0$. Since $C_{qK} < 0$, it follows that $-C_K(q_{(i+1)t}^{*}, K_i^*) < -C_K(q_{i(t-1)}, K_i^*)$ for $q_{i(t-1)} > 0$. The first equality in [23] follows by relabeling the indices, and the final equality follows from [19]. But the inequality in [23] implies that the marginal benefit of installing capacity K_i^* at site $i+1$ (the reduction in pumping costs) is less than the marginal capacity cost of K_i^*. Thus, optimal capacity is smaller at the smaller site, i.e., $K_i^* > K_{i+1}^*$ for every i.

Appendix: Simulation of Model 4

Optimal production, exploration, and capacity installation in Model 4 can be simulated using an algorithm of nested loops. For any given γ, s_1, K_1, and λ_1, the extraction path q_{1t} is determined by [18]. If total extraction from site 1 is greater (less) than $s_1 \times X$, then the marginal user cost, λ_1, must have been too small (large). Using this adjustment rule, the optimal shadow value, conditional on γ, s_1, and K_1, can be calculated by looping. Next [19] is used to determine whether too much or too little capacity has been installed in this site. This adjustment rule allows the optimal capacity to be calculated by looping where the optimal shadow value is calculated during each iteration. Once K_1 and λ_1 are computed optimally, [20] can be used to determine whether the given s_1 is too large or too small. This adjustment rule can be used, with nested loops for K_1 and λ_1, to compute the optimal site size, s_1, for a given γ. To determine whether γ is too high or too low, the optimal site size must be computed for each site. If the sum of the sites is greater (less) than S, then γ was too low (high). This adjustment rule, with nested loops for s_i, K_i, and λ_i, allows the optimal γ to be computed.

Figures

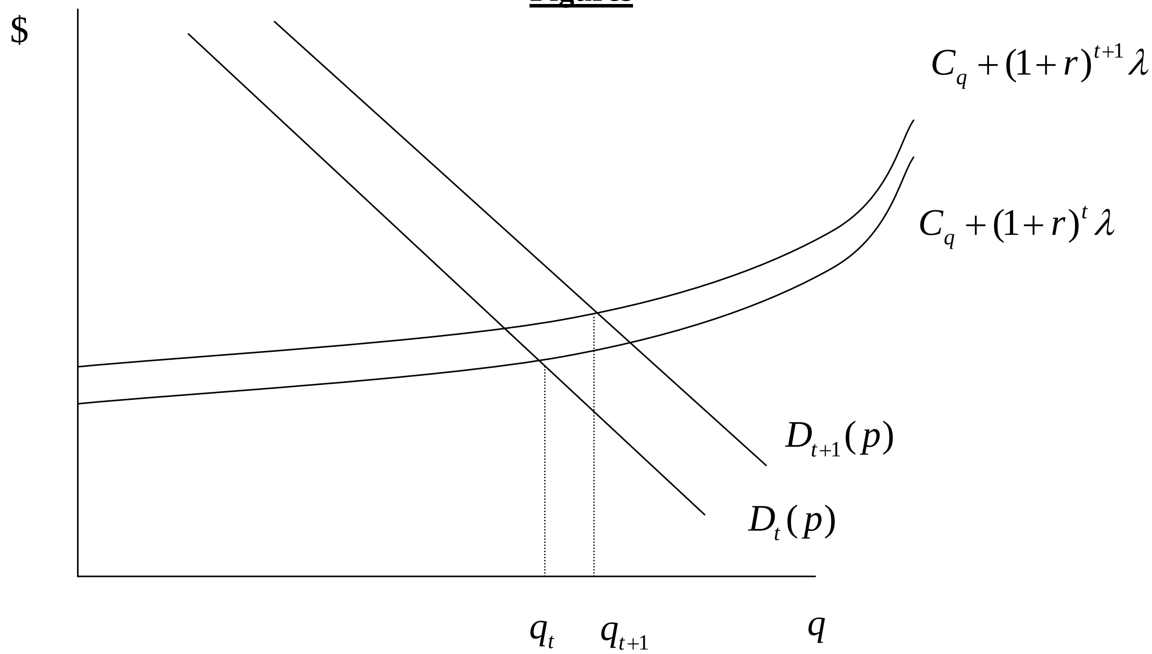

Figure 1: Illustration of a production increase in Model 1, the demand shift model.

