Set-up Costs and the Existence of Competitive Equilibria when Extraction Capacity Is Limited

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Abstract

Although set-up costs are prevalent and substantial in natural resource extraction, their analysis in decentralized markets is difficult. In particular, it is known that a Walrasian competitive equilibrium cannot exist in simple extraction models with set-up costs thus rendering standard competitive equilibrium analysis inapplicable. However, this paper demonstrates that this result is sensitive to the assumption of unlimited extraction capacity. When extraction capacity is limited, a competitive equilibrium can exist if each firm earns sufficient surplus to cover its set-up costs. Moreover, if firms choose extraction capacity, the optimal capacity choice ensures existence of a competitive equilibrium. The resulting competitive equilibrium price either grows at the rate of interest when total extraction is below capacity or is constant when capacity is fully utilized. In the equilibrium, identical deposits are opened simultaneously, and set-up costs for new deposits are incurred when the industry has excess capacity rather than when capacity is fully utilized.

Key words: Natural resource extraction, extraction constraints, set-up costs, competitive equilibrium, nonconvexity, capacity installation.

1 Introduction

Set-up costs of natural resource extraction include costs of tunneling, drilling wells, building pipelines, searching for deposits, and removing overburden from strip mines.[1] These "lumpy" costs must be incurred before any production can take place and thus induce non-convexities in the production possibilities set.[2] With non-convexities, the well-known theorems on the existence of competitive equilibria first developed by Arrow and Debreu (1954) are no longer applicable. Although competitive equilibria may exist in some non-convex economies,[3] Hartwick *et al.* (1986) and Fischer (1998) have shown that a competitive equilibrium *cannot* exist if extraction requires set-up costs.[4] Without existence of a competitive equilibrium, analysis of the functioning of these competitive natural resource markets is virtually impossible.[5]

The literature demonstrating non-existence has modeled set-up costs as exogenous and unrelated to extraction capacity. This paper demonstrates that if set-up costs arise from the installation of extraction capacity, then—despite non-convex production sets—the existence of a competitive equilibrium can be saved by recognizing that extraction capacity

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[1] Set-up costs are significant empirically. In Cicchetti's (1972) study of the Trans-Alaska oil pipeline, the initial capital cost was $2 billion while annual operating costs were only $95 million. Similarly, in Holland and Moore's (1999) study of imported water use in Arizona, the set-up cost for building dams, aqueducts, and pumping stations was $2.9 billion while the total annual operating costs were only $54 million.

[2] This paper analyzes set-up costs which are sunk. See Farzin (1984) and Lozada (1993) for analysis of reversible capital investment.

[3] Farrell (1959) first showed that a competitive equilibrium can exist in a non-convex economy with many agents, since the non-convexities yield only small discontinuities in aggregate demand or supply. Starr (1969) later provided a rigorous treatment of this result. Aumann (1966) showed the existence of a competitive equilibrium in a non-convex economy with a continuum of agents.

[4] In a related literature, Eswaran, Lewis and Heaps (1983) showed that a competitive equilibrium does not exist if there are fixed costs of extraction which can be avoided by extracting nothing in a given period. See also Kimmel (1984), Mumy (1984), and Lozada (1996).

[5] See Hogan and Holland (2000) on the difficulties of game-theoretic analysis of natural resource extraction. Empirical research has used a variety of techniques to analyze set-up costs. Cicchetti simply assumes the set-up cost is spread evenly across the entire deposit. Nordhaus (1973) and Chakravorty *et al.* (1997) do not explicitly model set-up costs.

may be limited. In particular, a competitive equilibrium exists if capacity is constrained such that producers' surplus is sufficiently large or if firms choose their capacity.

In the literature on non-existence, Hartwick *et al.* and Fischer argue that extraction from identical deposits with set-up costs and constant marginal extraction costs should be strictly sequential.[6] However, strictly sequential extraction requires each firm to install extraction capacity large enough to fill demand. If capacity installation is costly, simultaneous extraction from multiple deposits with set-up costs may well be efficient. Since the arguments demonstrating non-existence by Hartwick *et al.* and Fischer rely on the efficiency of strictly sequential extraction, their results do not demonstrate non-existence when extraction capacity is limited.

There is, however, a more fundamental difficulty for existence which does not depend on the strict sequencing of extraction, namely: a price-taking firm has an incentive to delay incurring the set-up cost beyond the efficient date. To illustrate, consider a market with a single, price-taking firm owning one deposit requiring a set-up cost.[7] If exploiting the deposit is efficient, the set-up cost should be incurred immediately. Once the set-up cost is sunk, consumption in each period should be exactly as if there were no set-up cost, i.e., the marginal benefit should grow at the rate of interest as discovered by Hotelling (1931). The First Welfare Theorem then implies that a competitive equilibrium price path—if one

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[6] Strictly sequential extraction is efficient since any extraction plan with simultaneous extraction from two deposits is dominated by a plan which delays the set-up cost of one of the deposits and reallocates extraction between the two deposits such that consumption is unchanged. This reallocation dominates the initial plan since extraction costs and consumption are unchanged but the set-up cost is delayed. With non-constant marginal extraction costs, the proposed reallocation would change the present value of extraction costs and thus would not necessarily dominate the plan with simultaneous extraction. Non-constant marginal extraction costs will be discussed in Section 4.

[7] Clearly, a single, unregulated firm would be expected to exert market power and would not be a price taker. Although this example is intended purely for illustrative purposes, the fact that each firm has an incentive to delay set-up costs also is true for multiple firms and has the same consequences for existence of a competitive equilibrium.

exists—must yield this efficient allocation.[8] The consumer's optimization implies that the Hotelling price path is the only potential competitive equilibrium price path. Are the Hotelling price path and efficient extraction consistent with a profit maximizing firm? Recall that under the Hotelling rule, the firm is indifferent between extraction in any two periods. If the set-up cost were delayed and extraction increased in later periods, the present value of revenue would be unchanged and the set-up cost would be delayed. Since this deviation would yield higher profit, the firm is not maximizing profit in the proposed equilibrium. Therefore the proposed price path is not an equilibrium price, and a competitive equilibrium does not exist.[9]

Analysis of multiple deposits with set-up costs introduces an additional difficulty for existence: discontinuities in the efficient marginal benefit path. Hartwick *et al.* showed that the marginal benefit path should jump down each time a new deposit is opened and then should grow at the rate of interest.[10] Facing a price path identical to the efficient marginal benefit path, price-taking firms now have an incentive to deviate from the efficient extraction program by extracting too early at the higher price. This deviation can be profitable if the set-up cost is incurred an instant too early and a large amount is extracted at the higher price. Note that each of these profitable deviations may not be feasible

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[8] Recall that both welfare theorems require price-taking behavior and complete markets, and the Second Welfare Theorem additionally requires convexity. Thus in the models under consideration here, the First Theorem holds, but the Second Theorem is not applicable.

