

# Simple Static Labor Supply Model

## A. Quick Review of Consumer Demand Theory

### 1. Rationale

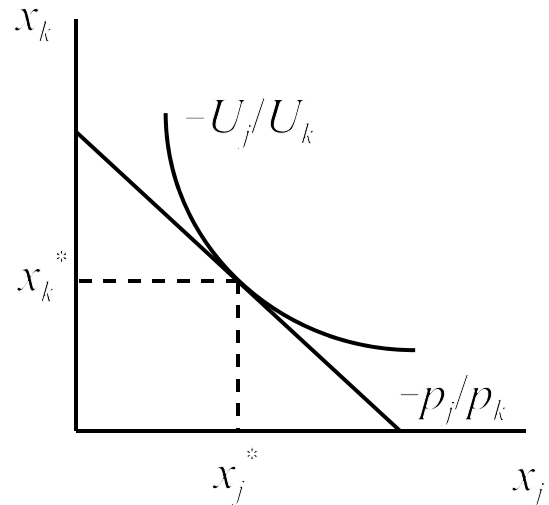
- a. the first couple of labor supply models that we will consider are simple extensions of consumer demand models
- b. before looking at the labor supply models, it's useful to review the some of the general properties of consumer demand

### 2. General model

- a. consider a person with preferences over  $N$  goods,  $x_1, x_2, \dots, x_N$
- b. assume that preferences can be described by a utility function  $U(x_1, x_2, \dots, x_N)$  where  $U_j > 0$  and  $U_{jj} < 0$  for  $j = 1, N$
- c. assume that the person has a total income,  $Y$ , and faces prices for each of the goods,  $p_j$ ; the budget constraint can be written  $Y \geq \sum p_j x_j$
- d. the person's problem is to choose quantities of the goods to maximize the utility function subject to the budget constraint
- e. we can incorporate the budget constraint by rewriting the maximization problem as a Lagrangian

$$\text{Max}_{x_1, \dots, x_N} \mathcal{L} = U(x_1, \dots, x_N) + \lambda \left( Y - \sum p_j x_j \right)$$

- f. for an interior solution the first order conditions are  $U_j = \lambda p_j$  for all  $j$ ; the second order conditions are guaranteed if the budget constraint is convex and the utility function is strictly concave
- g. the interior first-order conditions imply that  $U_j/U_k = p_j/p_k$  for all  $j$  and  $k$
- h. we obtain the direct demand functions  $x_j^* = f_j(p_1, \dots, p_N, Y)$  for all  $j$ ; a great amount of empirical work focuses on estimating the parameters of these demand functions



3. Properties of demand functions and related functions
- a. direct demand functions satisfy the following properties
- 1) adding up
  - 2) homogeneity of degree 0 in prices and income
  - 3) symmetric Slutsky matrix (matrix of substitution terms)
  - 4) negative semi-definite Slutsky matrix
- b. recall that

$$\frac{\partial x_j}{\partial p_k} = \frac{\partial h_j}{\partial p_k} - \frac{\partial x_j}{\partial Y} x_k$$

c. indirect utility function:

$$V(p_1, \dots, p_N, Y) = U(f_1(p_1, \dots, p_N, Y), \dots, f_N(p_1, \dots, p_N, Y))$$

1)  $V$  is nonincreasing in prices and nondecreasing in income

2)  $V$  is quasi-convex in  $p$

3)  $V$  is homogeneous of degree 0 in prices and income

d. expenditure function,  $e(p_1, \dots, p_N, \bar{u})$ , is the solution to

$$\text{Min}_{x_1, \dots, x_N} \sum p_j x_j$$

$$\text{subject to: } U(x_1, \dots, x_N) \geq \bar{u}$$

1)  $e$  is homogeneous of degree 1 in prices

2)  $e$  is concave in prices

e. Hicksian (compensated) demand functions,  $h_j(p_1, \dots, p_N, \bar{u})$ , are the set of demands which satisfy expenditure minimization

#### 4. functional forms

a. Cobb-Douglas utility specification:  $U = \sum \beta_j \ln x_j$

1) demand functions are  $x_j = \beta_j Y / p_j$

2) convenient but very restrictive

b. Stone-Geary (Linear Expenditure System) utility specification:  $U = \sum \beta_j \ln(x_j - \alpha_j)$

