Life-cycle Labor Supply

A. Elementary discrete-time, perfect foresight model

1. initial assumptions

   a. consider a model with T discrete periods (t = 0, T)

   b. there is a maximum amount of time in each period which can be divided between market and non-market activities, $L^* = H_t + L_t$ for $t = 0, T$

   c. wages in each period, W_t, and the person's initial endowment of wealth, A, are given exogenously

   d. there is perfect foresight in the model (the values for all economic variables are known in period 0)

   e. there are perfect capital markets, the person can borrow and save any amount at the market interest rate

   f. there is a constraint on ending wealth; at the end of period T, the person's wealth must equal zero (note: could easily extend the model to include bequests)

2. utility function

   a. person has preferences defined over consumption and non-market time in all periods such that

$$U = U\left(C_0, C_1, \ldots, C_T, L_0, L_1, \ldots, L_T\right)$$

   b. person chooses consumption and hours of work in all periods to maximize this utility specification subject to the time constraints described above and the life-time budget constraint described below

3. life-time budget constraint
   a. let Z_t denote the person's net worth in period t
   b. net worth evolves according to

$$Z_{t+1} - Z_t = r\,Z_t + W_t\,H_t - P_t\,C_t$$

   where r denotes the interest rate (assumed constant over time) and P_t denotes the period-specific price of consumption goods
   c. given the initial and terminal wealth conditions, the life-time budget constraint can be expressed

$$A + \sum_{t=0}^{T} (1+r)^{-t}\, W_t\, H_t = \sum_{t=0}^{T} (1+r)^{-t}\, P_t\, C_t$$

   d. a "full-income" equivalent can be written

$$A + \sum_{t=0}^{T} (1+r)^{-t}\, W_t\, L^{*} = \sum_{t=0}^{T} (1+r)^{-t} \left(P_t\, C_t + W_t\, L_t \right)$$

$$F = \sum_{t=0}^{T} (1+r)^{-t} \left(P_t\, C_t + W_t\, L_t \right)$$

4. hours functions
   a. this is a standard consumer choice problem; we will obtain demand-type solutions
   b. let $R_t = (1 + r)^{-t}$
   c. the solutions can be written

$$H_t = H_t\left(R_0 W_0, \ldots, R_T W_T, R_0 P_0, \ldots, R_T P_T, A\right)$$
$$C_t = C_t\left(R_0 W_0, \ldots, R_T W_T, R_0 P_0, \ldots, R_T P_T, A\right)$$

d. these solutions will satisfy standard demand conditions

e. consider the Slutsky equation

$$\left.\frac{\partial H_t}{\partial R_\tau W_\tau}\right|_U = \left.\frac{\partial H_t}{\partial R_\tau W_\tau}\right|_A + H_\tau \frac{\partial H_t}{\partial A}$$

compensated own wage effect should be positive;
could examine other properties of Slutsky matrix

f. increase in *A*: income effect which leads to decreases in labor supply in all periods

g. increase in W_t leads to

- income effect: decrease in labor supply which is spread (diluted) across all periods
- standard substitution effect: change in relative price of consumption and leisure in period *t* leads to an increase in labor supply in period *t*
- intertemporal substitution effects: work in period *t* becomes more valuable (leisure in *t* becomes more expensive) relative to work in other periods, substitute away from work in other periods and toward work in period *t*

5. empirical issues

   a. what happened to nonlabor income? Nonlabor income in each period is represented by rZ_t which is clearly endogenous. Exogenous equivalent to nonlabor income is the initial endowment, A.

   b. per-period labor supply and consumption are now functions of current wages and prices but also of wages and prices in <u>all</u> other periods; this represents a tremendous data burden

   c. for estimation, we need to consider restrictions that reduce the data burden

B. Time-separable utility specifications

1. utility specification

   a. let the utility function be given by

$$U = \sum_{t=0}^{T} (1+\rho)^{-t} U(C_t, H_t) = \sum_{t=0}^{T} D_t U(C_t, H_t)$$

   where $D_t = (1+\rho)^{-t}$

   b. can rewrite the maximization problem as a Lagrangian

$$\underset{C_0,\ldots,C_T,H_0,\ldots,H_T}{\text{Max}} \quad L = \sum_{t=0}^{T} D_t U(H_t, C_t) + \lambda\left[A + \sum_{t=0}^{T} R_t (W_t H_t - P_t C_t)\right]$$

