Some Basics on Life-cycle Consumption
(Notes from Deaton, 1992)

A. Introduction

1. Will examine consumption over time
2. A principal property of these models is that they allow people to transfer resources over time through borrowing and saving, which leads to consumption smoothing
3. To introduce these models, we will consider some simple cases
    a. starting with a two-period model with no uncertainty
    b. multiple-period certainty case with additive preferences
    c. multiple-period case with uncertainty

B. Simple two-period model with certainty

1. Consider a person who faces a choice of how much consumption to allocate across two periods
    a. preferences: person's preferences are defined over period 1 consumption, C_1, and period 2 consumption, C_2, by the utility function $U(C_1, C_2)$, which is increasing in both of its arguments
    b. resources: person has three resources that are received with certainty
        1) an initial endowment of wealth (inheritance), A_1, that is received at the start of period 1
        2) income or earnings that are received in each

period, y_1 and y_2

3) interest on savings (or debt) from period 1 that are received in period two; let the interest rate be r_2

c. <u>two-period budget constraint</u>: consider how the person's budget evolves

1) to simplify the model, let's ignore prices of consumption (equivalent to assuming that consumption is expressed as real expenditures and all other values are expressed in real terms)

2) in the first period, person begins with $A_1 + y_1$

3) savings or debts at the end of the first period are the difference between first period consumption and resources; so, wealth at the end of the first period is $A_2 = A_1 + y_1 - C_1$

4) assume there are no liquidity constraints, that is, assume there are no restrictions on savings or debts

5) resources in the second period are $(1+r_2)A_2 + y_2$

6) assume that the person cannot leave debts at the end of period 2

7) with these assumptions the (two-period) life-time budget constraint is

$$C_1 + \frac{C_2}{1+r_2} \leq A_1 + y_1 + \frac{y_2}{1+r_2}$$

a) left side of expression is the present discounted value of life-time consumption; notice how $(1+r_2)^{-1}$ serves as a price on period two consumption

b) right side of expression is the present discounted value of life-time income

2. The person's allocation problem is to choose C_1 and C_2 to maximize utility subject to the life-time budget constraint
   a. straightforward extension of standard consumer choice problem
   b. consider the optimal responses, C_1^* and C_2^*
   c. what happens if there is an increase in one of the income terms, say y_1
      1) increases resources available
      2) will increase C_1 and increase C_2 (extra resources transferred from period 1 to period 2 through savings); standard income effect
      3) note that the increases in each period's consumption are necessarily less than the increase in income; consumption is smoother than in the static model
   d. are things any different if there is an increase in y_2?
   e. now consider an increase in the interest rate, r_2
      1) $r_2\uparrow \rightarrow (1+r_2)^{-1}\downarrow$
         a) the increase in the interest rate effectively reduces the price of consumption in period two
         b) consumption in period two becomes less expensive relative to consumption in period one
         c) substitution effect
         d) however, because the substitution effect refers to consumption in different periods, we refer to

it as an *intertemporal substitution effect*

e) from this effect period 2 consumption increases while period 1 consumption decreases

2) the fall in the effective price also means that more consumption is available generally

a) standard income effect

b) leads to increases in consumption in both periods

3) despite some slightly different terminology, both of these effects are "standard" effects found in other consumption models; note, however there is an additional effect

a) in standard consumer choice models, income is fixed; in this model, however, full income (the complete right-side expression in the life-time budget constraint) changes with the interest rate

b) the decrease in $(1+r_2)^{-1}$ means that the present discounted value of life-time resources is lower

c) thus, the income effect from the effective price increase in consumption is offset by the decrease in the present discounted value of period 2 earnings

d) Deaton refers to this as the "human capital effect"

C. Model with multiple periods and intertemporal separability

1. Instead of two periods, now consider a general model with

a finite but arbitrary number of periods T

a. use t to denote the specific periods

b. the person's preferences are defined over consumption in each of the periods $U(C_1, C_2, \ldots, C_T)$

c. to simplify the analysis, assume that the interest rate is a constant, r, over time

d. we will maintain all of our other assumptions

  1) interest rate, (implicit) prices, and preferences are known with certainty

  2) unrestricted borrowing and saving

  3) initial wealth is A_1 and terminal wealth is zero

e. the life-time budget constraint is then

$$\sum_{t=1}^{T} \frac{C_t}{(1+r)^{t-1}} = A_1 + \sum_{t=1}^{T} \frac{y_t}{(1+r)^{t-1}}$$

f. analysis of this model is similar to the two-period model but more complicated

  1) still a generalization of consumer choice model

  2) income effects now distributed across multiple periods (in general, income effects in any one period become very small)

  3) intertemporal substitution effects are now also distributed across multiple periods

g. consumption in any one period depends on conditions in ALL periods (e.g., life-time pattern of income); for empirical analyses this represents a huge data burden

h. few predictions possible because of the myriad possibilities for substitutability and complementarity among the goods from the unrestricted preference function

