Simple Static Labor Supply Model

A. Quick Review of Consumer Demand Theory

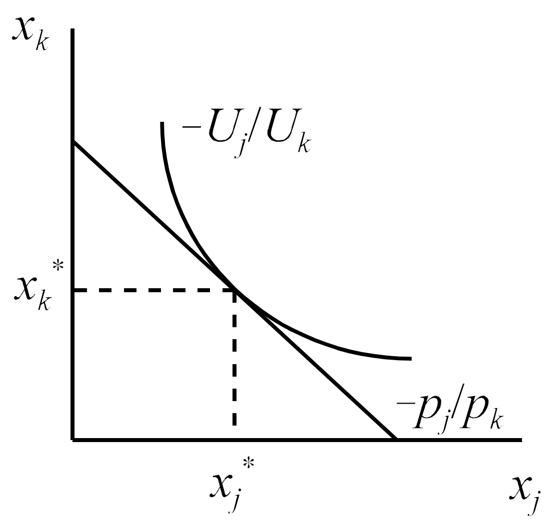
1. Rationale
   a. the first couple of labor supply models that we will consider are simple extensions of consumer demand models
   b. before looking at the labor supply models, it's useful to review the some of the general properties of consumer demand

2. General model
   a. consider a person with preferences over N goods, x_1, x_2, ..., x_N
   b. assume that preferences can be described by a utility function $U(x_1, x_2, ..., x_N)$ where $U_j > 0$ and $U_{jj} < 0$ for $j = 1, N$
   c. assume that the person has a total income, Y, and faces prices for each of the goods, p_j; the budget constraint can be written $Y \geq \Sigma p_j x_j$
   d. the person's problem is to choose quantities of the goods to maximize the utility function subject to the budget constraint
   e. we can incorporate the budget constraint by rewriting the maximization problem as a Lagrangian

$$\operatorname*{Max}_{x_1,\dots,x_N} \mathcal{L} = U(x_1,\dots,x_N) + \lambda\left(Y - \sum p_j x_j\right)$$

f. for an interior solution the first order conditions are $U_j = \lambda p_j$ for all j; the second order conditions are guaranteed if the budget constraint is convex and the utility function is strictly concave

g. the interior first-order conditions imply that $U_j/U_k = p_j/p_k$ for all j and k



h. we obtain the direct demand functions $x_j^* = f_j(p_1, ..., p_N, Y)$ for all j; a great amount of empirical work focuses on estimating the parameters of these demand functions

3. Properties of demand functions and related functions

a. direct demand functions satisfy the following properties

1) adding up

2) homogeneity of degree 0 in prices and income

3) symmetric Slutsky matrix (matrix of substitution terms)

4) negative semi-definite Slutsky matrix

b. recall that

$$\frac{\partial x_j}{\partial p_k} = \frac{\partial h_j}{\partial p_k} - \frac{\partial x_j}{\partial Y} x_k$$

c. indirect utility function:
$V(p_1, ..., p_N, Y) = U(f_1(p_1, ..., p_N, Y), ..., f_N(p_1, ..., p_N, Y))$

1) V is nonincreasing in prices and nondecreasing in income
2) V is quasi-convex in p
3) V is homogeneous of degree 0 in prices and income

d. expenditure function, $e(p_1, ..., p_N, \bar{u})$, is the solution to

$$\underset{x_1,...,x_N}{\text{Min}} \sum p_j x_j$$

$$\text{subject to:} \quad U(x_1,...,x_N) \geq \bar{u}$$

1) e is homogeneous of degree 1 in prices
2) e is concave in prices

e. Hicksian (compensated) demand functions, $h_j(p_1, ..., p_N, \bar{u})$, are the set of demands which satisfy expenditure minimization

4. functional forms

a. Cobb-Douglas utility specification: $U = \sum \beta_j \ln x_j$

1) demand functions are $x_j = \beta_j Y / p_j$
2) convenient but very restrictive

b. Stone-Geary (Linear Expenditure System) utility specification: $U = \sum \beta_j \ln(x_j - \alpha_j)$

1) demand functions are

$$x_j = (1-\beta_j)\alpha_j + \beta_j {Y}/{p_j} - \sum_{k \neq j} \beta_j \alpha_k {p_k}/{p_j}$$

2) still relatively convenient

3) less restrictive than Cobb-Douglas, though still relatively restrictive

c. there are lots of other functional forms

d. in general, functional form specifications involve trade-offs of convenience and flexibility

e. a substantial amount of empirical work in consumer demand theory focuses on testing the restrictions inherent in both the theory and these specific functional forms

B. Static Labor/Leisure Model

1. Assumptions

a. assume that preferences are defined over two goods: consumption, C, and time spent outside the labor market (leisure), L; we can write the utility function $U = U(C, L)$

b. assume that there is a maximum total amount of time available, K, and that time is divided between two mutually exclusive uses: market work, H, and time outside the labor market; we can write the time constraint $K = L + H$

c. assume that the individual receives an hourly wage, W, has non-labor income, N, and faces a price on consumption goods, P

1) budget constraint can be written $WH + N \geq PC$

2) the budget constraint does not exactly conform to our earlier consumer demand constraint because total income is no longer exogenous (depends on the amount of work)

3) note, however, that we can rewrite the budget constraint as $WK + N \geq WL + PC$; let $WK + N = F$ be "full income"; this is similar to the budget constraint from consumer demand theory