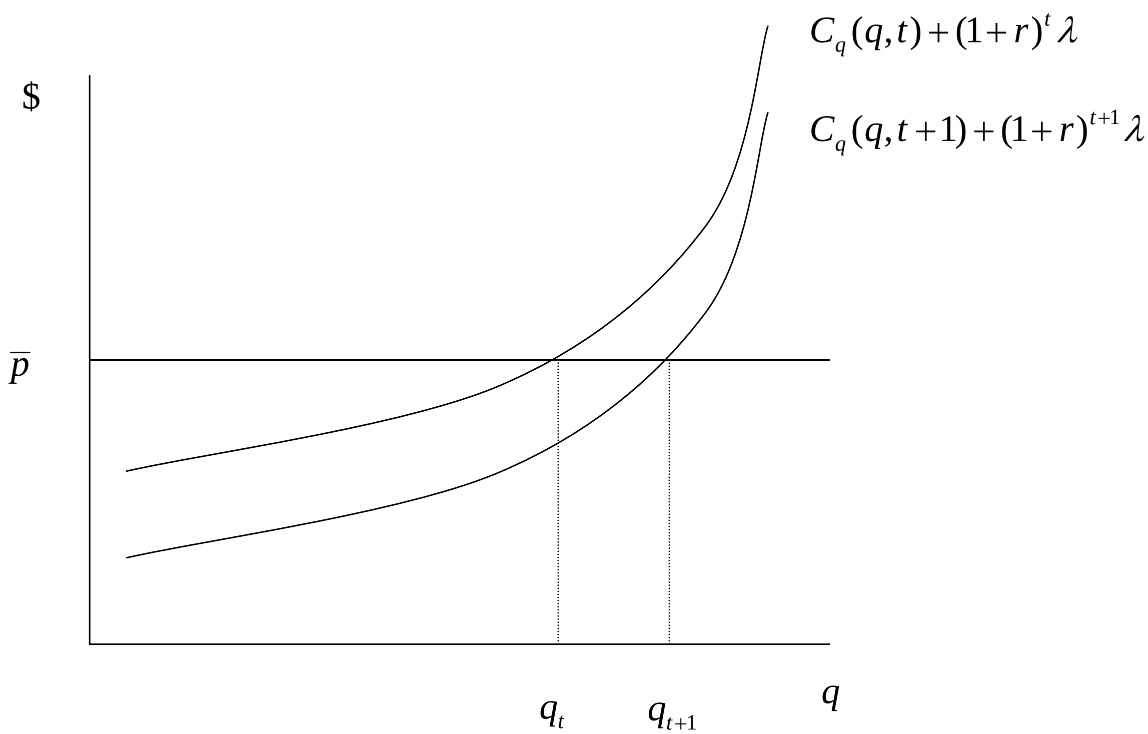

Figure 2: Illustration of increasing production in Model 2, the technological change model.

