

Compensating Wage Differentials

A. Hedonic Wage Theory – Labor Supply

1. General issues

- a. assume that goods, and more specifically jobs, are valued for a variety of utility-bearing qualities
- b. how do we find shadow prices and demands for the separate attributes when only the bundled good (job) and its total price (wage) are observed?

2. Assumptions

- a. a job can be described by a vector of N characteristics $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)$
 - 1) these possible job attributes include the risk of injury, location (e.g., lots of urban amenities), working conditions, risk of layoff, required hours
 - 2) separate from “fringe benefits” such as insurance coverage, pension, or vacation; these typically have separate explicit prices
- b. for convenience, we will describe all of the Z s as goods (e.g., job safety vs. risk of injury)
- c. there are a sufficiently large number of available jobs that people face a continuum of \mathbf{Z}
- d. workers and firms are atomistic and take the available wage as given
- e. characteristics of jobs cannot be unbundled; jobs are offered on a “take it or leave it” basis
- f. will consider only a single job per individual

g. people have preferences defined over the wage rate (general consumption) and the attributes such that $U = U(W, Z_1, Z_2, \dots, Z_N) = U(W, \mathbf{Z})$

h. wages are a function of the attributes such that $W = f(Z_1, Z_2, \dots, Z_N) = f(\mathbf{Z})$; we define $\partial W / \partial Z_i = f_i$

3. Worker's maximization problem

a. with these assumptions, we can write the worker's problem as

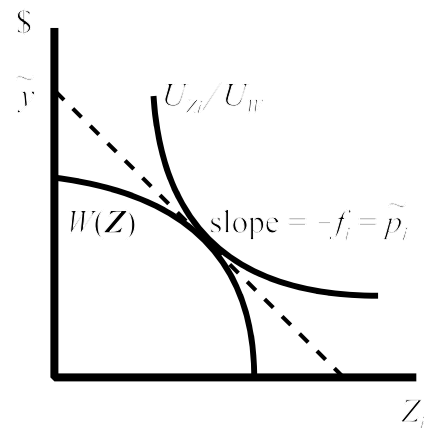
$$\text{Max}_{Z_1, Z_2, \dots, Z_N} U[f(Z_1, Z_2, \dots, Z_N), Z_1, Z_2, \dots, Z_N]$$

b. the first-order condition for an interior solution is

$$\frac{\partial U}{\partial Z_i} = U_W f_i + U_{Z_i} = 0 \quad \text{or} \quad \frac{U_{Z_i}}{U_W} = -f_i \quad \text{for } i = 1, N$$

c. graphically,

- 1) the worker's budget constraint is depicted as $W(\mathbf{Z})$
- 2) preferences can be described in terms of indifference curves between consumption and attribute i



- 3) slope at the tangency between the budget constraint and indifference curve can be interpreted as the “virtual price” of the attribute
- 4) can extend the budget line from the tangency to define a “virtual full income”

d. rewriting the problem in terms of virtual prices and incomes

1) at the solution, let

$$\tilde{p}_i = -f_i \quad \text{and} \quad \tilde{y} = w + \tilde{p}\mathbf{Z} = f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i$$

2) worker's problem becomes

$$\text{Max}_{W, \mathbf{Z}} U(W, \mathbf{Z})$$

$$\text{subject to } \tilde{y} = W + \tilde{p}\mathbf{Z}$$

3) holding \tilde{y} and \tilde{p} constant, the solutions can be written

$$Z_i = g(\tilde{p}, \tilde{y}) = g(f_1, f_2, \dots, f_N, f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i)$$

$$W = h(\tilde{p}, \tilde{y}) = h(f_1, f_2, \dots, f_N, f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i)$$

while this system is clearly simultaneous in the Z s, the properties of $g(\cdot)$ and $h(\cdot)$ are well-known

4) this approach is useful for describing behavior at the maximum (at a single point); however, it does not address selectivity of W and \mathbf{Z} combinations