4) for simplicity, we normalize prices at $P = 1$

2. Individual's choice

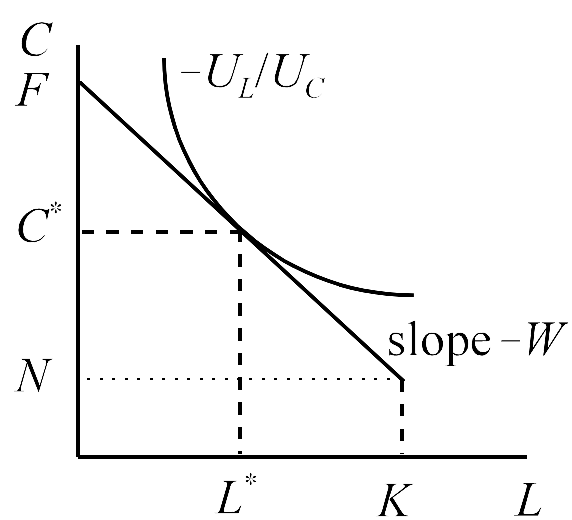
a. individual solves

$$\underset{C,L}{\text{Max}}\, U(C, L)$$

$$\text{subject to:} \quad F \geq WL + C$$

b. obtain "demand-type" solutions $C = C(W, F)$ and $L = L(W, F)$

c. these demand functions are subject to the standard restrictions



1) homogeneity – imposed by the price normalization

2) adding up – imposed if we

define the consumption function as
$C(W, F) = F - W \cdot L(W, F)$

3) symmetry of the Slutsky matrix – also imposed by construction
4) Slutsky matrix negative semi-definite (i.e., $\partial L/\partial W|_u < 0$) – testable restriction

3. Testing the Slutsky restriction
   a. recall

$$\left.\frac{\partial L}{\partial W}\right|_U = \left.\frac{\partial L}{\partial W}\right|_F + L\left.\frac{\partial L}{\partial F}\right|_W$$

   want to test whether this is negative

   b. practical problems
      1) how do we define *L* and *F* (i.e., how do we define *K*)?
      2) what does it mean to hold *F* constant while changing *W*?
   c. note that we can eliminate *F* because

$$\left.\frac{\partial L}{\partial W}\right|_F = \left.\frac{\partial L}{\partial W}\right|_N - \left.\frac{\partial L}{\partial F}\right|_W \frac{\partial F}{\partial W} = \left.\frac{\partial L}{\partial W}\right|_N - \left.\frac{\partial L}{\partial N}\right|_W K$$

   d. substituting

$$\left.\frac{\partial L}{\partial W}\right|_U = \left.\frac{\partial L}{\partial W}\right|_N + (L - K)\left.\frac{\partial L}{\partial N}\right|_W = \left.\frac{\partial L}{\partial W}\right|_N - H\left.\frac{\partial L}{\partial N}\right|_W$$

e. finally note that dH = −dL; so that

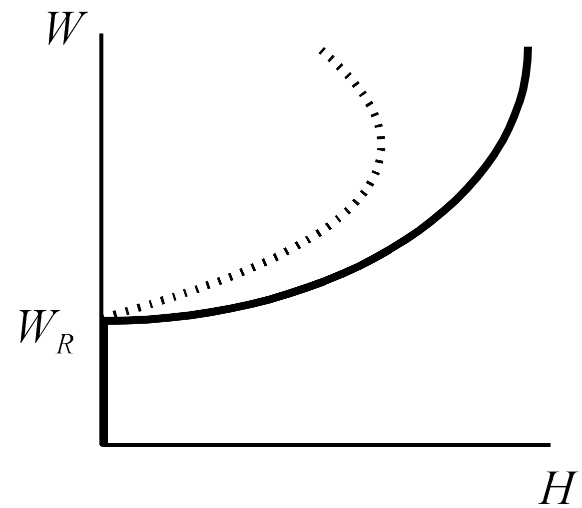
$$\left.\frac{\partial H}{\partial W}\right|_U = \left.\frac{\partial H}{\partial W}\right|_N - H\left.\frac{\partial H}{\partial N}\right|_W$$

f. can obtain the two partial derivatives on the RHS from a regression of hours on wages and non-labor income (hours, wages and non-labor income can all be directly observed)

4. Comparative statics of the labor supply function
   a. define the labor supply function as $H = H(W, N)$
   b. income effect: if non-market time is a normal good, an increase in non-labor income leads to an increase in non-market time and a decrease in labor supply
   c. wage increase leads to two types of effects
      1) income effect: increases income and decreases labor supply
      2) substitution effect: increases the effective price of leisure – increases labor supply
      3) effects are offsetting, the net effect is ambiguous

5. Shape of the labor supply curve
   a. labor supply curve maps different wages to different levels of labor supply

b. for very low wages, no labor is supplied ($H = 0$)



c. at some wage (the reservation wage, W_R), positive hours are supplied

d. initially, the labor supply curve slopes upward and bends forward (at $H = 0$ there is no income effect; near there the income effect is small); the substitution effect dominates the income effect

e. thereafter, the labor supply curve may bend backward if the income effect dominates the substitution effect

f. in the diagram, the solid line depicts a labor supply function that is forward bending at all wages; the dotted line depicts a labor supply function that is backward bending for some wages

6. Determination of the reservation wage

a. consider the following Lagrangian problem

$$\underset{C,L}{\text{Max}}\, \mathcal{L} = U(C, L) + \lambda(F - WL - C)$$

b. the Kuhn-Tucker conditions imply that $U_L - \lambda W \geq 0$

c. evaluated at $L = K$, the condition is $U_L(N, K) / \lambda \geq W$; the LHS of this expression defines the reservation wage
d. reservation wage is wage at which person is indifferent between working and not working
e. intuitively, the reservation wage is the slope of the indifference curve evaluated at $L = K$ ($H = 0$)