

Compensating Wage Differentials

A. Hedonic Wage Theory – Labor Supply

1. General issues

- a. assume that goods, and more specifically jobs, are valued for a variety of utility-bearing qualities
- b. how do we find shadow prices and demands for the separate attributes when only the bundled good (job) and its total price (wage) are observed?

2. Assumptions

- a. a job can be described by a vector of N characteristics $\mathbf{Z} = (Z_1, Z_2, \dots, Z_N)$
 - 1) these possible job attributes include the risk of injury, location (e.g., lots of urban amenities), working conditions, risk of layoff, required hours
 - 2) separate from “fringe benefits” such as insurance coverage, pension, or vacation; these typically have separate explicit prices
- b. for convenience, we will describe all of the Z s as goods (e.g., job safety vs. risk of injury)
- c. there are a sufficiently large number of available jobs that people face a continuum of \mathbf{Z}
- d. workers and firms are atomistic and take the available wage as given
- e. characteristics of jobs cannot be unbundled; jobs are offered on a “take it or leave it” basis
- f. will consider only a single job per individual

g. people have preferences defined over the wage rate (general consumption) and the attributes such that $U = U(W, Z_1, Z_2, \dots, Z_N) = U(W, \mathbf{Z})$

h. wages are a function of the attributes such that $W = f(Z_1, Z_2, \dots, Z_N) = f(\mathbf{Z})$; we define $\partial W / \partial Z_i = f_i$

3. Worker's maximization problem

a. with these assumptions, we can write the worker's problem as

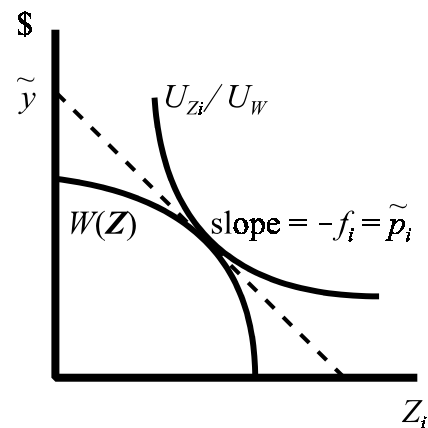
$$\text{Max}_{Z_1, Z_2, \dots, Z_N} U[f(Z_1, Z_2, \dots, Z_N), Z_1, Z_2, \dots, Z_N]$$

b. the first-order condition for an interior solution is

$$\frac{\partial U}{\partial Z_i} = U_W f_i + U_{Z_i} = 0 \quad \text{or} \quad \frac{U_{Z_i}}{U_W} = -f_i \quad \text{for } i = 1, N$$

c. graphically,

- 1) the worker's budget constraint is depicted as $W(\mathbf{Z})$
- 2) preferences can be described in terms of indifference curves between consumption and attribute i



- 3) slope at the tangency between the budget constraint and indifference curve can be interpreted as the “virtual price” of the attribute
- 4) can extend the budget line from the tangency to define a “virtual full income”

d. rewriting the problem in terms of virtual prices and incomes

1) at the solution, let

$$\tilde{p}_i = -f_i \quad \text{and} \quad \tilde{y} = w + \tilde{p}\mathbf{Z} = f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i$$

2) worker's problem becomes

$$\text{Max}_{W, \mathbf{Z}} U(W, \mathbf{Z})$$

$$\text{subject to } \tilde{y} = W + \tilde{p}\mathbf{Z}$$

3) holding \tilde{y} and \tilde{p} constant, the solutions can be written

$$Z_i = g(\tilde{p}, \tilde{y}) = g(f_1, f_2, \dots, f_N, f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i)$$

$$W = h(\tilde{p}, \tilde{y}) = h(f_1, f_2, \dots, f_N, f(\mathbf{Z}) - \sum_{i=1}^N f_i Z_i)$$

while this system is clearly simultaneous in the Z s, the properties of $g(\cdot)$ and $h(\cdot)$ are well-known

4) this approach is useful for describing behavior at the maximum (at a single point); however, it does not address selectivity of W and \mathbf{Z} combinations

B. Labor Demand

1. Firm's problem

a. assume that the only factor of production is labor

b. assume that job attributes can be provided at a cost $C = C(\mathbf{Z})$

- c. firm's maximize profits; so, the firm's problem can be written

$$\begin{aligned} \text{Max}_{L,Z} PQ(L) - WL - C(\mathbf{Z}) \\ = PQ(L) - f(Z_1, \dots, Z_N)L - C(Z_1, \dots, Z_N) \end{aligned}$$