2. first-order conditions are

$$H_t: \quad D_t \frac{\partial U}{\partial H_t} + \lambda R_t W_t = 0$$

$$C_t: \quad D_t \frac{\partial U}{\partial C_t} - \lambda R_t P_t = 0$$

$$\lambda: \quad A + \sum_{t=0}^{T} R_t (W_t H_t - P_t C_t) = 0$$

3. the solutions are

$$H_t = H_t\left(\frac{R_t}{D_t} W_t, \frac{R_t}{D_t} P_t, \lambda\right)$$

$$C_t = C_t\left(\frac{R_t}{D_t} W_t, \frac{R_t}{D_t} P_t, \lambda\right)$$

these are referred to as λ-constant supply and demand functions or Frisch-type functions

a. note that the labor supply and consumption functions depend on (adjusted) within-period prices and wages and λ

b. everything from other periods is summarized by λ; if we could obtain an estimate for λ or control for λ, this would greatly simplify the empirical requirements

4. interpretation of wage effects for these functions; can decompose effects of a wage change as follows

$$\frac{\partial H_t}{\partial W_t} = \left.\frac{\partial H_t}{\partial \lambda}\right|_W \frac{\partial \lambda}{\partial W_t} + \left.\frac{\partial H_t}{\partial W_t}\right|_\lambda$$

a. evolutionary wage changes (represented by second term above); this corresponds to a change in the wage profile that leaves λ unchanged (e.g., planned wage growth)

- this effect should be positive

- implies hours profile and wage profile should have same general shape
- similar to own-period substitution effect from earlier discussion but holding marginal utility of wealth constant

b. parametric wage changes; this is a shift in the entire wage profile; it produces evolutionary effects as well as wealth effects; the net effect is ambiguous

c. one-period parametric wage change; this is special type of parametric shift in which the wage change is limited to one period

- it leads to effects that are similar to a parametric change but with diminished wealth effects
- similar to income, own-period, and intertemporal substitution effects described earlier (because of separability, all intertemporal effects operate through λ)

5. MaCurdy (1981)

a. utility specification (CES per-period utility functions)

$$U_i = \sum_{t=0}^{T} D_t \left(\gamma_{it} C_{it}^{\omega_1} - \phi_{it} H_{it}^{\omega_2} \right)$$

where $0 < \omega_1 \leq 1$ and $\omega_2 > 1$

b. specification for hours function

1) assuming an interior solution, the labor supply function is

$$\ln H_{it} = \frac{1}{\omega_2 - 1}\left[\ln\lambda_i - \ln\phi_{it} - \ln\omega_2 + \ln {R_t}/{D_t} + \ln W_{it}\right]$$

2) let $\ln \phi_{it} = X_i\beta + \eta_i - \varepsilon_{it}$, $\delta = (\omega_2 - 1)^{-1}$, and $b = \delta \ln(\rho - r)$

3) then we can write an empirical counterpart

$$\ln H_{it} = F_i + bt + \delta \ln W_{it} + \varepsilon^*_{it}$$

c. estimation procedure

1) estimate b and δ using fixed effects procedure (i.e., differencing or mean-differencing observations for the same individual across years) to condition out F_i

- advantage: can obtain estimates with as few as two longitudinal observations

- disadvantage: this only gives us estimates of the evolutionary effect; we cannot examine general effects or make predictions without an estimate of F_i

2) approximate F_i by regressing estimated fixed effects on life-cycle variables

d. data

1) 1967-76 annual labor supply data on 513 prime-age, white, married males from the Panel Study of Income Dynamics

2) men were 25-46 in 1967, continuously married to the same spouse, and had labor supply data available for all ten years

e. results

1) evolutionary elasticity (δ): .10 to .23

2) own period elasticity: .10 to .50

3) cross-period elasticity: nearly zero

C. Extensions in the time-separable framework

1. Heckman and MaCurdy (1980)

a. important extensions to MaCurdy (1981)

1) consider female labor supply

2) consider attendant problems of zero hours and missing wages

b. employ fixed effects Tobit procedure

1) because of the Tobit procedure is nonlinear, the fixed effects cannot be "conditioned out" of the estimating equation; this leads to the incidental parameters problem

2) estimates of the fixed effects <u>and</u> the other coefficients are only consistent as $T \to \infty$ (instead of $N \to \infty$).