[9] In a static natural monopoly, a competitive equilibrium price cannot exist since firms have negative profit under marginal-cost pricing. In natural resource extraction, firms earn scarcity rents, and thus may earn positive profits from efficient pricing if the set-up cost is small. However, the preceding argument implies that a competitive equilibrium does not exist, even if firms could earn positive profits by pricing competitively.

[10] The Hartwick *et al.* result is that a set-up cost should be incurred when it generates a sufficient increase in surplus to cover the interest payment on the set-up cost. Since producers never receive any surplus in the constant marginal cost model, it is consumer surplus which must increase discontinuously, and thus the marginal benefit path jumps down.

if extraction capacity is limited. This paper derives conditions under which extraction capacity is sufficiently limited that there are no profitable deviations from efficiency, and a competitive equilibrium exists.

The competitive equilibrium is characterized by a price path which grows at the rate of interest when total extraction is below capacity or is constant when capacity is fully utilized. Set-up costs for new deposits are incurred when the price reaches a threshold at which the producer's surplus covers the interest payment on the set-up cost and still pays the firm the scarcity rent of the deposit. Since the capacity is not fully utilized when the price is rising, set-up costs for new deposits are incurred when the industry has excess capacity rather than when capacity is fully utilized. The price threshold insures that capacities in identical deposits are installed simultaneously.

If extraction requires significant set-up costs, it is likely that firms will have some market power. Even if there are many firms with many deposits, the model still predicts that there will only be a relatively small number of firms with excess capacity at any given time. Thus, the ability of firms to respond to an attempt to drive up the price may be limited by the lack of excess capacity. However, firms might be able to respond by installing capacity in additional deposits. On the other hand, firms might use capacity installation decisions to attempt to restrict output. Clearly, analysis of the many possible strategic interactions between firms is desirable, but is well beyond the scope of this paper. Given the likelihood of market power, the equilibria identified in this paper might be best understood as a regulatory equilibrium. A regulator with sufficient information could impose the equilibrium price path as a price ceiling. This ceiling would then induce firms to install the efficient amount of capacity at the proper time and to extract efficiently.

The paper proceeds as follows: Section 2 shows that a competitive equilibrium exists if extraction capacity is exogenously constrained such that i) the capacity price is high enough that producers can cover extraction and interest costs and ii) the deposit is large enough that costs are recovered before the deposit is exhausted. Section 3 then shows an equilibrium always exists when extraction capacity is chosen endogenously. Section 4 extends the model and Section 5 concludes.

2 Exogenous Extraction Capacity

Consider extraction of a commodity from multiple deposits with exogenous extraction capacities and set-up costs. Consumption is $Q(t)$ in period t, and consumer surplus in each period is $U(Q)$, where $U' > 0$ and $U'' < 0$.[11] Consumers and producers discount at the common rate, r. Let one of the deposits have sufficient extraction capacity installed to satisfy the entire market at constant marginal extraction cost c_0. Call it the *unconstrained deposit*, and let it have initial stock S_0.[12] Capacity must be installed for each of the remaining N deposits before they can be utilized. Call these N deposits the *constrained deposits*. For simplicity, assume that the constrained deposits are identical, i.e., have the same initial stock, S, marginal extraction cost, c, set-up cost, F,[13] and extraction constraint, $\bar{q}$.[14]

To find a competitive equilibrium, consider the price path, $p(t)$, and production and

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[11] If the choke price, $U'(0)$, is finite, then efficiency requires that the deposit be exhausted when the price reaches the choke price. If the choke price is infinite, then exhaustion need not occur in finite time.

[12] The unconstrained deposit can be thought of as all deposits—possibly owned by many owners—for which no further capacity installation is necessary. To compare with the models of Hartwick *et al.* or Lozada set $S_0 = 0$.

[13] Set-up costs may be incurred over a period of time. However, the important timing issue is when extraction can begin from the constrained deposits. Thus, F is the present value of all set-up costs at the time extraction begins.

[14] Note that any strict convex combination of the two feasible (money input, output) pairs, $(0, 0)$ and $(c\bar{q} + F, \bar{q})$, is not feasible. Therefore, the production set is not convex, and as a result, the existence of a competitive equilibrium is not guaranteed.

consumption allocations defined from the *constrained planner's problem* where the planner is constrained to install all N capacities simultaneously.[15] The optimization is:

$$\max_{q_0(t),q(t),T} \int_0^T e^{-rt}[U(q_0(t))-c_0q_0(t)]dt - e^{-rT}NF + \int_T^\infty e^{-rt}[U(Q(t))-c_0q_0(t)-cq(t)]dt \quad (1)$$

subject to the constraints

$$\int_0^\infty q_0(t)dt \leq S_0$$

$$\int_T^\infty q(t)dt \leq NS$$

$$q(t) = 0 \quad \forall t < T$$

$$q(t) \leq N\bar{q} \quad \forall t \geq T$$

where $q_0(t)$ is extraction from S_0, $q(t)$ is total extraction from all the constrained deposits, $Q(t) = q_0(t) + q(t)$ is consumption,[16] and extraction from the constrained deposits begins at T. The objective function is the discounted sum of consumer surplus less costs. The two stock constraints ensure that the total extracted from all deposits is less than the initial stocks, and the extraction constraints prevent extraction in any period from exceeding the installed capacity. Define Lagrange multipliers: λ_0 for the first stock constraint, λ for the second stock constraint, and $\mu(t)$ for the extraction constraint. The Lagrangian for the constrained planner's problem can then be written:

$$L = \int_0^T \left\{e^{-rt}[U(q_0) - c_0q_0] - \lambda_0 q_0\right\} dt \; - e^{-rT}NF + \lambda_0 S_0 + \lambda NS$$
$$+ \int_T^\infty \left\{e^{-rt}[U(Q) - c_0q_0 - cq] - \lambda_0 q_0 - \lambda q + \mu(t)(N\bar{q} - q)\right\} dt \quad (2)$$

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[15] The planner may wish to install capacities at different times. Therefore, the solution to the constrained planner's problem may not be efficient. Conditions will be derived below which ensure that the solution to this problem is indeed efficient.

[16] Note that extraction from the unconstrained deposit can be positive both before and after the capacities are installed.

