Equilibrium Job Search

A. Diamond's critique

1. Partial equilibrium model takes the distribution of wages $F(w)$ as given; but how does this distribution arise?
2. If workers are identical (have the same search strategies, arising from the same costs and benefits of search and the same expectations) and if employers know this,
    a. employers will simply set their wages at the reservation wage level;
    b. the wage distribution collapses to a single point, and
    c. the rationale for searching disappears
3. For a general equilibrium model with a non-degenerate wage distribution, we will need to modify the model of job search behavior; two useful and more realistic modifications are to account for job terminations and on-the-job search

B. Job search model with job terminations

1. Assumptions
    a. Continue with most of the assumptions of basic job search model in a stationary environment with variation in the arrival rate of job offers
    b. However, instead of assuming that jobs last forever, assume that they can be dissolved at any given time with probability q

c. If the job is dissolved, the person returns to unemployment

d. Consideration of job terminations is important in an equilibrium framework because it provides a supply of unemployed people

2. The present discounted value of work, V_e,

a. defined in terms of intervals of time, Δ, is

$$V_e = \frac{1}{1 + r\Delta}\left[w\Delta + (1 - q\Delta)V_e + q\Delta V_u\right]$$

expression reflects the wage that will be received, the expected future value of employment or unemployment, V_u, (depending on whether the job continues)

b. If we multiply both sides of the equation by $(1+r\Delta)$, rearrange terms and divide through by Δ, we get the continuous time expression

$$rV_e(w) = w + q\left(V_u - V_e(w)\right)$$

c. Further rearranging terms, we get

$$V_e(w) = \frac{w}{r+q} + \frac{q}{r+q}V_u$$

the value of employment reflects

1) the future value of wages, discounted by both the interest rate and the probability of job loss

2) the possible value of future unemployment also discounted by the interest rate and probability of job loss

3. The present discounted value of job search from unemployment in continuous time is

$$rV_u = b - c + \lambda_u \int_{rV_u}^{\infty} \left(V_e(w) - V_u\right) f(w)dw$$

(student's should verify that this is the appropriate expression even with job separations)

4. Substituting the term for the value of work and rearranging terms, we obtain the expression

$$\begin{aligned} w_r &= b - c + \frac{\lambda_u}{r+q} \int_{w_r}^{\infty} \left(w - w_r\right) f(w)dw \\ &= b - c + \frac{\lambda_u}{r+q} \int_{w_r}^{\infty} \left(1 - F(w)\right) dw \end{aligned}$$

5. Expression is nearly the same as in the standard model; the only difference is that the wage benefits of search are now discounted by both the interest rate and the probability of job loss

C. Extension of model to include on-the-job search

1. Assumptions
    a. We will maintain the assumptions from the previous model; however, we will also assume that job offers continue to arrive *after* a person accepts a job
    b. The job offers during employment come from the same wage distribution as offers during unemployment; the only difference is that the arrival rate of offers during employment, λ_e, is lower than

the rate during unemployment, reflecting different intensities of employed and unemployed search

c. There are no costs to switching jobs; so, workers accept any job offer that pays more than they are currently making

2. With these assumptions, the value of employment is

$$rV_e(w) = w + q\left(V_u - V_e(w)\right) + \lambda_e \int_w^\infty \left[V_e(\varepsilon) - V_e(w)\right] f(\varepsilon) d\varepsilon$$

this is the same as our previous expression, except for the last term, which reflects the value of higher wage job offers

3. The value of unemployed search remains

$$rV_u = b - c + \lambda_u \int_{rV_u}^\infty \left(V_e(w) - V_u\right) f(w) dw$$

4. Define the value of working at the reservation wage by substituting w_r in for w in the value of employment equation; rearranging terms, we get

$$w_r = rV_e(w_r) - q\left(V_u - V_e(w_r)\right) - \lambda_e \int_{w_r}^\infty \left[V_e(w) - V_e(w_r)\right] f(w) dw$$

5. However, at the reservation wage, $V_u = V_e(w_r)$; thus, the expression simplifies to

$$w_r = b - c + \left(\lambda_u - \lambda_e\right) \int_{w_r}^\infty \left[V_e(w) - V_u\right] f(w) dw$$

6. It can be shown that the expression further simplifies to

$$w_r = b - c + \left(\lambda_u - \lambda_e\right) \int_{w_r}^\infty \frac{1 - F(w)}{r + q + \lambda_e\left(1 - F(w)\right)} dw$$

