Nonlinear Budget Constraints

A. Introduction
   1. So far, we have considered a very simple description of the budget constraint
      a. wages are constant as hours increase
      b. non-labor income is fixed and does not depend on other characteristics such as income

   2. In this section, we are going to consider more realistic budget constraints that account for taxes, government subsidies, work costs and the like

   3. A common feature of these modifications is that they lead to non-linear budget constraints, which in turn lead to estimation issues

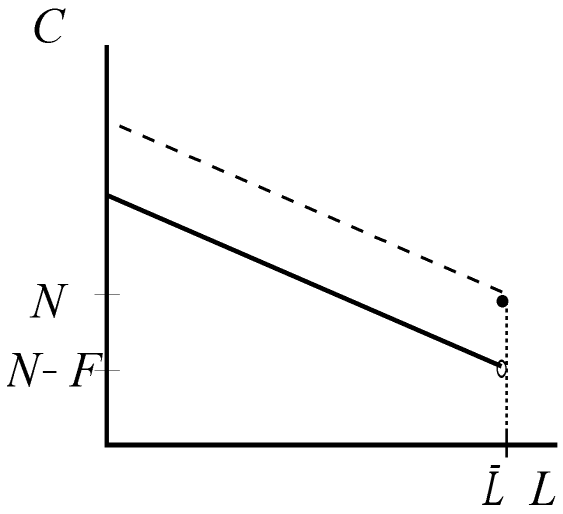
B. Examples of non-linear budget constraints
   1. Fixed money costs of work
      a. consider several possible expenses associated with work such as child care, union dues, uniforms
      b. sometimes these expenses do not vary with the number of hours worked (i.e., are lump-sum expenses)
      c. effect on budget constraint
         1) let the (initial) budget constraint without fixed costs of work be $Y = WH + N$

2) with fixed costs, the (new) budget constraint is

$Y = N$ if the person does not work

$Y = WH + N - F$ if the person works

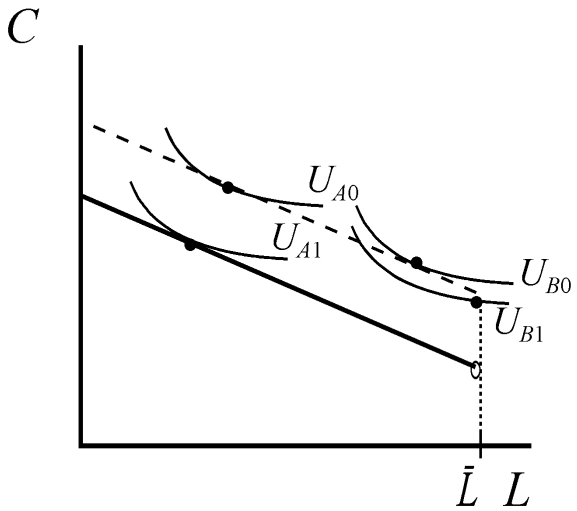
3) the resulting budget constraint has a notch at the no work point



d. effect on labor supply

1) compare behavior for people who do and do not face fixed costs of work

2) for people who work many hours (like person A on right), the imposition of fixed costs is similar to a drop in non-labor income; it will cause these people to work more



3) for people who work few hours (like person B above), it may be optimal to stop working

4) imposition of a fixed cost increases the reservation wage and results in a discontinuous labor supply function

2. Effect of taxes on labor supply

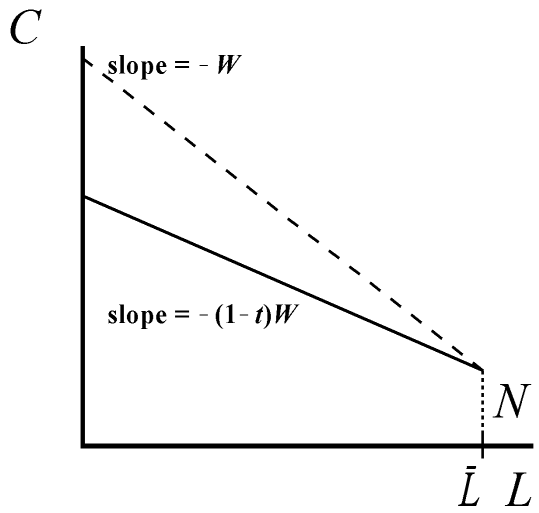
   a. effect of a flat proportional tax on earnings