1) demand functions are

$$x_j = (1 - \beta_j) \alpha_j + \beta_j \frac{Y}{p_j} - \sum_{k \neq j} \beta_j \alpha_k \frac{p_k}{p_j}$$

- 2) still relatively convenient
- 3) less restrictive than Cobb-Douglas, though still relatively restrictive
- c. there are lots of other functional forms
- d. in general, functional form specifications involve trade-offs of convenience and flexibility
- e. a substantial amount of empirical work in consumer demand theory focuses on testing the restrictions inherent in both the theory and these specific functional forms

## B. Static Labor/Leisure Model

### 1. Assumptions

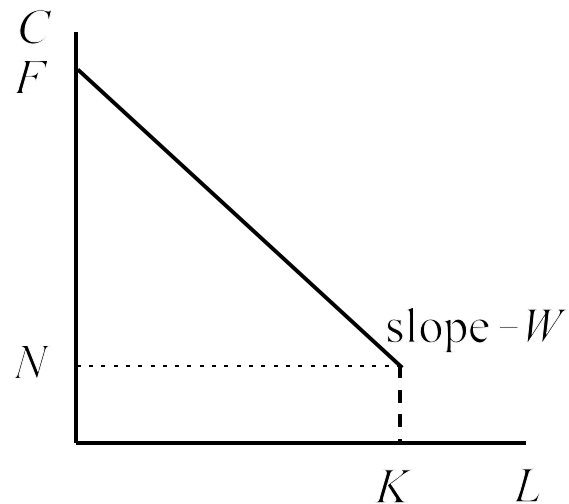
- a. assume that preferences are defined over two goods: consumption,  $C$ , and time spent outside the labor market (leisure),  $L$ ; we can write the utility function  $U = U(C, L)$
- b. assume that there is a maximum total amount of time available,  $K$ , and that time is divided between two mutually exclusive uses: market work,  $H$ , and time outside the labor market; we can write the time constraint  $K = L + H$
- c. assume that the individual receives an hourly wage,  $W$ , has non-labor income,  $N$ , and faces a price on consumption goods,  $P$ 
  - 1) budget constraint can be written  $WH + N \geq PC$

- 2) the budget constraint does not exactly conform to our earlier consumer demand constraint because total income is no longer exogenous (depends on the amount of work)
- 3) note, however, that we can rewrite the budget constraint as

$$WK + N \geq WL + PC$$

let  $WK + N = F$  be “full income”; this is similar to the budget constraint from consumer demand theory

- 4) for simplicity, we normalize prices at  $P = 1$
- 5) a graph of the budget constraint appears on the right



2. Individual's choice
  - a. individual solves

$$\text{Max}_{C,L} U(C, L)$$

$$\text{subject to: } F \geq WL + C$$

- b. obtain “demand-type” solutions  $C = C(W, F)$  and  $L = L(W, F)$

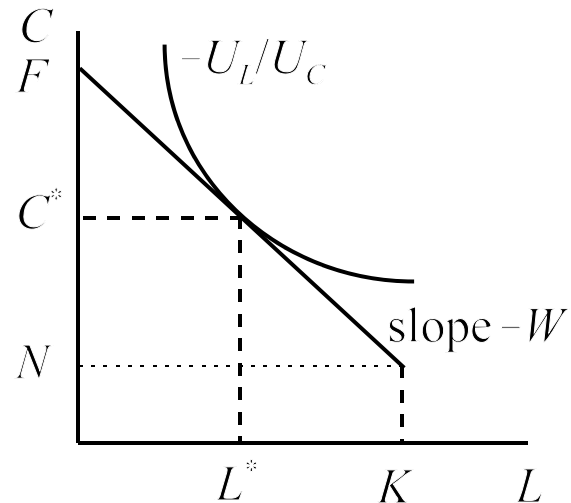
c. these demand functions are subject to the standard restrictions

1) homogeneity – imposed by the price normalization

2) adding up – imposed if we define the consumption function as  $C(W, F) = F - W \cdot L(W, F)$

3) symmetry of the Slutsky matrix – also imposed by construction

4) Slutsky matrix negative semi-definite (i.e.,  $\partial L / \partial W|_u < 0$ ) – testable restriction



### 3. Testing the Slutsky restriction

a. recall

$$\frac{\partial L}{\partial W} \Big|_U = \frac{\partial L}{\partial W} \Big|_F + L \frac{\partial L}{\partial F} \Big|_W$$

want to test whether this is negative

b. practical problems

1) how do we define  $L$  and  $F$  (i.e., how do we define  $K$ )?