7. Reservation wage for accepting a job from unemployed job search is lower when on-the-job search is allowed

a. in the case where $\lambda_u = \lambda_e$, the reservation wage collapses to $b - c$
   1) people accept the first job that is offered that exceeds the net benefits/costs of unemployment
   2) if the net costs are zero or if $b - c$ defines the floor of the wage distribution, spells of unemployment are only determined by the arrival rate of job offers
b. in the case where $\lambda_e = 0$, the reservation wage returns to the simple case with job terminations

D. Equilibrium model with an endogenous wage distribution
1. Assumptions
   a. We will assume that the number of workers and firms is fixed; for convenience, we will set the size of each group at one
   b. Continue to use the stationary search framework with job terminations and on-the-job search
      1) terminations refresh the pool of unemployed people and help to anchor the wage distribution
      2) separations from on-the-job search help to produce a non-degenerate wage distribution
   c. We will treat the separation and offer arrival rates, q, λ_u, and λ_e, as fixed (which is admittedly unrealistic); equilibrium will be achieved through the wage distribution (i.e., through the wages that firms offer)
2. Some additional notation

a. let $\ell(w)$ represent the number (proportion) of workers working at a firm that pays w

b. let $L(w) = \int_0^w \ell(\varepsilon) f(\varepsilon) d\varepsilon$ represent the proportion of all workers who work in firms paying less than w

c. let u denote the proportion of workers who are unemployed (unemployment rate)

3. Flows into jobs

a. consider the flows of workers into jobs that pay wages of w or more

b. the proportion of these jobs is $1 - F(w)$

c. rate of flows from unemployment is $\lambda_u u[1 - F(w)]$

d. rate of flows from employment is $\lambda_e L(w)[1 - F(w)]$

4. Flows out of jobs that pay w or more are $q[1 - u - L(w)]$

5. In equilibrium, flows into and out of jobs at each wage are equal

$$[\lambda_u u + \lambda_e L(w)][1 - F(w)] = q[1 - u - L(w)]$$

a. because this relationship holds at all w and because u is constant at all w,

$$\lambda_e L'(w)[1 - F(w)] - [\lambda_u u + \lambda_e L(w)] f(w) = -qL'(w)$$

which is obtained by differentiating both sides of the previous expression by w

b. noting that $L'(w) = \ell(w) f(w)$, we can rewrite this as

$$[q + \lambda_e (1 - F(w))] \ell(w) f(w) = [\lambda_u u + \lambda_e L(w)] f(w)$$

$$[q + \lambda_e (1 - F(w))] \ell(w) = \lambda_u u + \lambda_e L(w)$$

c. since this relationship also holds at all w and u is constant at all w, we can differentiate again to obtain

$$[q+\lambda_e(1-F(w))]\ell'(w)-\lambda_e\ell(w)f(w)=\lambda_e\ell(w)f(w)$$

$$\frac{\ell'(w)}{\ell(w)}=\frac{2\lambda_e f(w)}{q+\lambda_e(1-F(w))}$$

d. the employment distribution is related to the wage distribution and the rates of job offers and separations

6. Firm behavior

a. assumptions

1) each firm produces an exogenous amount y

2) rate of profit of a firm that pays w is $(y-w)\,\ell(w)$

3) each firm takes the behavior of others as given

b. profit maximization implies

$$\frac{\ell'(w)}{\ell(w)}=\frac{1}{y-w}$$

c. in addition, firms must offer wages that at least meet the reservation wage associated with unemployed job search; this sets a floor on the wage distribution

7. The wage conditions for firms give us a differential equation in $\ell(w)$ ($\ell'(w) / \ell(w) = \mathrm{d}\ln \ell(w) / \mathrm{d}w$); solving,

a. we get $\ell(w) = A / (y-w)$

b. we can use the equilibrium at w_r to determine that $\ell(w_r) = \lambda_u u/(q+\lambda_e)$ and that $A = \lambda_u u\,(y-w_r) / (q+\lambda_e)$

c. also, the wage floor implies that in equilibrium

$u = q / (q + \lambda_u)$

d. thus, the employment distribution is

$$\ell(w) = \frac{\lambda_u q}{(q+\lambda_e)(q+\lambda_u)} \frac{(y-w_r)}{(y-w)}$$