      1) consider an individual with a pre-tax budget-constraint $Y = WH + N$

      2) assume that the government imposes a flat proportional tax of t on earnings; the after-tax budget constraint is

$$\begin{aligned} Y &= WH + N - tWH \\ &= (1-t)WH + N \end{aligned}$$

      3) graphically, proportional tax decreases the effective wage from W to $(1-t)W$



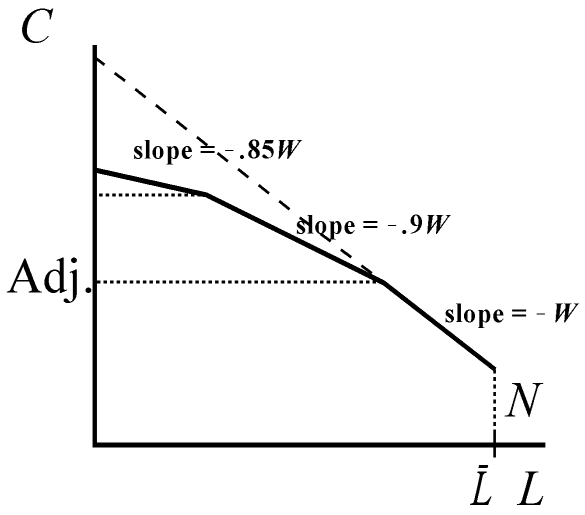
      4) equivalent to a wage decrease

      5) will cause substitution

(–) and income (+) effects; net effect on labor supply will be ambiguous

6) is this a realistic description of the tax system?
   - describes social security & some state systems
   - poor description of federal system

b. progressive income tax
   1) the federal income tax is mostly *progressive*
   2) progressive means that the marginal tax rate increases with income (what would regressive mean?)
   3) in 2007, the marginal tax rates for a single taxpayer are:
      - 10% for adjusted incomes below $7,825
      - 15% on the next increment up to $31,850
      - 25% on the next increment up to $77,100
      - 28% on the next increment up to $160,850
      - 33% on the next increment up to $349,700
      - 35% on adjusted incomes thereafter
   4) tax rates only apply to “adjusted income” (income less deductions, exemptions)
   5) initial tax rate is 0 up to adjustment amount
   6) the effect of a progressive tax on the budget constraint is shown below



- first segment corresponds to income below adjusted amount
- next segment corresponds to first taxable bracket (10% marginal tax rate)
- final segment corresponds to second taxable bracket (15% marginal tax rate)

7) general concave shape

c. 2001 Tax Relief Act

1) cuts tax rates over next several years
2) for instance, introduced the 10% bracket into the 15% bracket
3) 28% bracket from 2000 has been reduced to 25%
4) highest tax bracket has been reduced from 40% to 35%
5) graphically, budget constraint has swivelled out

6) what are the effects on labor supply?

d. other features of the federal income tax system
    1) these rates aren't the only feature of the federal system; there are other exclusions, credits, etc.
    2) retirement related features: exclusion of pension contributions from income; deferment of earnings on pension savings; exclusion of some individual retirement savings and Social Security benefits
    3) poverty related: Earned Income Tax Credit
    4) health related: exclusion of employer contributions for insurance and medical care; exclusion of Medicare benefits; deductibility of large medical expenses
    5) employment related: dependent care tax credit; exclusion of employer-provided child care; targeted jobs tax credits; exclusion of certain employee benefits
    6) housing related: mortgage interest deduction
    7) these features make the actual tax system much more complicated to model
e. state and local taxes add to the complexity