3) some other Monte Carlo work by Heckman suggests that resulting bias is likely to be small

c. estimation of conditional fixed effects procedure

1) procedure requires at least one "working" observation per individual

2) total sample of 672 white women 30 to 65 years; 452 (2/3) worked at least once in eight years

3) adopt a simple procedure to account for sample selection bias from "ever-worked" criteria

2. Incorporating uncertainty into a family life-cycle model (Browning Deaton and Irish 1985)

   a. utility specification

      1) household chooses leisure for two individuals (l_{1t} and l_{2t}) and consumption q_t in period t

      2) to maximize

$$u_t(l_{1t}, l_{2t}, q_t) = \mathrm{E}_t \sum_{k=t+1}^{T} u_k(l_{1k}, l_{2k}, q_k)$$

      where E_t is the expectations operator and the sub-period utilities incorporate the effects of discounting

      3) subject to a per period budget process

$$A_{t+1} = (A_t + w_{1t}h_{1t} + w_{2t}h_{2t} - p_t q_t)/(1+r_t)$$

      and terminal conditions for A_0 and A_T.

   b. let $v_t(A_t, w_{1t}, w_{2t}, p_t)$ be the indirect sub-period utility function at t, and let $v_t^*(A_t)$ be the sum of current and expected future utilities as perceived at t given assets A_t

c. optimization gives

1) at $t = T$, $v_T^*(A_T) = v_T(A_T, w_{1T}, w_{2T}, p_T)$

2) at $t < T$,

$$v_t^*\left(A_t\right) = \max\left\{v_t\left(A_t, w_{1t}, w_{2t}, p_t\right) + E_t\left[v_{t+1}^*\left(A_t + w_{1t}h_{1t} + w_{2t}h_{2t} - p_t q_t\right)\right]\right\}$$

3) with labor supply solutions

$$h_{1t} = f_{1t}(\lambda_t, w_{1t}, w_{2t}, p_t)$$
$$h_{2t} = f_{2t}(\lambda_t, w_{1t}, w_{2t}, p_t)$$

where λ_t is the *expected* marginal utility of wealth at t (very similar to λ-constant demands from MaCurdy but with stochastic marginal wealth controls)

4) estimate model using an approximate technique that essentially comes down to appending an error term to λ

C. Intertemporally nonseparable preferences (Hotz, Kydland and Sedlacek 1988)

1. Theoretical model

a. individual maximizes $E_t \sum_{\tau=t}^{T} \beta^{\tau-t} U\left(Z_\tau, C_\tau\right)$ where C_τ is consumption at τ and Z_τ is a distributed lag in nonmarket time

b. numerous studies suggest that preferences for leisure may be intertemporally nonseparable (e.g., current benefits from previous nonmarket production and experience; alternatively, could represent habit formation and consumption capital)

c. specifically, assume Z_t evolves as $Z_t = l_t + \alpha a_t$ where $a_t = l_{t-1} + (1-\eta)a_{t-1}$

c. individuals face a per-period budget constraint

$$A_{t+1} = (A_t + w_t h_1 - C_t) / (1 + r_t)$$

with terminal conditions on A_0 and A_T.

d. leads to a dynamic programming problem

$$V^t(A_t,\ a_t,\ w_t) = \max\Big\{U(Z_t,\ C_t) + \beta E_t V^{t+1}\big[\gamma_t(A_t + w_t h_t - C_t),(1-\eta)a_t + l_t,\ w_{t+1}\big]\Big\}$$

e. can show that the solutions satisfy

$$E_t\{U_C(t) - \beta\gamma_t U_C(t+1)\} = 0$$

$$E_t\Big\{-\big[U_Z(t) - w_t U_C(t)\big] - \beta\Big(\alpha U_Z(t+1) - (1-\eta)\big[U_Z(t+1) - w_{t+1}U_C(t+1)\big]\Big)\Big\} = 0$$

f. actual realizations will follow

$$U_C(t) - \beta\gamma_t U_C(t) = u_{1,t+1}$$

$$-\big[U_Z(t) - w_t U_C(t)\big] - \beta\Big(\alpha U_Z(t+1) - (1-\eta)\big[U_Z(t+1) - w_{t+1}U_C(t+1)\big]\Big) = u_{2,t+1}$$

g. rational expectations implies $E_t(u_{t+1}) = 0$ and that these errors are orthogonal to all information available at and up to time t

2. Generalized Method of Moments procedure

   a. parameterize utility specification (Hotz et al. use translog specification)

   b. form sample moments that are the product of the realization errors and the information set variables

   c. minimize a weighted quadratic $O_N(\theta)' \; W_N \; O_N(\theta)$ where the optimal (most efficient) weighting matrix is chosen according to Hansen (1982)

   d. note that Hotz et al. base most of their results on the first optimality condition in consumption; structural estimates and tests for nonseparability here are independent of the precise evolution of wages (not sensitive to assumptions about exogeneity of wages)