

# Family Labor Supply

## A. Unitary preference approach

### 1. Model

#### a. assumptions

- 1) consider a two-person family with preferences  $U = U(x, L_m, L_f)$  where
  - $x$  is household consumption, and
  - $L_i$  is non-market time for member  $i$  ( $i = m, f$ )
- 2) let the household budget constraint be
$$x = w_m H_m + w_f H_f + N$$
- 3) and let the time constraints be  $K = H_i + L_i$  for  $i = m, f$
- 4) household chooses  $H_m$  and  $H_f$  to maximize preferences subject to time and money constraints
- 5) this is a straightforward extension of the simple static model that incorporates the market and non-market contributions of an additional person

#### b. solution to the model (determinants of joint labor supply)

- 1) like the simple static model, the two-person model is solved as a standard consumer demand problem
- 2) maximization yields labor supply functions

$$H_m = H_m(w_m, w_f, N)$$

$$H_f = H_f(w_m, w_f, N)$$

for each household member, labor supply depends on nonlabor income, own wages and other member's wages

- 3) comparative static results: an increase in  $N$ 
  - assuming  $L_m$  and  $L_f$  are normal, the increase in  $N$  leads to decreases in  $H_m$  and  $H_f$
  - same result as simple static model
- 4) comparative static results: an increase in  $w_m$ 
  - produces an income effect:  $H_m$  and  $H_f$  decrease
  - produces a standard "own" substitution effect for  $m$ :  $H_m$  increases
  - produces a cross-substitution effect
    - if  $L_m$  and  $L_f$  are substitutes,  $H_f$  decreases
    - if  $L_m$  and  $L_f$  are complements,  $H_f$  increases

## 2. testable implications

### a. Consumer demand restrictions

#### 1) consider the Slutsky decomposition

$$\left. \frac{\partial H_i}{\partial w_j} \right|_U = \left. \frac{\partial H_i}{\partial w_j} \right|_N - H_j \frac{\partial H_i}{\partial N}$$

2) three implications of consumer demand theory

$$\left. \frac{\partial H_i}{\partial w_i} \right|_U > 0 \quad (\text{own subst. positive})$$

$$\left. \frac{\partial H_i}{\partial w_j} \right|_U = \left. \frac{\partial H_j}{\partial w_i} \right|_U \quad (\text{cross subst. symmetric})$$

$$\left. \frac{\partial H_i}{\partial w_i} \right|_U \left. \frac{\partial H_j}{\partial w_j} \right|_U - \left. \frac{\partial H_i}{\partial w_j} \right|_U \left. \frac{\partial H_j}{\partial w_i} \right|_U > 0 \quad (\text{neg. semi - definite})$$

3) Killingsworth reports that while own substitution effects are generally found to be positive, most empirical research does not support the other two implications (important exception is 1974 study by Ashenfelter & Heckman)

b. Pooled income restrictions

- 1) unearned income attributable to different members has the same effect on labor demand (i.e., unearned income is effectively pooled)
- 2) this restriction has also been rejected by empirical studies

B. Bargaining approach

1. General Issues

a. more natural approach to modeling household behavior because

- 1) it incorporates the utility specifications of each individual in the household

- 2) it is capable of describing how differences between individuals are resolved
- b. substantially more complicated than the single-preference neoclassical model; in particular,
- 1) these models specify bargaining power within marriage in terms of options outside of marriage; thus, they must provide a description of those options
  - 2) a proper description of the options outside of marriage further requires a model marriage and/or household formation
  - 3) bargaining models must also specify equilibrium conditions
- c. economists have examined both cooperative and non-cooperative household bargaining models
2. Cooperative bargaining models
- a. McElroy and Horney model
- 1) consider a two-person household with individuals  $m$  and  $f$  who each have preferences defined over
    - $x_0$  — a pure public good within the household
    - $x_i$  — a private good consumed by  $i$  ( $i = m, f$ )
    - $L_i$  — leisure enjoyed by  $i$
  - 2) these goods have associated prices  $p_0, p_m, p_f, w_m,$  and  $w_f$

3) if  $m$  and  $f$  were unmarried, each would have a utility function defined over the goods that are specifically of interest to them

$$U_m^U = U_m^U(x_0, x_m, L_m) \quad \text{and} \quad U_f^U = U_f^U(x_0, x_f, L_f)$$

and individual budget constraints

$$N_i + w_i T = p_0 x_0 + p_i x_i + w_i L_i \quad i = m, f$$

leading to indirect utility functions of the form

$$V_i^U = V_i^U(p_0, p_i, w_i, N_i) \quad i = m, f$$

these indirect utility functions describe  $m$ 's and  $f$ 's opportunities outside of marriage