Let $p(t) \equiv U'(Q)$ be the optimal marginal benefit path.[17] The Kuhn-Tucker first order and complementary slackness conditions for optimal extraction can be written using $p(t)$ as:

$$\begin{array}{rrll}
q_0(t) \geq 0 & e^{-rt}[p(t) - c_0] - \lambda_0 \leq 0 & C.S. & \forall t \\
q(t) \geq 0 & e^{-rt}[p(t) - c] - \lambda - \mu(t) \leq 0 & C.S. & \forall t \geq T \\
\lambda_0 \geq 0 & S_0 - \int_0^\infty q_0(t)dt \geq 0 & C.S. & \\
\lambda \geq 0 & NS - \int_0^\infty q(t)dt \geq 0 & C.S. & \\
\mu(t) \geq 0 & N\bar{q} - q(t) \geq 0 & C.S. & \forall t \geq T
\end{array} \tag{3}$$

Define the *augmented marginal cost* as the marginal extraction cost plus the scarcity cost of the stock, i.e., $AMC_0(t) \equiv c_0 + \lambda_0 e^{rt}$ and $AMC(t) \equiv c + \lambda e^{rt}$.[18] The first equation implies that extraction from the unconstrained deposit is optimal if $p(t)$ equals the augmented marginal cost when extraction is positive. The second equation implies that extraction from a constrained deposit is optimal if extraction is at capacity when $p(t)$ exceeds the augmented marginal cost and the capacity has been installed. Thus $p(t) = \max\{U'(N\bar{q}), c + \lambda e^{rt}\}$ when production is optimal and only from the constrained deposits. An application of the Herfindahl (1967) argument[19] to the first order conditions implies that $p(t) = \min\{c_0 + \lambda_0 e^{rt}, \max\{U'(N\bar{q}), c + \lambda e^{rt}\}\}$ after the capacities have been installed.

To compute the optimal time to install the capacities, first note that the optimal extraction paths need not be continuous at T. Let $q_0^- \equiv \lim_{t \uparrow T} q_0(t)$, $q_0^+ \equiv \lim_{t \downarrow T} q_0(t)$,

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[17] $p(t)$ is defined from the optimization of a function over a compact domain. The optimum exists, and $p(t)$ is well defined. Later, I will show that $p(t)$ is a competitive equilibrium price path. Note that the definition of $p(t)$ implies that consumers are optimizing, thus i will only need to show that producers are maximizing profits taking $p(t)$ as given.

[18] Note that price equal to augmented marginal cost is the well-known Hotelling rule that net price grows at the rate of interest.

[19] The Herfindahl result implies that for two deposits with different marginal costs, c_1 and c_2, and shadow values, λ_1 and λ_2, the price path is continuous and is given by $p(t) = \min\{c_1 + \lambda_1 e^{rt}, c_2 + \lambda_2 e^{rt}\}$.

and $q^+ \equiv \lim_{t \downarrow T} q(t)$ be extraction immediately before and after T. Similarly, let $Q^+ \equiv \lim_{t \downarrow T} Q(t)$ be consumption immediately after T. Differentiation of Equation 2 yields the first order condition for T:

$$U(q_0^-) - (c_0 + \lambda_0 e^{rT})q_0^- + rNF = U(Q^+) - (c_0 + \lambda_0 e^{rT})q_0^+ - (c + \lambda e^{rT})q^+ \tag{4}$$

Define *augmented net surplus* as gross consumer surplus less augmented marginal costs, i.e., $ANS(t) = U(Q) - AMC_0 q_0 - AMCq$.[20] Equation 4 states that augmented net surplus after T must exceed the augmented net surplus before T by the interest payment on the set-up cost,[21] i.e., $ANS^- + rNF = ANS^+$. This condition implies that there must be a discontinuous increase in augmented surplus when the capacities are installed. In general, this jump can occur in augmented producer or consumer surplus. The following lemma derives conditions under which $p(t)$ is continuous. If $p(t)$ is continuous, consumer surplus is continuous and the jump occurs solely in augmented producer surplus.

Lemma 1 *$p(t)$ is continuous if and only if $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$. Furthermore, if $p(t)$ is not continuous, capacities are installed after the unconstrained deposit is exhausted.*

Proof:
When extraction is from the unconstrained deposit, $p(t) = c_0 + \lambda_0 e^{rt}$ is continuous. When extraction is from the constrained deposits, $p(t) = \min\{c_0 + \lambda_0 e^{rt}, \max\{U'(N\bar{q}), c + \lambda e^{rt}\}\}$ is continuous. Thus if $p(t)$ is not continuous, capacities must be installed after S_0 is exhausted, and the discontinuity occurs when the capacities are installed. At T, $p(t)$ cannot jump up since if $p(t)$ jumped up at exhaustion of S_0, the objective function could be increased by increasing λ_0. Thus, the only possible discontinuity is for $p(t)$ to jump down at T.

If $p(t)$ jumps down at T, then $U'(q_0^-) > U'(Q^+)$ and $q_0^- < Q^+$. If $c + \lambda e^{rT} > U'(N\bar{q})$, then the result follows trivially. If $c + \lambda e^{rT} \leq U'(N\bar{q})$, then consumption is at capacity after

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[20] "Augmented" net surplus is smaller than the usual surplus measure since the costs are augmented by the scarcity cost. Note that ANS measures the flow of surplus and thus the set-up costs are not included in the definition of surplus.

[21] This condition on augmented surplus is identical to the Hartwick *et al.* condition on the Hamiltonian, namely: $H^- + rNF = H^+$

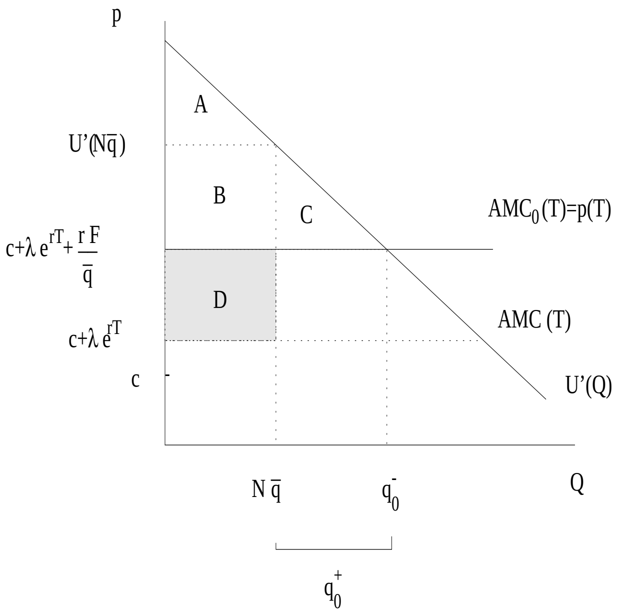

Figure 1: Augmented marginal cost and net surplus at T if the set-up cost is small and $p(t)$ is continuous. The increase in augmented surplus, Area(D)=$ANS^{+} - ANS^{-} = rNF$, accrues entirely to producers and consumer surplus does not change at T.

T, i.e., $Q^{+} = N\bar{q}$. Then by equation 4, $ANS^{-} = ANS^{+} - rNF = U(N\bar{q}) - (c + \lambda e^{rT} + \frac{rF}{\bar{q}})N\bar{q}$. Since consumer surplus is increasing $ANS^{-} = U(q_0^{-}) - U'(q_0^{-})q_0^{-} < U(N\bar{q}) - U'(N\bar{q})N\bar{q}$. Therefore $c + \lambda e^{rT} + \frac{rF}{\bar{q}} > U'(N\bar{q})$.