3. Means-tested transfers
    a. consider a *transfer program* (program that transfers money from the government to people) that
        1) provides a guaranteed benefit *G* for people without income, and

2) reduces the benefit by a rate *BRR* for each dollar of income they receive

3) total benefits would be

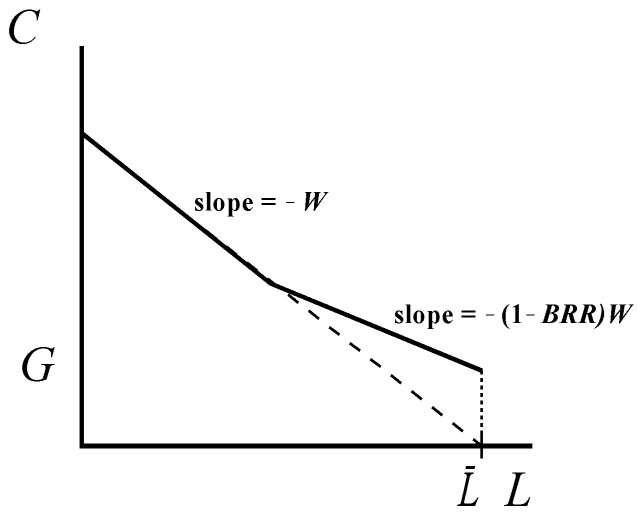
$$B = \text{Max}(G - BRR \times Y, 0)$$

b. graphically,

1) assume pre-transfer budget constraint is $Y = WH$ (i.e., no non-labor income); also, ignore other taxes

2) people with no income receive G

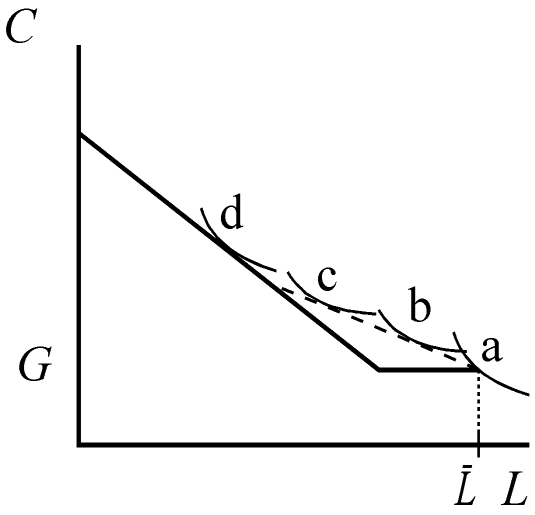
3) earnings are effectively taxed at a rate *BRR* until benefits are exhausted



c. big work disincentive

d. what effect would an increase in the benefit reduction rate to 100% have on labor supply and program participation?

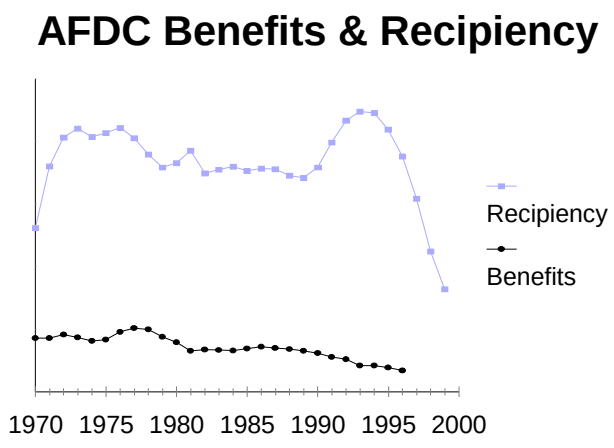
1) people who don't work or work little initially (like a and b) will not work