B. Labor Demand

1. Firm's problem

a. assume that the only factor of production is labor

b. assume that job attributes can be provided at a cost $C = C(\mathbf{Z})$

- c. firm's maximize profits; so, the firm's problem can be written

$$\begin{aligned} \text{Max}_{L, \mathbf{Z}} PQ(L) - WL - C(\mathbf{Z}) \\ = PQ(L) - f(Z_1, \dots, Z_N)L - C(Z_1, \dots, Z_N) \end{aligned}$$

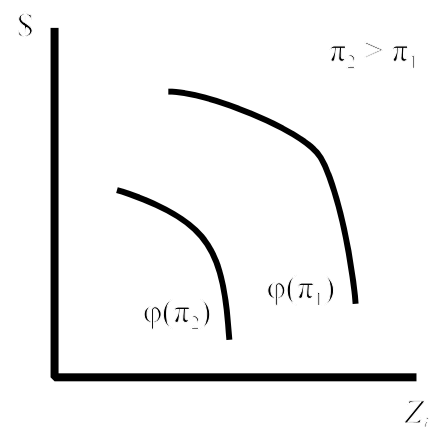
- d. for an interior solution, the first-order conditions are

$$\begin{aligned} L: \quad PQ_L = W \\ Z_i: \quad f_i = -C_i / L \end{aligned}$$

2. Offer functions

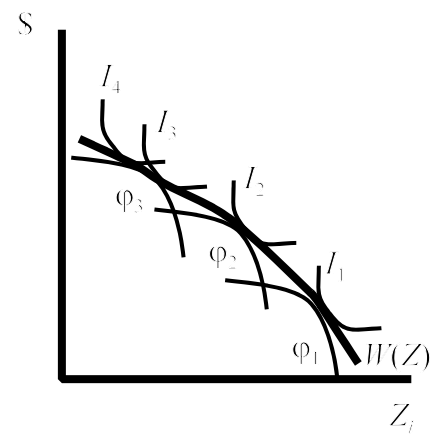
- define an offer function $\varphi(\mathbf{Z}, \pi)$ which satisfies $\pi = PQ(L) - \varphi L - C(\mathbf{Z})$ and $PQ_L = \varphi$
- the offer function represents all of the combinations of Z which the firm can offer and maintain the same profit level; this is similar to an iso-cost line
- graphically,

- for any given firm, we can define a family of offer functions
- firm will minimize the offer curves subject to the prevailing wage rate



C. Wage Determination

1. Assume
 - a. that there are several firms with offer curves $\varphi_1, \varphi_2, \dots, \varphi_K$
 - b. and several workers with indifference curves I_1, I_2, \dots, I_J
2. The equilibrium wage will be an envelop between the offer and indifference curves
3. Graphically,
 - a. there are a family of indifference curves
 - b. there are a family of offer curves
 - c. and assuming a continuum of people and firms, there is a smooth wage function



D. Estimation Issues

1. General issues
 - a. want to identify how differentiated qualities affect wages
 - b. also want to identify demand and supply functions for the individual attributes
2. Epple's critique
 - a. let prices be given by $p(\mathbf{Z})$
 - b. for each attribute Z_i , there is
 - 1) a demand function: $p_i(\mathbf{Z}) = f_i(\mathbf{Z}, \mathbf{X}_D, y, p(\mathbf{Z}))$

2) a supply function: $p_i(\mathbf{Z}) = g_i(\mathbf{Z}, \mathbf{X}_S, p(\mathbf{Z}))$