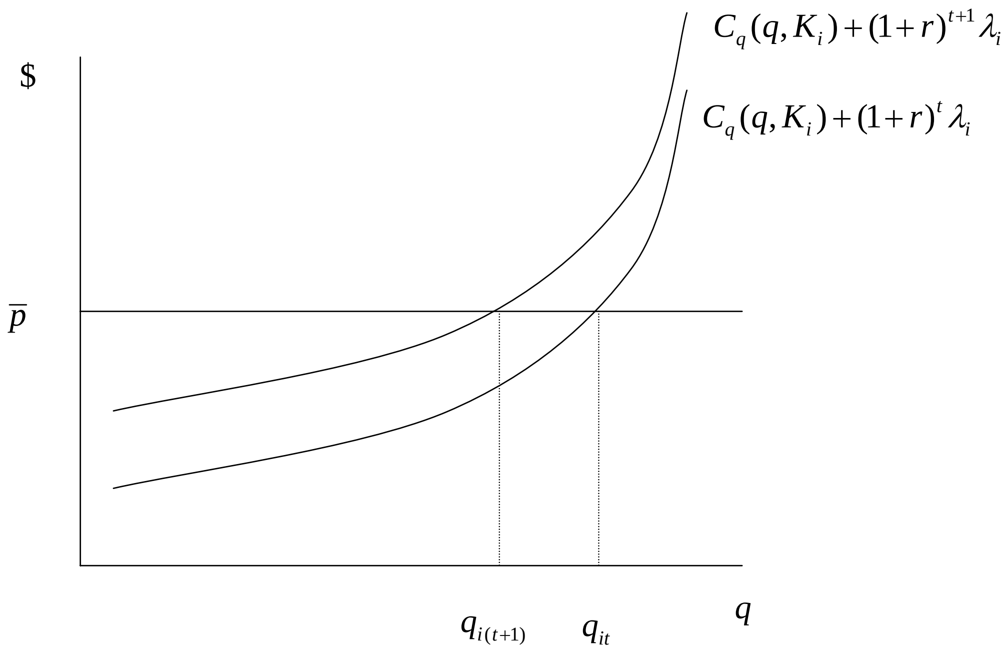

Figure 3: Graphical demonstration of declining production at a site for Model 4, the site development model.