2) people were participating but working (like c) near the cut-off will work more
3) people who were not participating (like d) will not be affected

e. AFDC and TANF

1) AFDC – Aid to Families with Dependent Children; old welfare program
2) TANF – Temporary Assistance for Needy Families; program after 1996 reforms
3) trends in programs
   - real value of benefits peaked in the 1970s; benefits steadily eroded



thereafter (when Food Stamps and Medicaid are added, benefits still fall through the 1980s and 90s)

- recipiency rose sharply during the late 1960s and early 1970s then declined
- recipiency also rose sharply during the late 1980s and early 1990s but has fallen dramatically since; recipiency is now lower than in 1970

Differences between AFDC and TANF

	AFDC	TANF
Federal funding	Unlimited matching	Fixed grant
State funding	Matching required	Maintenance of effort requirement
Eligibility	Single parent families with children; two parent families with unemployed parent	Set by state
Income limits	Set by state	Set by state
Benefit levels	Set by state	Set by state
Entitlement	States required to assist all families eligible under income standards	No entitlement
Work requirement	Participation in JOBS program	By 2002, half of state's caseload must be in specified work activities
Exemptions from work requirement	Parents with children under 3 (or 1 at state's discretion)	None, though states may exempt single parents with children under 1
Work trigger	None	Required after 2 years
Time limits	None	5 year limit (20% hardship exemptions)

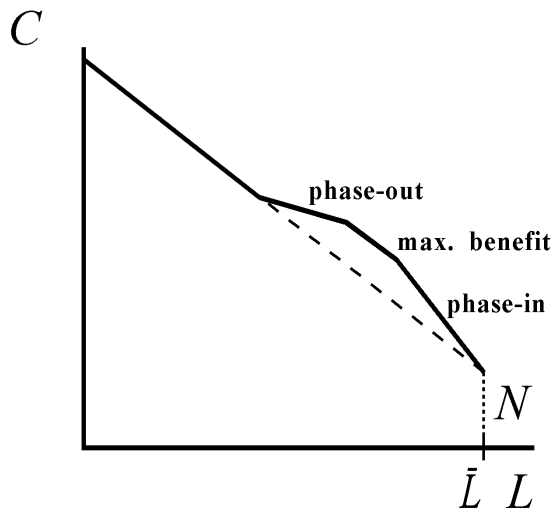
Source: 1998 *Green Book*, Table 7-1.

State-by-state Welfare Benefits 2000

State	TANF	+ Food Stamps	% pov. stand.	State	TANF	+ Food Stamps	% pov. stand.
Ala.	164	490	42	Mont.	469	703	61
Alaska	923	1101	95	Neb.	364	630	54
Arizona	347	618	53	Nevada	348	618	53
Ark.	204	518	45	N. Ham.	575	777	67
Calif.	626	813	70	N. Jer.	424	672	58
Colo.	357	625	54	N. Mex.	439	682	59
Conn.	636	820	71	N. York	577	779	67
Del.	338	611	53	N. Car.	272	565	49
D.C.	379	640	55	N. Dak.	457	695	60
Florida	303	587	51	Ohio	373	636	55
Georgia	280	571	49	Okla.	292	579	50
Hawaii	712	1061	92	Oregon	460	697	60
Idaho	293	580	50	Penn.	421	669	58
Illinois	377	639	55	R. I.	554	763	66
Indiana	288	576	50	S. Car.	204	518	45
Iowa	426	673	58	S. Dak.	430	676	58
Kansas	429	675	58	Tenn.	185	504	44
Kent.	262	558	48	Texas	201	515	45
Louis.	190	508	44	Utah	451	690	60
Maine	461	697	60	Ver.	708	870	75
Mary.	417	667	58	Virginia	354	623	54
Mass.	579	780	67	Wash.	546	757	65
Mich.	459	696	60	W. Va.	328	604	52
Minn.	532	789	68	Wisc.	673	846	73
Missi.	170	494	43	Wyom.	340	613	53
Misso.	292	579	50				

Source: 2000 *Green Book,* Tables7-7, 7-8 and U.S. Census Bureau.