4) within marriage, each individual has preferences

$$U_i^M = U_i^M(x_0, x_m, x_f, L_m, L_f) \quad i = m, f$$

(note, the individuals have interdependent preferences—goods going to the other spouse enter their own utility) and a joint budget constraint

$$N_m + N_f + (w_m + w_f)T = p_0 x_0 + p_m x_m + p_f x_f \\ + w_m L_m + w_f L_f$$

5) we can define the gains from marriage for each spouse as

$$U_i^M(x_0, x_m, x_f, L_m, L_f) - V_i^U(p_0, p_i, w_i, N_i) \quad i = m, f$$

6) we assume a cooperative symmetric Nash solution; specifically, we assume that the couple chooses an allocation of goods that maximizes

$$J = [U_m^M - V_m^U][U_f^M - V_f^U]$$

subject to the budget constraint within marriage (defined above); the terms in brackets are the gains to marriage for each individual, and the  $V_i^U$  terms represent the threat points

7) taking derivatives and solving, we obtain labor supply and goods demand functions

$$H_i = H_i(p_0, p_m, p_f, w_m, w_f, N_m, N_f) \quad i = m, F$$

$$x_j = x_j(p_0, p_m, p_f, w_m, w_f, N_m, N_f) \quad j = 0, m, F$$

8) comparative statics

a) increase in  $N_m$

- produces a standard neoclassical income effect (shifts  $H_m$  and  $H_f$  downward)

- changes relative bargaining positions of spouses as  $m$ 's opportunities outside of marriage improve; depending on whether the couple is selfish or altruistic, will shift away from  $m$ 's or  $f$ 's labor supply

b) increase in  $w_m$

- produces standard neoclassical income effects
- produces standard own and cross-substitution effects from neoclassical model
- produces bargaining effects
- “adjustment” effect (necessary because monotone transformations of Nash determined indifference curves cannot affect choices)

b. Key results and insights

- 1) opportunities outside of marriage affect choices within marriage – this occurs even if there is no direct effect on the opportunities within marriage
- 2) changes in the distribution of unearned income within marriage that *do not affect the opportunities outside of marriage* have no affect on behavior
- 3) substitution effects are now greatly generalized (simple symmetry from neoclassical model is removed)

- 4) leads to some simple tests, e.g., can examine whether changes in nonlabor income attributable to each spouse have different effects on household labor supply and consumption

c. qualifications

- 1) cooperative framework is restrictive
- 2) may be unrealistic to think that divorce is a relevant threat point for minor disagreements

3. Collective Models (Chiappori 1988)

a. Generalization of collective decision-making approach

- 1) allows for separate utility functions for spouses
- 2) instead of assuming an explicit bargaining framework, assumes that collective decisions satisfy feasibility conditions and are Pareto optimal (cooperative bargaining leads to Pareto efficient solutions but other approaches do as well)
- 3) practically, this is accomplished by assuming that each spouse maximizes his or her own utility subject to a budget constraint that includes an income-sharing rule
- 4) income sharing rule depends on each person's wages, contributions of incomes and other characteristics

b. Approach nests unitary and cooperative bargaining approaches as special cases

- c. An advantage of the approach is that, in principle, one can use this approach back out individual consumption shares (i.e., identify the sharing rule) even if only aggregate consumption is observed

#### 4. Non-cooperative Models

##### a. General issues

- 1) some researchers have argued that the threat point in the static cooperative model (divorce) is unrealistic
- 2) these researchers have suggested instead that non-cooperative equilibria within marriage be considered; for example,
  - i) Woolley (1988) suggested a “consistent conjectural equilibrium” (e.g., “harsh words and burnt toast”)
  - ii) Lundberg and Pollak (1993) suggested that couples might revert to “socially recognized and sanctioned gender roles
- 3) in these static models, the different threat points have very different behavioral implications
  - i) in the cooperative model, a change in the distribution of resources within marriage which does not change the total level of resources or the resources outside of marriage does not change behavior

- ii) a similar change in the non-cooperative model changes the partners' relative bargaining positions and, therefore, affects behavior
  - iii) an example of such a change would be a child care allowance or tax subsidy paid only to married mothers
- 4) theory provides little guidance as to how to specify the appropriate threat point
- b. Multi-period non-cooperative models (from Bergstrom 1996)
- 1) Rubinstein's (1982) division model
    - i) consider two agents who wish to partition some surplus; agents take turns proposing a division ( $w$  to agent 1,  $(1-w)$  to agent 2); at each turn, agent receiving offer may either accept the offer and the surplus is divided at that point or reject the offer and make an offer of his/her own
    - ii) agents are impatient (discount the future with discount rates  $\delta_1$  and  $\delta_2$ , respectively)
    - iii) case #1: if utility for each player of receiving  $w$  is simply  $U_i = \delta_i^t w$ , then as the time between offers approaches zero, the only subgame perfect equilibrium is for agent  $i$  to propose initially  $\alpha_i = \delta_i / (\delta_1 + \delta_2)$
    - iv) case #2: more generally, if utility is  $U_i = \delta_i^t u_i(w)$ , then the only subgame perfect

equilibrium is the allocation which maximizes the Nash product  $u_1^{\alpha_1} u_2^{\alpha_2}$  on the utility possibility set  $\{(u_1(w), u_2(1-w)) \mid 0 \leq w \leq 1\}$ ; if the agents have the same discount rates, this is the same as the symmetric Nash equilibrium with threat points  $(0, 0)$