If $p(t)$ is continuous at T, then $p(t) = U'(q_0^{-}) = U'(Q^{+})$. The assumptions on U then imply that $U(q_0^{-}) = U(Q^{+})$ and also $q_0^{-} = Q^{+}$. Since this implies that $q_0^{-} = q_0^{+} + N\bar{q}$, equation 4 can be written:

$$c_0 + \lambda_0 e^{rT} = c + \lambda e^{rT} + \frac{rF}{\bar{q}} \tag{5}$$

Since $q_0^{+} \geq 0$, it follows that $Q^{+} \geq N\bar{q}$ and $U'(Q^{+}) \leq U'(N\bar{q})$ and thus $c + \lambda e^{rT} + \frac{rF}{\bar{q}} = c_0 + \lambda_0 e^{rT} = p(T) = U'(Q^{+}) \leq U'(N\bar{q})$. ■

Lemma 1 implies that the increment to augmented surplus at T is captured entirely by producers if the marginal benefit of extraction at capacity exceeds the augmented marginal cost plus the interest payment per unit, $\frac{rF}{\bar{q}}$.[22] This result is illustrated in Figures 1 and 2 for different set-up costs. In Figure 1, the set-up cost is small such that

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[22] The static ($\lambda = 0$) intuition for the condition $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ is that the average cost be less than the marginal benefit when production is at capacity. Note that this is precisely the condition which ensures existence of a competitive equilibrium in the static model with fixed costs.

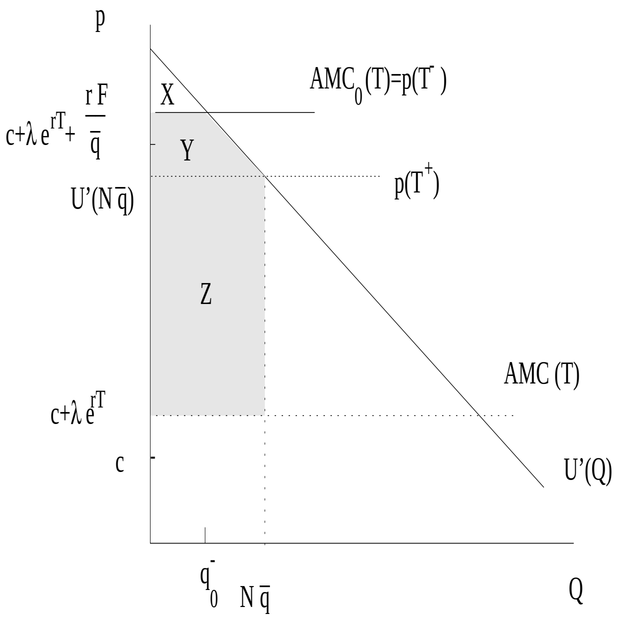

Figure 2: Augmented marginal cost and net surplus at T if the set-up cost is large and $p(t)$ is discontinuous. The increase in augmented surplus, Area(Y+Z)=$ANS^+ - ANS^- = rNF$, accrues partly to producers and partly to consumers. Therefore, the marginal benefit jumps down at T.

$c + \lambda e^{rT} + \frac{rF}{\bar{q}} < U'(N\bar{q})$. The augmented net surplus before T, ANS^-, is the area above the augmented marginal cost curve and below the demand curve, i.e., Area (A+B+C). Since $AMC_0(T) > AMC(T)$, production from the constrained deposits generates additional surplus. At T, this additional surplus is Area(D)=rNF and accrues entirely to the producers. In Figure 2, the set-up cost is large such that $c + \lambda e^{rT} + \frac{rF}{\bar{q}} > U'(N\bar{q})$. In this case the increment to surplus at T, Area(Y+Z)=rNF, is split by producers and consumers. This jump in consumer surplus implies that the marginal benefit path is discontinuous at T.

If the marginal benefit path is continuous, it can be written

$$p(t) = \begin{cases} c_0 + \lambda_0 e^{rt} & \text{if } t \in [0, \bar{T}] \\ U'(N\bar{q}) & \text{if } t \in [\bar{T}, \hat{T}] \\ c + \lambda e^{rt} & \text{if } t \in [\hat{T}, \infty) \end{cases} \tag{6}$$

where S_0 is exhausted at $\bar{T}$, and $\hat{T}$ is the time when the augmented marginal cost of the

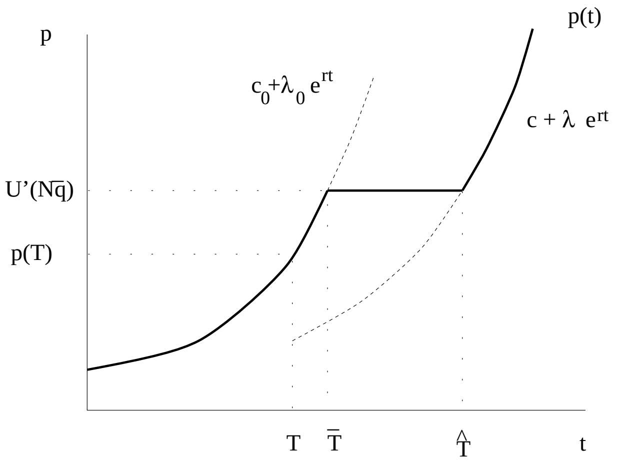

Figure 3: Continuous marginal benefit path $p(t)$ for the constrained planner's problem if $c + \lambda e^{rT} + \frac{rF}{\bar{q}} < U'(N\bar{q})$. Note that $p(T)$ exceeds the augmented marginal cost of the constrained deposits by $\frac{rF}{\bar{q}}$.

constrained deposits rises above $U'(N\bar{q})$ so that extraction is below capacity.[23] This marginal benefit path is illustrated in Figure 3. Let $D(p)$ be the inverse marginal utility schedule, i.e., the demand curve. Optimal extraction from the unconstrained deposit is

$$q_0(t) = \begin{cases} D(c_0 + \lambda_0 e^{rt}) & \text{if } t \in [0, T] \\ D(c_0 + \lambda_0 e^{rt}) - N\bar{q} & \text{if } t \in (T, \bar{T}] \\ 0 & \text{if } t \in (\bar{T}, \infty) \end{cases} \tag{7}$$

and extraction from the constrained deposits is

$$q(t) = \begin{cases} 0 & \text{if } t \in [0, T] \\ N\bar{q} & \text{if } t \in (T, \hat{T}] \\ D(c + \lambda e^{rt}) & \text{if } t \in (\hat{T}, \infty) \end{cases} \tag{8}$$

These extraction paths are illustrated in Figure 4. Note that this program has simultaneous extraction, not only from all the constrained deposits, but also from all producers between T and $\bar{T}$.