3) where X_D and X_S are observable demand and supply shifters

c. assume that there are K distinct markets with linear demand and supply relationships

$$p_k = \gamma_k + \psi_k' Z_k + \frac{Z_k' \pi_k Z_k}{2} + \zeta_k$$

$$\text{demand: } \frac{\partial p_k}{\partial Z_k} = A_D Z_k + H_D X_{Dk} + v_{Dk}$$

$$\text{supply: } \frac{\partial p_k}{\partial Z_k} = A_S Z_k + H_S X_{Sk} + v_{Sk}$$

d. consider linear attribute demand and supply functions

e. OLS in the attribute demand and supply equations is not consistent

1) to see this in the demand equation, solve for Z_k

$$Z_k = (\pi_k - A_D)^{-1} \left(H_D X_{Dk} + v_{Dk} - \psi_k' \right)$$

2) either Z_k or X_{Dk} depends on v_{Dk} and OLS applied to the attribute demand equation is inconsistent

f. consistent estimation of the price equation requires

$$E(\zeta_k v_{Dk}) = 0 \text{ and } E(\zeta_k v_{Sk}) = 0$$

3. Goddeeris (JPE 1988)

a. examines differences between public-interest and private attorneys

- b. hedonic characteristic is a dummy variable (similar to introduction in *Handbook* chapter by Rosen); greatly simplifies choice problem
 - c. uses endogenous dummy variable model to estimate the wage premium associated with choice of law firm
 - d. actual estimation procedure is complicated by the use of a choice-based sample
4. Hwang et al. (1992) use simulated data and show that estimates of compensating differentials can be very sensitive to unobserved characteristics such as productivity
5. Moffitt (JoLE, 1984)
- a. examines relationship between labor supply and a wage function which depends on hours worked (wage function in which hours are an endogenous determinant)
 - b. motivation for endogenous wages
 - 1) labor imposes quasi-fixed costs to firm; thus, low hours are associated with a low marginal rate (Oi, 1962)
 - 2) marginal productivity declines at high hours leading to lower wages (Barzel, 1973)
 - 3) result is an S-shaped budget constraint
 - c. considers
 - 1) linear labor supply function
 - 2) quadratic (in hours) earnings function
 - d. this specification leads to a nonlinear budget constraint

- e. solves estimation problem by estimating the budget constraint at 15 discrete points
 - 1) sensitivity check with different numbers of points
 - 2) also compares results with a linear budget constraint
- f. results
 - 1) rejects linear wage relationship
 - 2) use of a nonlinear hours function leads to lower wage elasticities
 - 3) nonlinear model provides a much better fit to the data

E. Other empirical applications

1. Gender differences in wages (Macpherson & Hirsch 1995)
 - a. Macpherson & Hirsch examined whether occupational differences in skills requirements and job disamenities could account for gender differences in wages
 - b. use “longitudinal” data from the CPS (CPS households can be linked over time)
 - c. match occupational information to characteristics from the Dictionary of Occupational Titles and other sources
 - 1) training requirements
 - 2) computer use
 - 3) strength requirements
 - 4) hazards

- 5) physical & environmental conditions
 - d. find differences in these characteristics between men's and women's occupations
 - e. also find that measured differences account for up to 2/3 of the occupational gender differences in wages
2. Local amenities
- a. Roback (1982) examined compensating differentials in wages and rents that were associated with local amenities
 - b. used CPS data for wages and FHA data for housing
 - c. obtained city-specific measures of
 - 1) crime
 - 2) pollution
 - 3) density
 - 4) climate (heating degree days, snow fall, cloudiness)
 - d. amenities are able to explain some of the regional differences in wages

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Homework

Consider a person who has preferences defined over her own consumption, C , and her safety on the job, S . Assume that preferences can be represented by the following Cobb-Douglas utility function

$$U = (1 - \beta) \ln C + \beta \ln S$$

where $0 < \beta < 1$. Consumption is constrained by the amount of earned income, $W(S)$, and unearned income, N . Earnings depend on the level of safety, such that

$$W(S) = \kappa - \tau S$$

where $\kappa > 0$ and $\tau > 0$.

- a. What level of safety will this person choose?
- b. How does the choice of safety vary with (i) the shadow price of safety, τ , (ii), the level of unearned income, N , and (iii) preferences for safety, β ?