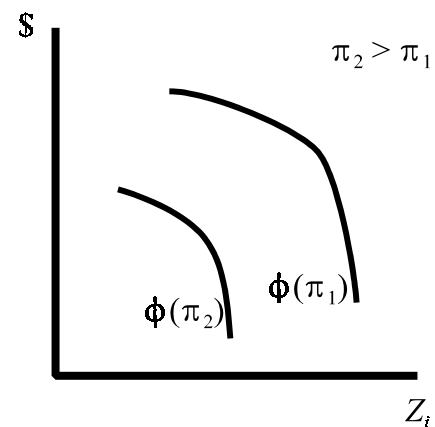
- d. for an interior solution, the first-order conditions are

$$\begin{aligned} L: \quad PQ_L = W \\ Z_i: \quad f_i = -C_i / L \end{aligned}$$

2. Offer functions

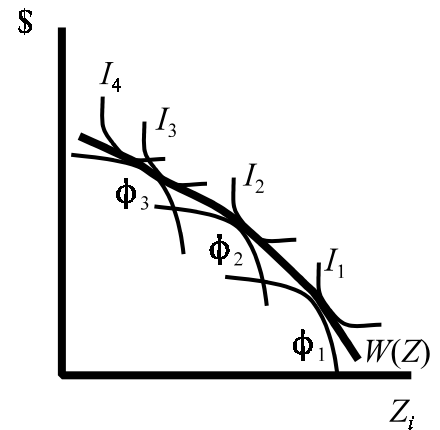
- define an offer function $\phi(\mathbf{Z}, \pi)$ which satisfies $\pi = PQ(L) - \phi L - C(\mathbf{Z})$ and $PQ_L = \phi$
- the offer function represents all of the combinations of Z which the firm can offer and maintain the same profit level; this is similar to an iso-cost line
- graphically,

- for any given firm, we can define a family of offer functions
- firm will minimize the offer curves subject to the prevailing wage rate



C. Wage Determination

1. Assume
 - a. that there are several firms with offer curves $\phi_1, \phi_2, \dots, \phi_K$
 - b. and several workers with indifference curves I_1, I_2, \dots, I_J
2. The equilibrium wage will be an envelop between the offer and indifference curves
3. Graphically,
 - a. there are a family of indifference curves
 - b. there are a family of offer curves
 - c. and assuming a continuum of people and firms, there is a smooth wage function



D. Estimation Issues

1. General issues
 - a. want to identify how differentiated qualities affect wages
 - b. also want to identify demand and supply functions for the individual attributes
2. Epple's critique
 - a. let prices be given by $p(\mathbf{Z})$
 - b. for each attribute Z_i , there is
 - 1) a demand function: $p_i(\mathbf{Z}) = f_i(\mathbf{Z}, \mathbf{X}_D, y, p(\mathbf{Z}))$

- 2) a supply function: $p_i(\mathbf{Z}) = g_i(\mathbf{Z}, \mathbf{X}_S, p(\mathbf{Z}))$
 - 3) where X_D and X_S are observable demand and supply shifters
- c. assume that there are K distinct markets with linear demand and supply relationships

$$p_k = \gamma_k + \psi_k' Z_k + \frac{Z_k' \pi_k Z_k}{2} + \zeta_k$$

$$\text{demand: } \frac{\partial p_k}{\partial Z_k} = A_D Z_k + H_D X_{Dk} + \mathbf{v}_{Dk}$$

$$\text{supply: } \frac{\partial p_k}{\partial Z_k} = A_S Z_k + H_S X_{Sk} + \mathbf{v}_{Sk}$$

- d. consider linear attribute demand and supply functions
- e. OLS in the attribute demand and supply equations is not consistent
- 1) to see this in the demand equation, solve for Z_k

$$Z_k = (\pi_k - A_D)^{-1} \left(H_D X_{Dk} + \mathbf{v}_{Dk} - \psi_k' \right)$$
 - 2) either Z_k or X_{Dk} depends on \mathbf{v}_{Dk} and OLS applied to the attribute demand equation is inconsistent
- f. consistent estimation of the price equation requires $E(\zeta_k \mathbf{v}_{Dk}) = 0$ and $E(\zeta_k \mathbf{v}_{Sk}) = 0$
3. Goddeeris (JPE 1988)
- a. examines differences between public-interest and private attorneys

- b. hedonic characteristic is a dummy variable (similar to introduction in *Handbook* chapter by Rosen); greatly simplifies choice problem
 - c. uses endogenous dummy variable model to estimate the wage premium associated with choice of law firm
 - d. actual estimation procedure is complicated by the use of a choice-based sample
4. Moffitt (JoLE, 1984)
- a. examines relationship between labor supply and a wage function which depends on hours worked (wage function in which hours are an endogenous determinant)
 - b. motivation for endogenous wages
 - 1) labor imposes quasi-fixed costs to firm; thus, low hours are associated with a low marginal rate (Oi, 1962)
 - 2) marginal productivity declines at high hours leading to lower wages (Barzel, 1973)
 - 3) result is an S-shaped budget constraint
 - c. considers
 - 1) linear labor supply function
 - 2) quadratic (in hours) earnings function
 - d. this specification leads to a nonlinear budget constraint
 - e. solves estimation problem by estimating the budget constraint at 15 discrete points
 - 1) sensitivity check with different numbers of points

2) also compares results with a linear budget constraint

f. results

1) rejects linear wage relationship

2) use of a nonlinear hours function leads to lower wage elasticities

3) nonlinear model provides a much better fit to the data