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[23] The endogenous shadow values are fully specified by the equations insuring continuity at $\bar{T}$ and $\hat{T}$, the first order condition in equation 5, and the stock constraints: $\int_0^\infty q_0(t)dt = S_0$ and $\int_0^\infty q(t)dt = NS$.

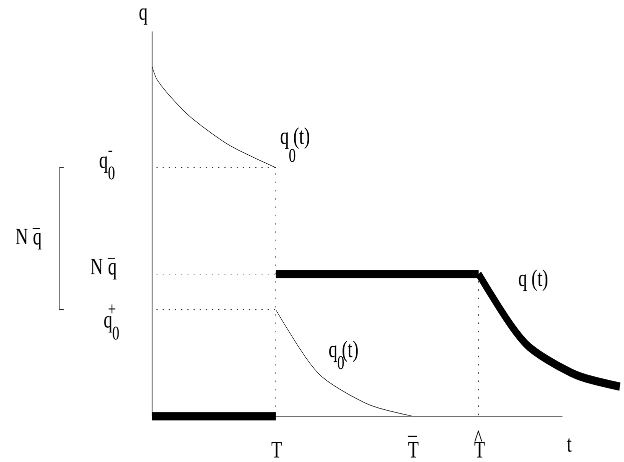

Figure 4: Optimal extraction paths if the marginal benefit path is continuous.

Lemma 1 shows that $p(t)$ is continuous if the marginal benefit at capacity exceeds the sum of the marginal extraction cost, a scarcity increment λe^{rT}, and the interest payment $\frac{rF}{\bar{q}}$. The following lemma restates this condition solely in terms of exogenous parameters.

Lemma 2 *If $c + \frac{rF}{\bar{q}} \le U'(N\bar{q})$ and $S \ge S^{min}$ then $c + \lambda e^{rT} + \frac{rF}{\bar{q}} \le U'(N\bar{q})$ where*

$$S^{min} = \frac{\bar{q}}{r}\left(\ln(U'(N\bar{q}) - c) - \ln(U'(N\bar{q}) - c - \frac{rF}{\bar{q}})\right) + \frac{1}{N}\int_0^{\infty} D(c + (U'(N\bar{q}) - c)e^{rt})dt \quad (9)$$

Proof:
When S is smaller, its shadow value, λ, and scarcity cost, λe^{rt}, are larger. Therefore, there is a minimum stock, S^{min}, for which the condition holds with equality, i.e., for which $c + \lambda e^{rT} + \frac{rF}{\bar{q}} = U'(N\bar{q})$. Using this equation and the fact that $c + \lambda e^{r\hat{T}} = U'(N\bar{q})$, it follows that $r(\hat{T} - T) = \ln(U'(N\bar{q}) - c) - \ln(U'(N\bar{q}) - c - \frac{rF}{\bar{q}})$. Next note that when extraction from the constrained deposits is below capacity, i.e., $t \ge \hat{T}$, the amount extracted from these deposits is independent of S. This amount, A, can be written as a function of the exogenous parameters:[24]

$$A = \int_{\hat{T}}^{\infty} D(c + \lambda e^{rt})dt = \int_0^{\infty} D(c + (U'(N\bar{q}) - c)e^{rt})dt.$$

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[24] If the choke price, $U'(0)$, is finite, then all deposits must be exhausted at time T^c when the price path reaches the choke price, i.e., $c + \lambda e^{rT^c} = U'(0)$. Thus the amount extracted while the constrained deposits produce below capacity can be written:

$$A = \int_0^{T^c - \hat{T}} D(c + (U'(N\bar{q}) - c)e^{rt})dt$$

Note that since $e^{r(T^c - \hat{T})} = \frac{U'(0) - c}{U'(N\bar{q}) - c}$, A is still a function solely of exogenous parameters.

Noting that S^{min} equals $\bar{q}(\hat{T} - T) + \frac{1}{N}A$ completes the derivation. ■

Lemma 2 translates a condition on augmented producer surplus into two conditions: one on producer surplus and another on deposit size. The first condition ensures average cost is less than the price for each period in which firms extract. The second condition ensures that the stock is large enough that the producer surplus covers the entire set-up cost before the stock is exhausted.[25]

The first proposition demonstrates that the conditions of Lemma 2 are precisely those required for existence of a competitive equilibrium.

Proposition 1 *A competitive equilibrium exists if* $c + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ *and* $S \geq S^{min}$ *where* S^{min} *is defined in equation 9.*

Proof:

Consider the price path, $p(t)$, as presented in equation 6. Lemmas 1 and 2 imply that $p(t)$ is continuous, and the production allocations are as in equations 7 and 8. Since $p(t)$ equals the marginal benefit of consumption in every period, consumers are optimizing. Next consider the owner of the unconstrained deposit S_0. The present value of the profit per unit equals λ_0 in each period with positive extraction and would be smaller in periods with no extraction. Therefore, the owner of the unconstrained deposit is maximizing profit given $p(t)$. Finally consider firm n, the owner of one of the constrained deposits, S_n. Can firm n increase profit by decreasing extraction in some period and increasing extraction in another period? Firm n would want to decrease extraction after $\hat{T}$, since the present value of profit per unit, λ, is lowest then. The firm cannot increase extraction in the interval $[T, \hat{T}]$ since extraction is already at capacity. In order to increase production before T, the capacity must be installed earlier. If firm n installs capacity at T_n, the Lagrangian for its profit maximization is[26]

$$L_n = \int_{T_n}^{\infty} e^{-rt}[p(t) - c]q_n(t) - \lambda q_n(t) + \mu(t)[\bar{q} - q_n(t)]\, dt - e^{-rT_n}F + \lambda S_n \qquad (10)$$

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[25] To understand the intuition behind these two conditions, consider the case where the surplus condition holds with equality, i.e., $c + \frac{rF}{\bar{q}} = U'(N\bar{q})$. In this case, the maximum producer surplus that a competitive firm can earn while producing at capacity is rF. If the firm is to earn sufficient producer surplus to cover the entire set-up cost, the firm would need to produce forever. Thus the "minimum" deposit size would need to be infinite, i.e., a backstop.

[26] The marginal profit from an extra unit of stock is λ since the unit must be extracted when capacity is not binding. Thus the firm's shadow value equals the shadow value from the optimization of the constrained planner's problem.