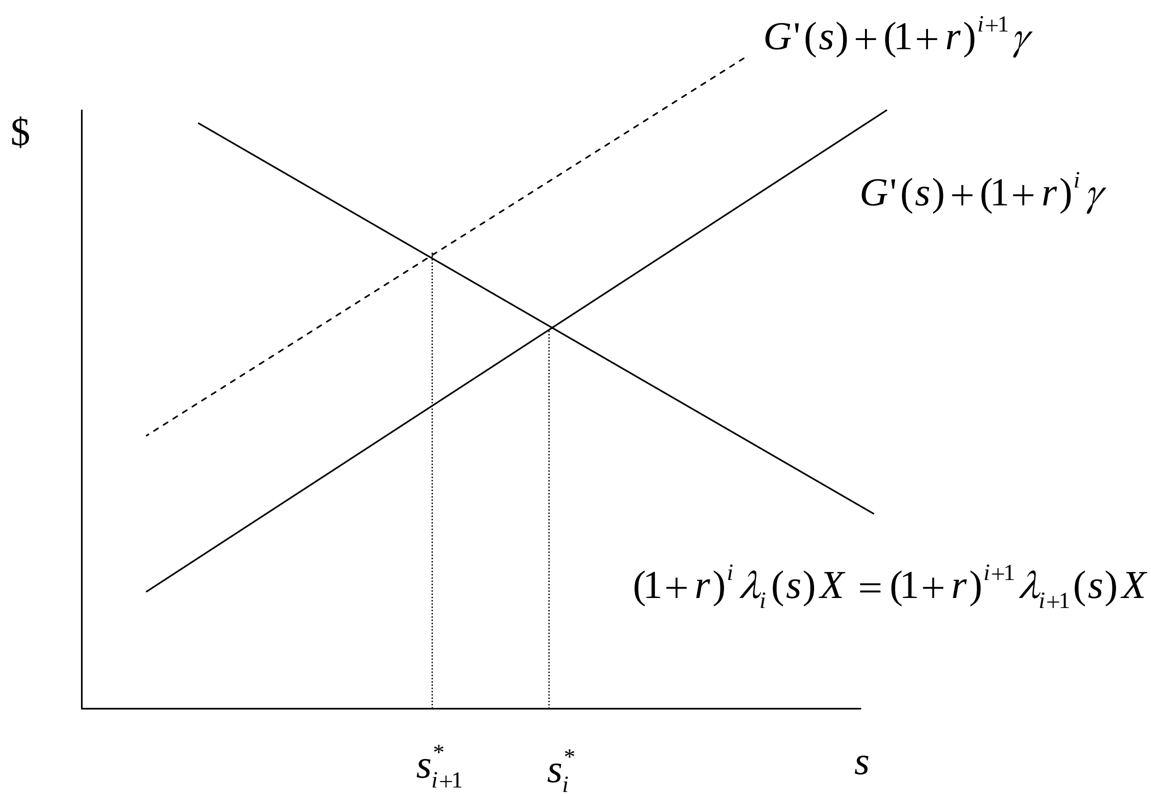

Figure 4: Graphical demonstration of declining site size for Model 4, the site development model.

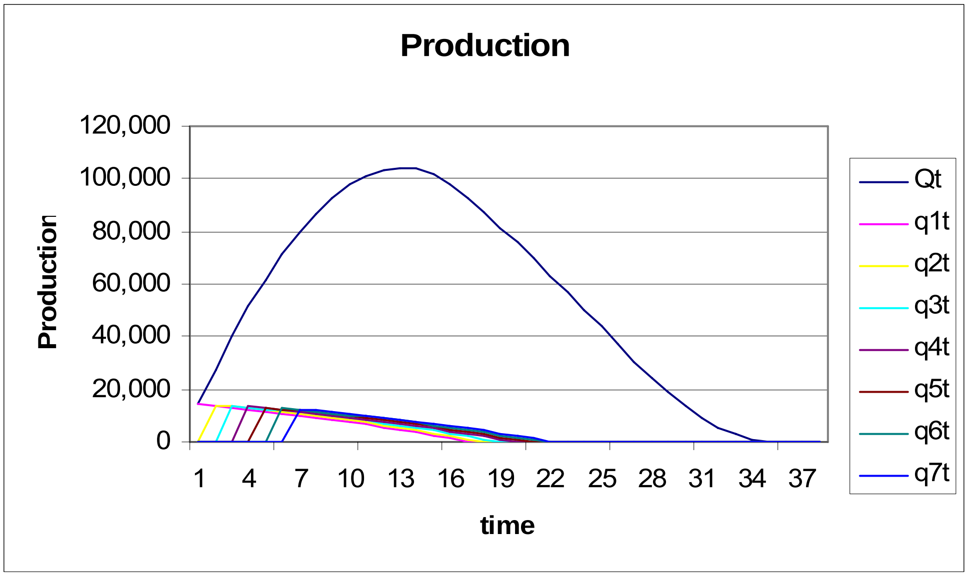

Figure 5: Simulation of aggregate production and production from individual sites in Model 4, the site development model.