2. To put additional structure on the model, assume that
   a. preferences are strongly (additively) intertermporally separable such that

$$U = u_1(C_1) + u_2(C_2) + \ldots + u_T(C_T)$$

      1) each of these sub-utility functions has the properties of a regular static utility function (e.g., is increasing in consumption, has an associated indirect sub-utility function, etc.)
      2) specification abstracts from habit formation and other time dependencies
   b. moreover, assume that each of the sub-utilities can be specified as

$$u_t(C_t) = (1+\delta)^{1-t}\, u(C_t, Z_t)$$

      1) δ is a subjective discount rate
      2) Z_t represents characteristics that shift preferences, such as age, family size, health conditions, etc.
   c. let λ be the Lagrange multiplier, we can rewrite the consumer's problem as choosing $C_1, C_2, \ldots, C_T$ to maximize

$$\sum_{t=1}^{T}\frac{1}{(1+\delta)^{t-1}}u(C_t, Z_t) + \lambda\left[A_1 + \sum_{t=1}^{T}\frac{y_t}{(1+r)^{t-1}} - \sum_{t=1}^{T}\frac{C_t}{(1+r)^{t-1}}\right]$$

   d. the first-order conditions for each period are simply

$$u'(C_t, Z_t) = \lambda\left(\frac{1+\delta}{1+r}\right)^{t-1}$$

1) consumption decisions are made to keep the marginal utility of consumption equal to the discounted constant marginal value of wealth
2) consumption in each period depends ONLY on
   a) characteristics from that period
   b) the subjective and economic discount rates, and
   c) the constant marginal value of wealth – changes in characteristics from other periods ALL enter through the marginal value of wealth, λ
3) refer to these as Frisch, or λ-constant, consumption/demand functions
4) this is an incredibly useful simplification

e. if we suppose further that the subjective and economic discount rates are the same ($\delta = r$), the first-order condition simplifies further to $u'(C_t, Z_t) = \lambda$
   1) conditional on Z_t, people in this model behave to keep the marginal utility of consumption constant
   2) if Z_t were constant, consumption would also be constant across the life-cycle, regardless of the life-cycle path of income – complete consumption smoothing

f. more generally, if we assume that δ and r are close to each other and if we condition on Z_t, consumption evolves according to

$$\frac{\dot{C}_t}{C_t} = \frac{-u'}{C_t u''}(r-\delta)$$

g. if we further assume that preferences follow a CES specification, $u(C_t) = (1-\rho)\, C_t^{1-\rho}$ with $\rho > 0$, then percentage changes in consumption follow $\rho^{-1}\,(r-\delta)$

D. Uncertainty

1. The previous models help to illustrate consumption smoothing and life-cycle consumption patterns but rely on the unrealistic assumption that future incomes, interest rates, needs, and the like are known with certainty
   a. doesn't matter whether you solve the model at period 1, t, or T; the solutions are all the same
   b. no updating in the model
2. The standard way to incorporate uncertainty is to replace the life-time utility function with an expected utility function
   a. maintain the additivity assumption
   b. rewrite the function from the current period (t) going forward
   c. function can be written

$$U^* = \mathrm{E}\left[\sum_{s=t}^{T} u_s(C_s, Z_s) \middle| I_t\right]$$

      1) here E is the expectations operator
      2) I_t represents the information available at period t
3. Person's decision comes down to choosing a level of

consumption this period, knowing

   a. that decision will affect subsequent outcomes through the level of wealth, A_{t+1}, that is passed on
   b. the person will be making subsequent consumption decisions

4. Let $V_t(A_t)$ be the value of expected life-time utility associated with the optimal choice of savings S_t
   a. we can rewrite $V_t(A_t)$ as

   $$V_t(A_t) = \max_{S_t} \left[u_t(A_t + y_t - S_t) + \mathrm{E}_t\left(V_{t+1}((1+r)S_t)\right)\right]$$

   b. for simplicity we are abstracting from Z_t and other things that could enter V and u
5. The optimal level of consumption satisfies

   $$u_t'(C_t) = (1+r)\mathrm{E}_t\left[V_{t+1}'(A_{t+1})\right] = (1+r)\mathrm{E}_t\left[u_{t+1}'(C_{t+1})\right]$$

6. If we make the additional assumptions that utility is the same over time and the subjective discount rate equals the interest rate, the optimal condition becomes

   $$u'(C_t) = \mathrm{E}_t\left[u'(C_{t+1})\right]$$

7. Once again we get consumption smoothing
   a. this time in expectations
   b. smoothing involves the marginal utilities of consumption
   c. depending on risk preferences, expected consumption might also be smoothed
   d. for some preference specifications this would imply $C_{t+1} = C_t + e_{t+1}$ where is e_{t+1} a random error

References:

Deaton, Angus. *Understanding Consumption*. Oxford: Clarendon Press, 1992.