The derivative with respect to T_n of the Lagrangian is

$$\frac{dL_n}{dT_n} = -e^{-rT_n}\left[(p(T_n) - c - \lambda e^{rT_n})\bar{q} - rF\right]$$

Therefore, the profit maximizing time to install the capacity is when $p(T_n) = c + \lambda e^{rT_n} + \frac{rF}{\bar{q}}$ which implies that $T_n = T$. Thus firm n cannot increase profits by reallocating extraction. Can the firm increase profit by never incurring the set-up cost and earning zero profit? Using a calculation from the appendix shows that firm n's profit:

$$\begin{aligned}\pi_n = \int_T^\infty e^{-rt}[p(t) - c]\frac{1}{N}q(t)dt - e^{-rT}F &= \int_T^\infty [\lambda + \mu(t)]\frac{1}{N}q(t)dt - e^{-rT}F \\ = \lambda S + \left[e^{-rT}\frac{F}{\bar{q}} + (\bar{T} - T)\lambda_0 - (\hat{T} - T)\lambda\right]\bar{q} - e^{-rT}F \\ = \lambda\frac{1}{N}A + \lambda_0\bar{q}(\bar{T} - T) \geq 0\end{aligned}$$

is non-negative. Therefore, firm n is maximizing profit given $p(t)$. Thus, $p(t)$, $q_0(t)$, and $q(t)$ form a competitive equilibrium, and a competitive equilibrium exists. ■

The competitive equilibrium in Proposition 1 is characterized by the price path in Equation 6 and the extraction paths in Equation 7 and 8. The price path is either constant or grows at the rate of interest net of extraction costs. Note that after S_0 is exhausted, industry capacity is fully utilized. However, the set-up costs are incurred before S_0 is exhausted. Thus the additional capacity is installed while the industry has excess capacity.

Since the above optimization problem constrained the planner to install all capacities simultaneously, the resulting allocation need not be efficient. However, under the sufficient conditions in Proposition 1, the allocation can be supported as a competitive equilibrium, and the First Welfare Theorem then ensures that the allocation is in fact efficient. Thus, these sufficient conditions imply that the constraint on the planner does not bind and we have the following corollary:

Corollary 1 *If* $c + \frac{rF}{\bar{q}} \leq U'(N\bar{q})$ *and* $S \geq S^{min}$ *then it is efficient to incur all set-up costs simultaneously.*

If the sufficient conditions in Proposition 1 do not hold, relaxing the constraint on the planner may increase welfare, i.e., simultaneous capacity installation may not be efficient. In this case, a competitive equilibrium cannot exist. To illustrate, consider the case where the constrained deposits are large (hence, the shadow values are small), $U'(\bar{q}) > c + \frac{rF}{\bar{q}}$, and production from a second firm would not generate enough surplus to cover its capacity installation costs.[27] As in the Hartwick *et al.* solution, the efficient shadow values of the identical, constrained deposits are not equal. The capacity for S_1 is efficiently installed when the marginal benefit path reaches $c + \lambda_1 e^{rT_1} + \frac{rF}{\bar{q}}$. The first deposit, S_1, is then extracted until extraction from the second deposit would generate the additional surplus rF, i.e., $ANS_2^- + rF = ANS_2^+$. As in Hartwick *et al.*, S_1 is exhausted when the new capacity is installed. Although augmented surplus must increase when each new capacity is installed, consumption does not change if the capacity is binding. However, if capacity is not binding, marginal benefit must jump down as in Hartwick *et al.* This marginal benefit path is illustrated in Figure 5. Once capacities no longer bind, the marginal benefit follows a sawtooth pattern.

3 Endogenous Extraction Capacity

The previous section demonstrated sufficient conditions for existence of a competitive equilibrium when extraction capacity is exogenous. Now the model is extended to include the choice of extraction capacity. For simplicity, again assume that the N firms which install capacity are identical, i.e., have the same capacity installation technology and the same initial stock. Let $F(\bar{q})$ be each firm's identical set-up cost of installing capacity $\bar{q}$ where F is

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[27] Production from a second firm would not generate enough surplus to cover capacity installation if $U(2\bar{q}) - U(\bar{q}) < rF + c\bar{q}$. This implies $c + \frac{rF}{\bar{q}} > U'(N\bar{q})$ for $N \geq 2$.

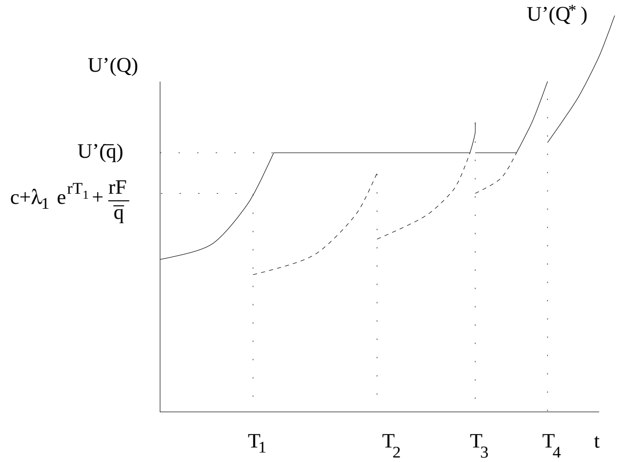

Figure 5: Marginal benefit path if capacities are large such that only one firm enters the market at a time. Note the sawtooth pattern of the augmented marginal costs.

differentiable, $F(0) = 0$, $F' > 0$, and $F'' > 0$. The production sets are still non-convex since each firm must choose a single time to install its capacity. To illustrate the non-convexity, consider the following two feasible, input-output extraction paths,[28] A and B, and their convex combination $\frac{1}{2}$A+$\frac{1}{2}$B

		Period			
Path	1	2	3	4	...
A	$(\frac{1}{2}cS + F(\frac{1}{2}S), \frac{1}{2}S)$	$(\frac{1}{2}cS, \frac{1}{2}S)$	$(0, 0)$	$(0, 0)$	....
B	$(0, 0)$	$(cS + F(S), S)$	$(0, 0)$	$(0, 0)$	....
$\frac{1}{2}$A + $\frac{1}{2}$B	$(\frac{1}{4}cS + \frac{1}{2}F(\frac{1}{2}S), \frac{1}{4}S)$	$(\frac{3}{4}cS + \frac{1}{2}F(S), \frac{3}{4}S)$	$(0, 0)$	$(0, 0)$	....

where the pairs represent the money input and output in each period. Path A installs extraction capacity $\frac{1}{2}S$ in period 1 and extracts $\frac{1}{2}S$ in periods 1 and 2 and nothing in all subsequent periods. Path B extracts nothing in period 1; then installs capacity S and extracts S in period 2; and finally extracts nothing in all subsequent periods. Is the convex

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[28] A feasible extraction plan consists of money input and output in each period such that: money input is sufficient to cover the extraction costs, capacity is installed before any extraction takes place, and extraction constraints are not violated.

combination of these two paths, $\frac{1}{2}$A+$\frac{1}{2}$B, feasible? Extraction in period 1 of the combination path is feasible since there are sufficient inputs to extract $\frac{1}{4}S$ and to install the capacity $\frac{1}{4}S$ since $F(\frac{1}{4}S) \leq \frac{1}{2}F(\frac{1}{2}S)$. The infeasibility is in the second period. The plan calls for extraction of $\frac{3}{4}S$, however there is not sufficient capacity to extract $\frac{3}{4}S$. Thus the plan is infeasible, and the production set is not convex.[29]