4. The Earned Income Tax Credit
    a. *refundable* tax credit available to people with low to moderate levels of earned income
    b. background
        1) first implemented in 1975 to reduce tax burden associated with social security
        2) expanded several times, most significantly in 1990 and 1993; now viewed as a major anti-poverty program
        3) costs increased from $1.25 billion (6.2 million families) in 1975 to $30 billion (19 million families) in 1999
        4) some states have similar credits in their tax systems
        5) important and often-overlooked component of welfare reform (one of the kinder provisions)
    c. general structure
        1) percentage supplement to earnings up to some maximum benefit (benefit increases with earnings)

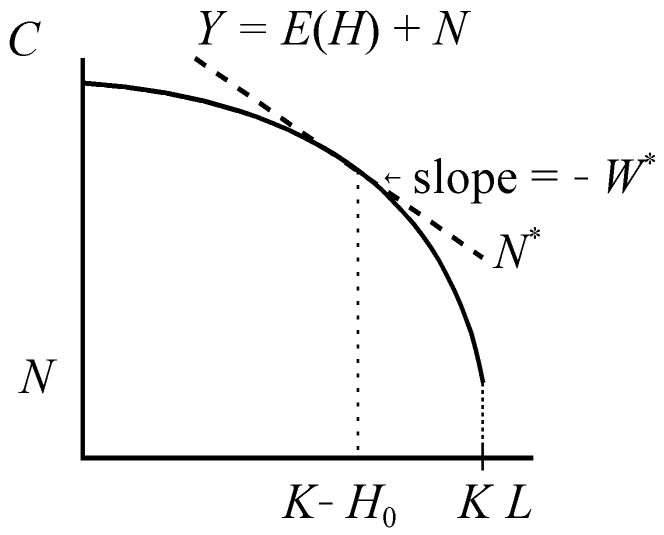


        2) benefit is fixed for some intermediate range of earnings

3) benefit is phased out over some other range
4) provisions vary across family size
5) limitations on unearned income and other factors

d. six schedules as of 2006 (three for married, three for single); for single people
   1) one child – 34% of earnings up to $8,080; maximum credit $2,747; phase-out rate 16% after $14,850
   2) two children – 40% of earnings up to $11,340; maximum credit $4,536; phase-out rate 21% after $14,850
   3) no children – 7.65% of earnings up to $5,390; maximum credit $412; phase-out rate 7.65 % after $8,750

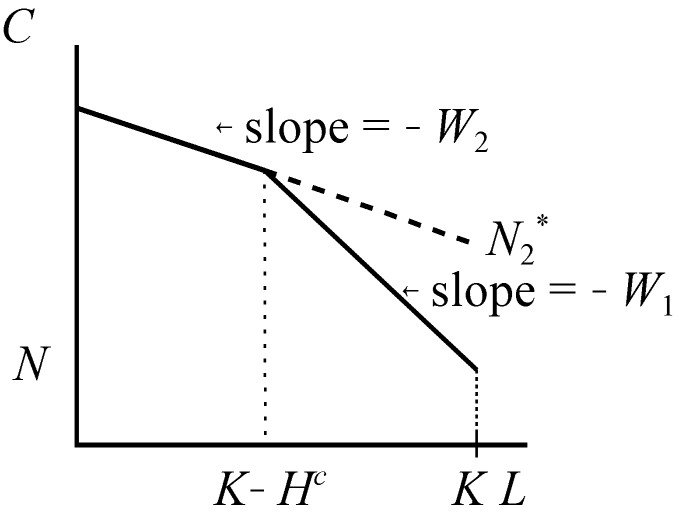
C. Virtual prices and incomes
   1. Consider a general budget constraint $Y = E(H) + N$ where $E(H)$ is a nonlinear earnings function
   2. Given this constraint, suppose that a person chooses some level of work H_0
   3. At this level of work, we can define a virtual or effective wage, $W^* = E'(H_0)$; this is simply the slope of the budget constraint at H_0
   4. We can also define a virtual nonlabor income $N^* = E(H_0) + N - W^*H_0$
   5. Graphically,