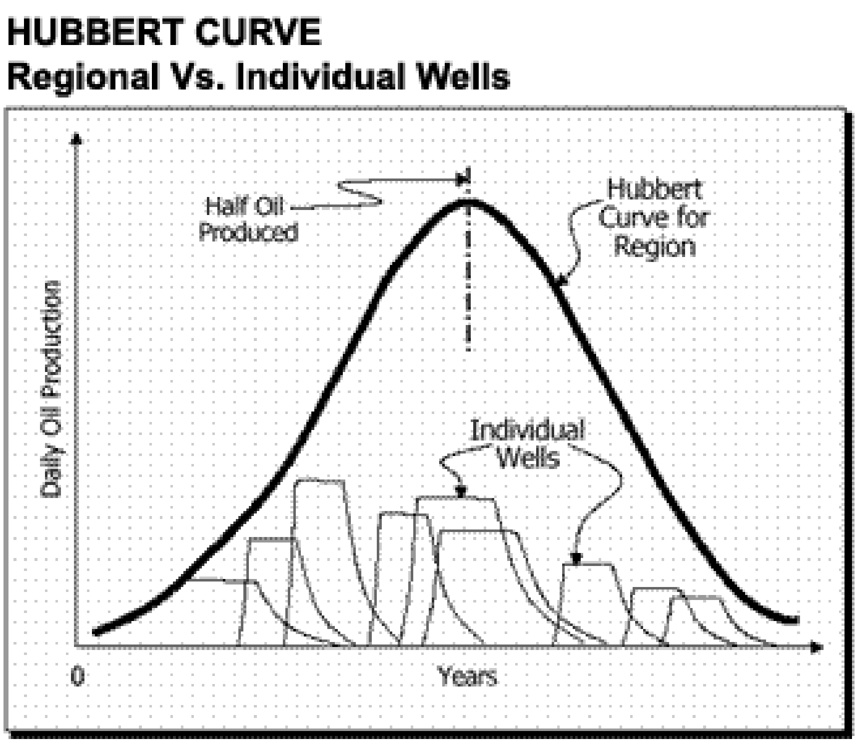

Figure 6: Schematic of Hubbert's curve for a region. Source: Peak oil primer from www.energybulletin.net/primer.php.

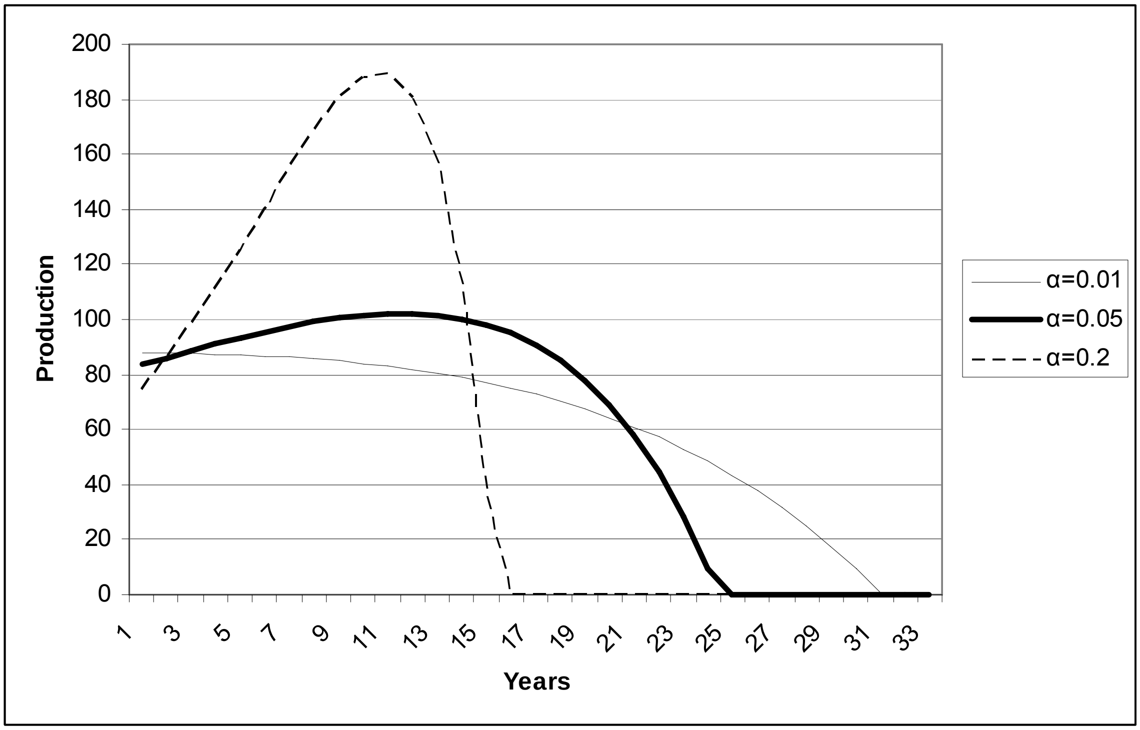

Figure 7. Simulated production peaks for different levels of demand growth.