To show existence of a competitive equilibrium, define a price path and allocation in terms of the constrained planner's problem where as above all capacities must be installed simultaneously. If capacities are installed simultaneously, each deposit should have identical capacity. Thus the constrained planner's problem is as given in equation 1 with the modifications that optimization is now also with respect to $\bar{q}$ and that the set-up cost, F, is a function of $\bar{q}$. As above, define $p(t)$ as the marginal benefit of optimal consumption, i.e., $p(t) \equiv U'(Q^*(t))$. As in Section 2, conditions for optimal extraction are as given in equation 3 and the condition for optimal capacity installation is $ANS^- + rNF(\bar{q}^*) = ANS^+$. Additionally, the Kuhn-Tucker condition for optimal capacity choice is

$$\bar{q}^* \geq 0 \quad -e^{-rT}NF'(\bar{q}^*) + \int_T^\infty N\mu(t)dt \leq 0 \quad C.S. \tag{11}$$

where $\bar{q}^*$ is the optimal capacity choice. Equation 11 implies that the marginal cost of installing capacity should equal the discounted sum of the shadow values of the extraction constraint over all periods in which it binds.[30]

As above, the following lemmas are used to describe the marginal benefit path:

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[29] In the technology assumed here, capacity must be chosen at the outset and cannot subsequently be expanded. Thus the extra inputs in period 2 cannot be used. Similar models by Switzer and Salant (1986), Olsen (1989) and Lozada (1993) allow for capacity to be expanded at any time and have convex production sets.

[30] In peak load pricing problems, a similar condition holds which requires that the capacity cost be spread across all the peak periods in which the capacity binds.

Lemma 3 *If the optimal capacity choice in the constrained planner's problem is positive, then* $c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} \leq U'(N\bar{q}^*)$.

Proof:
Suppose $c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} > U'(N\bar{q}^*)$. Lemma 1 then implies that $p(t)$ is discontinuous at T, and extraction is strictly sequential. The integral over the shadow value of the capacity constraint is then given by the second calculation in the appendix:

$$\int_T^\infty \mu(t)dt = \frac{1}{r}\left[e^{-rT}(U'(N\bar{q}^*) - c) - \lambda\right] - (\hat{T} - T)\lambda$$

But then

$$\int_T^\infty \mu(t)dt < \frac{1}{r}\left[e^{-rT}(c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} - c) - \lambda\right] - (\hat{T} - T)\lambda$$

$$= e^{-rT}\frac{F(\bar{q}^*)}{\bar{q}^*} - (\hat{T} - T)\lambda \leq e^{-rT}F'(\bar{q}^*)$$

This strict inequality implies, by the Kuhn-Tucker condition in equation 11, that $\bar{q}^* = 0$. ■

Lemma 4 *The optimal capacity choice in the constrained planner's problem is positive if and only if* $c + rF'(0) < U'(0)$.

Proof:
Note that in Lemma 2, the limit of the minimal stock as the capacity becomes smaller is zero, i.e., $\lim_{\bar{q}\to 0} S^{min} = 0$. Note also that $\lim_{\bar{q}\to 0} \frac{F(\bar{q})}{\bar{q}} = F'(0)$. Thus if $c + rF'(0) < U'(0)$, there exists a small capacity, $\bar{q}'$, such that $c + \frac{rF(\bar{q}')}{\bar{q}'} \leq U'(N\bar{q}')$ and $S \geq S^{min}$. But then Lemma 2 implies that installing capacity $\bar{q}'$ and extracting the constrained deposits yields a higher maximand to the constrained planner's problem than installing zero capacity. Hence the optimal capacity must be positive.

Conversely, if $c + rF'(0) \geq U'(0)$, then $\bar{q}^* = 0$. To show this, suppose the optimal capacity were greater than zero, i.e., $\bar{q}^* > 0$. But then

$$c + \lambda e^{rT} + \frac{rF(\bar{q}^*)}{\bar{q}^*} \geq c + \frac{rF(\bar{q}^*)}{\bar{q}^*} \geq c + rF'(0) \geq U'(0) > U'(N\bar{q}^*)$$

This contradicts Lemma 3, hence $\bar{q}^* = 0$. ■

If positive capacities are installed, Lemma 3 shows that the marginal benefit path is continuous. Lemma 4 shows that if the choke price is higher than the extraction cost plus the interest payment on installation of a marginal unit of capacity, then it is efficient to install positive capacity. The proposition on existence can now be shown:

Proposition 2 *If set-up costs are the endogenous costs of installing capacity, a competitive equilibrium exists.*

Proof:
If $c+rF'(0) \geq U'(0)$, the price path $p(t) = \min\{c_0+\lambda_0 e^{rt}, U'(0)\}$ is a competitive equilibrium price path, where no capacity is installed and the constrained deposits are not extracted.

If $c+rF'(0) < U'(0)$, consider the price path $p(t)$ defined from the above constrained planner's problem. Lemma 4 ensures that positive capacity is installed. Lemma 3 together with Lemma 1 imply that $p(t)$ is continuous and hence is as defined in equation 6 and illustrated in Figure 3. As above, consumers and the owner of the unconstrained deposit are optimizing given $p(t)$. The Lagrangian for an owner of a constrained stock is in equation 10 where the set-up cost is $F(\bar{q})$ and the firm chooses $q_n(t)$, $\bar{q}_n$, and T_n. As argued in the proof of Proposition 1, profits cannot be increased by changing $q_n(t)$ given the capacity and time of installation. The first order conditions for profit maximizing T_n and $\bar{q}_n$ are

$$\frac{dL_n}{dT_n} = -e^{-rT_n}\left[(p(T_n) - c - \lambda e^{rT_n})\bar{q}_n^* - rF(\bar{q}_n^*)\right] = 0$$

and

$$\frac{dL_n}{d\bar{q}_n} = \int_{T_n}^{\infty} \mu_n(t)dt - e^{-rT_n}F'(\bar{q}_n^*) = 0$$

These are precisely the first order conditions for the constrained planner's problem. Thus, $T_n = T$, $\bar{q}_n^* = \bar{q}^*$, and firms are maximizing profits given $p(t)$. Therefore, this price and allocation is a competitive equilibrium, and hence a competitive equilibrium exists. ■

Proposition 2 shows that a competitive equilibrium exists for the model with endogenous extraction capacity. In this equilibrium the capacities are installed while the price is rising and aggregate extraction is less than industry capacity. Furthermore, firms choose extraction capacities which are small enough that there is a period of simultaneous extraction with the unconstrained deposit. By choosing small capacity, a firm can decrease its capacity installation cost. However, with a small flow capacity, it takes longer to extract the stock. The optimal capacity choice balances these costs and benefits and thus ensures that profits are non-negative. As above, the First Welfare Theorem implies:

Corollary 2 *If set-up costs are the endogenous costs of installing capacity, it is efficient to incur identical set-up costs simultaneously.*

4 Extensions and Generalizations

The model can be generalized to analyze capacity installation for multiple deposits with different extraction and capacity-installation costs. Recall that in the competitive equilibrium, capacities were installed while the price was rising (see Figure 3), and aggregate extraction was below industry capacity. For multiple deposits with different extraction and capacity-installation costs, the equilibrium has the following properties: a) new capacities are installed while the price is rising and the industry has unutilized capacity; b) after some stocks are exhausted, extraction capacity becomes fully utilized and the price is constant; and c) after the price is constant for a time, the scarcity costs increase and it is again efficient to extract below capacity causing the price to rise and extraction capacity to be installed for other deposits. Thus, the price path follows a stair-step pattern: rising when industry capacity is not binding and flat when capacity is fully utilized. The results of Section 3 generalize to show that this price path is a competitive equilibrium price path.