6. With the virtual wage and virtual non-labor income, we have linearized the budget constraint at H_0
   a. consider an indirect utility function v and a labor supply function h defined for the general linear budget constraint problem (i.e., the same functions that we have considered previously)
   b. at H_0, the person's indirect utility is $V = v(W^*, N^*)$
   c. optimal hours are $H_0 = h(W^*, N^*)$
   d. that is, at H_0, the solution to the nonlinear budget constraint problem is the same as the linearized one

D. Estimation issues for a two-part convex budget set
   1. Consider a person who faces a budget constraint in which
      a. N is the non-labor income
      b. W_1 is the wage for hours worked up to H^c, and
      c. W_2 ($< W_1$) is the wage for hours worked beyond H^c

2. Assume that the person has a utility function $U(C, L)$ such that the optimal labor supply in the presence of a linear budget constraint with wage W^* and non-labor income N^* is $H = h(W^*, N^*)$
3. Graphically,
   a. for first segment, virtual wage is W_1 and virtual non-labor income is N
   b. for second segment, virtual wage is W_2 and virtual non-labor income is $N_2^* = N + (W_1 - W_2)H^c$



4. What would happen if we just regressed observed hours on observed virtual wages and non-labor incomes
   a. note that the person's choice follows
      1) segment 1: $H = h(W_1, N)$ if $h(W_1, N) < H^c$
      2) segment 2: $H = h(W_2, N_2^*)$ if $h(W_2, N_2^*) > H^c$
      3) at H^c, if $h(W_1, N) \geq H^c$ and $h(W_2, N_2^*) \leq H^c$
   b. consider a simple linear specification for h: $h(W^*, N^*) = \alpha + \beta W^* + \gamma N^*$
   c. assume that α varies across individuals but that β and γ do not
   d. run OLS on observations drawn from segment 1

$$E(H \mid H < H^c) = \beta W_1 + \gamma N + E(\alpha \mid H < H^c)$$
$$= \beta W_1 + \gamma N + E(\alpha \mid \alpha < H^c - \beta W_1 - \gamma N)$$

e. Clearly, OLS leads to biased results
   1) α is truncated from above
   2) the truncation depends on the independent variables

5. general problem with non-linear budget constraints is that people self-select onto particular segments of the budget constraint; observed virtual wages and non-labor incomes depend on budget factors but also preference factors
6. instead of OLS, could use a technique similar to the endogenous switching regression method (either MLE or 2-step) to estimate this model; Moffitt (1986) describes methods for two-part convex and concave budget constraints

E. Estimation issues for more complex budget constraints
1. two-part budget constraint is a restrictive and special case; in general, post-tax, post-transfer budget constraints are much more complicated
2. similar set of selection issues arise with more complicated models; however, the number of segments and kinks makes it impractical to model all of the possible choices
3. standard approach has been to "discretize" the budget constraint–that is, to restrict the number of choices of hours (e.g., no work, part-time, full-time)–and consider

discrete decisions to participate in programs or take-up particular sets of benefits

4. with just a few programs or elective tax features, the number of choices can still grow quickly
5. researchers typically specify a utility function; so, approach is structural (this is an advantage, as reactions to new types of budget constraints can be simulated)
6. utility function typically includes heterogeneity in tastes and perhaps program stigma
    a. error terms are sometimes given a simple and easily computed distribution like multivariate, independent extreme value distribution
    b. alternatively, more complex distributions can be used; however, this increases the computational burden of the model (see, e.g., Keane & Moffitt 1998)