Life-cycle Labor Supply (cont.)

E. Structural dynamic discrete-time model (Eckstein and Wolpin 1989a, b)

1. Theoretical model

a. Suppose that a person faces a finite life-time consisting of T+1 discrete periods; denote each period $t = 0, T$

b. In each period, the person faces two choices

1) $d(t)$=1 person does not work in labor market

2) $d(t)$=2 person works in labor market

c. Let $Y(t)$ denote the person's unearned income and let $w(t)$ denote earned income; the per-period rewards associated with each choice are

$$R_1(t) = \alpha_1 + Y(t)$$

$$R_2(t) = \alpha_2 + w(t) + Y(t)$$

d. Earnings depend on work experience $K(t)$ and a stochastic term $\varepsilon(t) \sim$ i.i.d. $N(0, \sigma^2)$ such that

$$\ln w(t) = \beta_0 + \beta_1 K(t-1) + \varepsilon(t) \quad \text{where}$$

$$K(t) = K(t-1) + I(d(t) = 2)$$

e. In this model, current wages are known but future wages are not (only the distribution of future wages is known); also model is dynamic because current work affects future work

f. In each period, the worker chooses $d(t)$ to maximize

$$R_i(t) + \delta E\left[V\left(\Omega(t+1) \mid d(t)\right)\right]$$

where $V(\Omega(t))$ is the maximal value at time t, conditional on the history up to that point summarized by $\Omega(t)$; $V(\Omega(t+1))$ represents what this value will be next period

g. Solution of the model in any period t is obtained by backwards recursion from T back to t

h. Solution from t forward consists of a series of reservation wages which in turn lead to a series of critical values on the stochastic terms

2. Model is estimated by Eckstein and Wolpin 1989b using maximum likelihood

   a. One modification that is helpful (necessary) for estimation is to assume that wages are measured with error; prevents lowest wage at a given period from serving as the de facto reservation wage

   b. So long as the stochastic terms are serially uncorrelated, the likelihood function is calculable, though complicated

   c. If the errors are serially uncorrelated, the model can be estimated starting anytime in a person's life-time and treating existing market experience as pre-determined

3. Estimation is substantially more complicated if the stochastic terms are serially correlated (likelihood would be characterized by a high dimension multiple integral)

4. Adding more choices in each time period or richer dynamics would also be a substantial complication

References

Eckstein, Zvi and Kenneth Wolpin. “The Specification and Estimation of Dynamic Stochastic Discrete Choice Models: A Survey.” *Journal of Human Resources* 24:4 (Autumn 1989a), 562-98.

_____. “Dynamic Labour Force Participation of Married Women and Endogenous Work Experience.” *Review of Economic Studies* 56:3 (July 1989b), 375-90.