A competitive equilibrium might not exist if extraction is unlimited and the set-up costs determine stock size, e.g., construction costs for lining a landfill determine the total amount of garbage which can be landfilled. To see this, modify the model presented in equation 1 such that the set-up cost is a convex function of the stock size, i.e., $F(S_n)$, and there are no flow constraints. The first order condition for the optimal stock choice, $\lambda_n = e^{-rT_n} F'(S_n^*)$, defines the efficient stock size S_n^* if F is convex. The efficient extraction plan is then identical to the plan for a model with exogenous stocks S_n^*, set-up costs $F(S_n^*)$, and unlimited extraction. As shown by Fischer, a competitive equilibrium cannot exist in this model.

With constant marginal extraction costs and no extraction constraints, firms never receive any augmented surplus. If marginal extraction costs are increasing in current extraction, then producers may capture sufficient surplus such that consumer surplus does not jump. Thus, increasing marginal extraction costs have the same implications as extraction constraints, namely: under certain conditions, efficient extraction will be simultaneous and a competitive equilibrium will exist.

Finally, an important limitation is that a competitive equilibrium may not exist in this model—even with endogenous capacity choice—if demand shifts over time. Consider the simple example of one firm with an infinite stock, i.e., a backstop. Let demand in all periods have the same choke price, but let demand increase dramatically after the first period and then be stationary thereafter. Suppose the surplus in the first period is small enough that it is efficient to wait until the second period to install capacity. Can this efficient production path be supported as a competitive equilibrium? The answer is no, because a price does not exist which clears the market in the first period. Since first-period consumption must be zero, the price must be greater than or equal to the choke price. However, firms seeing this high price would want to install capacities and produce in the first period. Thus the market does not clear, and a competitive equilibrium does not exist. In spite of this counter-example, a competitive equilibrium may still exist in models with demand shifts, if the demand shifts are not too extreme. Alternatively, if capacity can be expanded in multiple periods, then convexity ensures existence of a competitive equilibrium.

5 Conclusion

Although set-up costs are prevalent and substantial in natural resource extraction, the previous literature has shown that their inclusion in the analysis implies that a competitive equilibrium cannot exist. However, I show that this result is sensitive to the assumption of unlimited extraction capacity by showing that a competitive equilibrium can exist when capacity is constrained. Specifically, a competitive equilibrium exists if capacity is constrained such that: i) when firms are extracting at capacity, the price covers the extraction and interest costs, and ii) the firm's deposit is large enough that costs are recovered before the deposit is exhausted. Furthermore, a competitive equilibrium exists if firms choose extraction capacity provided demand does not shift dramatically. The equilibrium is characterized by a price path which grows at the rate of interest when some firms are extracting below capacity and is constant when capacity is fully utilized. Firms install capacity when the price reaches a level which allows firms to earn normal scarcity rents while covering their extraction and set-up costs. Firms install capacity when there is excess capacity (and the price is rising) rather than when capacity is fully utilized.

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Appendix

To calculate the integral of the shadow value of the capacity constraint if all capacities are installed simultaneously and $c+\lambda e^{rT}+\frac{rF}{\bar{q}} \leq U'(N\bar{q})$, first note that equation 3 implies $\mu(t)=e^{-rt}(p(t)-c)-\lambda$ when there is extraction from the constrained deposit. Substituting for $p(t)$, recalling that $p(T)=c+\lambda e^{rT}+\frac{rF}{\bar{q}}$, and noting that $\mu(t)$ is continuous yields:

$$\mu(t)=\begin{cases} e^{-rT}\frac{rF}{\bar{q}} & \text{if } t=T \\ e^{-rt}(c_0-c)+\lambda_0-\lambda & \text{if } t\in[T,\bar{T}] \\ e^{-rt}(\bar{p}-c)-\lambda & \text{if } t\in[\bar{T},\hat{T}] \\ 0 & \text{if } t\in[\hat{T},\infty] \end{cases}$$

where $\bar{p}\equiv U'(N\bar{q})$. Thus:

$$\int_T^{\infty}\mu(t)dt=\int_T^{\bar{T}} e^{-rt}(c_0-c)+(\lambda_0-\lambda)dt+\int_{\bar{T}}^{\hat{T}} e^{-rt}(\bar{p}-c)-\lambda\ dt$$

$$=\frac{1}{r}(e^{-rT}-e^{-r\bar{T}})(c_0-c)+(\bar{T}-T)(\lambda_0-\lambda)+\frac{1}{r}(e^{-r\bar{T}}-e^{-r\hat{T}})(\bar{p}-c)+(\hat{T}-\bar{T})(-\lambda)$$

$$=\frac{1}{r}\left[\mu(T)-\lambda_0+\lambda-[\mu(\bar{T})-\lambda_0+\lambda]\right]+\frac{1}{r}\left[\mu(\bar{T})+\lambda-[\mu(\hat{T})+\lambda]\right]+(\bar{T}-T)\lambda_0-(\hat{T}-T)\lambda$$

$$=\frac{1}{r}[\mu(T)]+(\bar{T}-T)\lambda_0-(\hat{T}-T)\lambda$$

$$=e^{-rT}\frac{F}{\bar{q}}+(\bar{T}-T)\lambda_0-(\hat{T}-T)\lambda$$

If $c+\lambda e^{rT}+\frac{rF}{\bar{q}}>U'(N\bar{q})$, Lemma 1 implies that optimal extraction has no phase of simultaneous extraction between the unconstrained deposit and all the constrained deposits, i.e., $T=\bar{T}$. The integral of the shadow values is then:

$$\int_T^{\infty}\mu(t)dt=\int_T^{\hat{T}} e^{-rt}(\bar{p}-c)-\lambda\ dt$$

$$=\frac{1}{r}(e^{-rT}-e^{-r\hat{T}})(\bar{p}-c)+(\hat{T}-T)(-\lambda)$$

$$=\frac{1}{r}[\mu(T)]-(\hat{T}-T)\lambda$$

$$=\frac{1}{r}\left[e^{-rT}(\bar{p}-c)-\lambda\right]-(\hat{T}-T)\lambda$$