2) what does it mean to hold  $F$  constant while changing  $W$ ?

c. note that if we differentiate  $L(W, F)$  w.r.t.  $W$  holding  $N$  constant we obtain

$$\left. \frac{\partial L}{\partial W} \right|_N = \left. \frac{\partial L}{\partial W} \right|_F + \left. \frac{\partial L}{\partial F} \right|_W \frac{\partial F}{\partial W} = \left. \frac{\partial L}{\partial W} \right|_F + \left. \frac{\partial L}{\partial N} \right|_W K$$

d. rearranging terms

$$\left. \frac{\partial L}{\partial W} \right|_F = \left. \frac{\partial L}{\partial W} \right|_N - \left. \frac{\partial L}{\partial N} \right|_W K$$

e. substituting into the Slutsky equation

$$\left. \frac{\partial L}{\partial W} \right|_U = \left. \frac{\partial L}{\partial W} \right|_N + (L - K) \left. \frac{\partial L}{\partial N} \right|_W = \left. \frac{\partial L}{\partial W} \right|_N - H \left. \frac{\partial L}{\partial N} \right|_W$$

f. finally note that  $dH = -dL$ ; so that

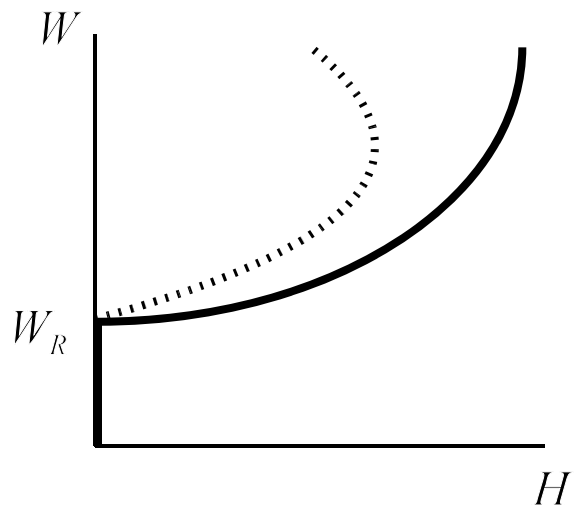
$$\left. \frac{\partial H}{\partial W} \right|_U = \left. \frac{\partial H}{\partial W} \right|_N - H \left. \frac{\partial H}{\partial N} \right|_W$$

g. can obtain the two partial derivatives on the RHS from a regression of hours on wages and non-labor income (hours, wages and non-labor income can all be directly observed)

4. Comparative statics of the labor supply function
  - a. define the labor supply function as  $H = H(W, N)$
  - b. income effect: if non-market time is a normal good, an increase in non-labor income leads to an increase in non-market time and a decrease in labor supply
  - c. wage increase leads to two types of effects
    - 1) income effect: increases income and decreases labor supply
    - 2) substitution effect: increases the effective price of leisure – increases labor supply
    - 3) effects are offsetting, the net effect is ambiguous

5. Shape of the labor supply curve

- a. labor supply curve maps different wages to different levels of labor supply holding all other characteristics (nonlabor income, preferences, etc.) constant
- b. for very low wages, no labor is supplied ( $H = 0$ )
- c. at some wage (the reservation wage,  $W_R$ ), positive hours are supplied
- d. initially, the labor supply curve slopes upward and bends forward (at  $H = 0$  there is no income effect; near there



the income effect is small); the substitution effect dominates the income effect

- e. thereafter, the labor supply curve may bend backward if the income effect dominates the substitution effect
- f. in the diagram, the solid line depicts a labor supply function that is forward bending at all wages; the dotted line depicts a labor supply function that is backward bending for some wages

6. Determination of the reservation wage

- a. consider the following Lagrangian problem

$$\text{Max}_{C,L} \mathcal{L} = U(C, L) + \lambda(F - WL - C)$$

- b. the Kuhn-Tucker conditions imply that  $U_L - \lambda W \geq 0$
- c. evaluated at  $L = K$  (i.e.,  $H = 0$ ), the condition is

$$U_L(N, K) / \lambda \geq W$$

the LHS of this expression defines the reservation wage

- d. reservation wage is wage at which person is indifferent between working and not working
- e. intuitively, the reservation wage is the slope of the indifference curve evaluated at  $H = 0$