- 2) Binmore's (1985) division model with outside options
  - i) Binmore extended the Rubinstein's model to give agents the option of breaking off negotiations at any time and receiving  $m_i$
  - ii) solution is the allocation which maximizes the Nash product  $u_1(w)^{\alpha_1} u_2(1-w)^{\alpha_2}$  on the utility possibility set  $\{(u_1(w), u_2(1-w)) \mid 0 \leq w \leq 1\}$  and subject to the constraint  $u_i \geq m_i$  for  $i = 1, 2$ ; in general, this is not the same as the solution to  $[u_1(w) - m_1]^{\alpha_1} [u_2(1-w) - m_2]^{\alpha_2}$  (the Nash solution at the threat points  $(m_1, m_2)$ )
  - iii) so long as the gains from marriage are divided in such a way that both parties are better off being married than unmarried, the threat of divorce is not credible

## C. Models of Marriage and Fertility

### 1. Demographic vs. economic models

- a. demographic models—schedules which summarize empirical regularities
    - 1) useful in the evaluation of the quality of data
    - 2) also useful as a building block in other procedures for estimating levels and trends of fertility, nuptiality, and mortality in developing countries
  - b. economic models—useful for explaining how these empirical regularities arise in the first place and especially how changes in constraints affect these regularities
2. Economic models of marital status
- a. Gale-Shapely (1962) Marriage Algorithm (general equilibrium model)
    - 1) men propose version: each person first ranks every other person; each man then makes an offer to his “highest ranked” woman; women either reject the offer or say maybe to the best offer received; men who are rejected make new offers; the procedure continues until each man has made offers to every “suitable” woman
    - 2) implications of men-propose version: assignment is at least as good for every man as any other stable assignment and at least as bad for every women as any other stable assignment

- 3) if women propose, assignment is at least as good for every woman and at least as bad for every man as any other stable assignment
  - 4) one problem with this model is that it does not allow for transferable utility (utility possibility frontier for any two potential spouses is a single point)
- b. simple (partial equilibrium) search model from Montgomery and Trussell (1986)
- 1) let characteristics of a prospective spouse be summarized by a single observable index  $\varepsilon$
  - 2) individual who searches pays search costs of  $c$  in each period and receives one draw from population of prospective spouses; this population is characterized by a known distribution  $F(\varepsilon)$  such that the sequence of draws is assumed to be i.i.d.
  - 3) marriages are assumed to last forever and the present value of a match at the time that it occurs is assumed to be  $\varepsilon / (1-\delta)$  where  $\delta$  is the searcher's discount rate
  - 4) following Lippman and McCall (1976), the solution can be described in terms of a reservation value on  $\varepsilon$ ,  $\varepsilon_r$  where  $\varepsilon_r$

$$\varepsilon_r = v^s - c + \frac{\delta}{1-\delta} \int_{\varepsilon_r}^{\infty} (\varepsilon - \varepsilon_r) dF$$

5) implications:

- reservation value rises (ave. search time increases) with utility during search
- reservation value falls (ave. search time decreases) with costs of search
- reservation value rises with mean and variance of distribution
- constant hazard for age at marriage

6) model can be modified to incorporate probabilistic offers, finite lives, etc.

## References

- Bergstrom, T. C. (1996). "Economics in a Family Way," *Journal of Economic Literature* 34: 1903-1934.
- Binmore, K. (1985). "Bargaining and Coalitions," in *Game-Theoretic Models of Bargaining*, A. Roth, ed., Cambridge: Cambridge University Press, 259-304.
- Chiappori, P. (1988). "Rational Household Labor Supply," *Econometrica* 56: 63-89.
- Lundberg, S. and R. Pollak. (1993). "Separate Spheres Bargaining and the Marriage Market," *Journal of Political Economy* 96: 1165-82.
- Montgomery, M. and J. Trussell. (1986). "Models of Marital Status and Childbearing," in *Handbook of Labor Economics*, O. Ashenfelter and R. Layard, eds., Amsterdam: North-Holland, 205-71.
- Rubinstein, A. (1982). "Perfect Equilibrium in a Bargaining Model," *Econometrica* 50: 97-109.
- Woolley, F. (1988). "A Non-cooperative Model of Family Decision Making," LSE mimeo.