8. Equilibrium wage distribution

a. The equilibrium employment flow and wage setting equations also lead to a differential equation in $F(w)$; it can be shown that the solution to this equation is

$$F(w) = \frac{q+\lambda_e}{\lambda_e}\left(1 - \sqrt{\frac{y-w}{y-w_r}}\right)$$

b. The maximum wage (determined at $F(w_{\max}) = 1$) is

$$w_{\max} = y - (y-w_r)\left(\frac{q}{q+\lambda_e}\right)^2$$

c. The reservation wage is

$$w_r = \frac{(b-c)(q+\lambda_e)^2 + (\lambda_u - \lambda_e)\lambda_e y}{(q+\lambda_e)^2 + (\lambda_u - \lambda_e)\lambda_e}$$

d. Can immediately see that in the absence of on-the-job search (i.e., with $\lambda_e = 0$), wage distribution collapses to one point $w_r = w_{\max} = b - c$

E. Other approaches for obtaining distributions of wages

1. There are other equilibrium approaches for obtaining wage distributions without resorting to on-the-job search

2. Heterogeneous search costs

a. Intuitively, firms may need to offer different wages if workers differ in their net costs of search

b. Rogerson et al. (2004) work through example with two types of workers (high and low costs of search)

c. With two types of workers, firms have an incentive to offer a higher wage—doing so speeds the expected recruiting time

d. In equilibrium, firms *may* offer two levels of wages

   1) workers with high costs of search accept any wage

   2) workers with low costs of search accept only the higher wage

e. Depending on the structure of the economy, there are three possible outcomes

   1) all firms offer the lower wage

   2) all firms offer the higher wage

   3) a fraction of firms offers each wage

3. Efficiency wages

   a. Assume that workers may shirk and that shirking increases the probability of job loss

   b. Higher wages deter workers from shirking because they increase the cost of job loss

   c. In this case, different wages affect the separation rate rather than the hiring rate

   d. Can arrive at an equilibrium with multiple wages

F. Simple matching model (Pissarides 2000)

1. Consider homogenous workers and firms and assume that there is no on-the-job search so that the wage distribution collapses to a single point

2. Job matches and separations

   a. assume that there are L workers and that unemployed workers are matched to vacant jobs according to a process that depends on the ratio of the vacancies as a proportion of workers (the vacancy rate), v, and the unemployment rate, u

      1) let $\theta = v / u$; θ is an indicator of labor market tightness

      2) denote the *matching function* as $m(\theta)$, where $m'(\theta) \leq 0$; this describes the rate at which vacancies are filled

      3) the rate at which unemployed workers find jobs is $\theta \cdot m(\theta)$; this is the offer arrival rate that we considered before

   b. also assume that jobs are terminated; continue to use q as the separation rate

   c. in equilibrium, $u = \dfrac{q}{q + \theta m(\theta)}$

   d. downward-sloping relationship between unemployment and θ is known as the *Beveridge curve*

3. Worker behavior

   a. value of unemployment is $rV_u = b + \theta m(\theta)[V_e - V_u]$

b. value of employment is $rV_e = w + q[V_u - V_e]$

c. the reduced form expressions are

$$rV_u = \frac{(r+q)b + \theta m(\theta)w}{r + q + \theta m(\theta)}$$

$$rV_e = \frac{qb + [r + \theta m(\theta)]w}{r + q + \theta m(\theta)}$$

4. Firm behavior

a. assume that each firm can offer exactly one job (makes jobs and firms synonymous)

b. value to a firm of opening a job vacancy is $rV_v = -\kappa + m(\theta)[V_j - V_v]$, where κ is the per-period cost of maintaining the vacancy

c. in equilibrium, the profits associated with creating new vacancies are zero so that

$$V_j = \frac{\kappa}{m(\theta)}$$

d. value of a filled job is $rV_j = y - w - qV_j$

e. in equilibrium, the marginal product of workers equals their wage and hiring costs

$$y = w + \frac{(r+q)\kappa}{m(\theta)}$$

5. Wage determination

a. jobs produce rents for both the worker and the firm

b. assume that wages are chosen so that the rents associated with jobs divided (shared) following generalized Nash cooperative bargaining, where β is the share of the rents going to the worker

c. this leads to the condition, $(1-\beta)(V_e - V_u) = \beta(V_j - V_v)$

d. solving, we get a reservation wage

$$rV_u = w_r = b + \frac{\beta}{1-\beta}\kappa\theta$$

e. and an aggregate wage

$$w = (1-\beta)b + \beta(y + \kappa\theta)$$

References

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