Static Models of Labor Demand

A. Introduction

1. Although the content of this course to date suggests otherwise, there are actually two sides to the labor market: households who supply labor and firms that demand it
2. A substantial amount of research in labor economics focuses on the demand or firm side
3. The study of labor demand goes beyond basic firm production and input decisions to consider special features of the labor market
4. We will start, however, with the basic decisions in a static (one period) framework

B. Competitive firm with one input

1. Consider a profit maximizing firm operating in competitive product and factor markets in which it takes the prices in those markets as given
2. The firm produces a single type of output, Q, with a single input, labor (L) according to a production function $\varphi(L)$ where $\varphi' > 0$ and $\varphi'' < 0$
3. Each unit of output is sold at a price, p, and each unit of labor is purchased at a price/wage, w
4. Profits for the firm are defined as the difference between revenues and costs and can be written

$$\Pi = p\,\varphi(L) - w\,L$$

5. The firm chooses amounts of its sole input, *L*, to maximize this profit function; stated another way, it maximizes profits subject to technological and cost constraints
6. At an interior solution, the marginal revenue product of labor equals the marginal cost, or wage rate, of labor

$$p\,\varphi'(L^*) = w$$

7. Implicit differentiation shows that the firm's demand for labor is downward sloping

$$\frac{\partial L^*}{\partial w} = \frac{1}{p\varphi''(L^*)} < 0$$

C. Firms with market power

1. If the firm has some power to set prices in its product market instead of being competitive, it will face a downward sloping price function, $p(Q)$ with $p' < 0$
2. The optimal (interior) labor demand will then satisfy

$$p'[\varphi(L^*)]\,\varphi'(L^*)\,\varphi(L^*) + p[\varphi(L^*)]\,\varphi'(L^*) = w$$

3. Let η represent the absolute value of the elasticity of output demand; the interior FOC can be re-expressed

$$\varphi'(L^*)\left[1 - \frac{1}{\eta}\right] = \frac{w}{p\left[\varphi(L^*)\right]}$$

4. The absolute wage elasticity of the demand for labor *increases* with the price elasticity of product demand (first law of derived demand)

5. Suppose instead that the firm has some power to affect prices in the labor market—that is, that it has some *monopsony* or buyer power
    a. the firm now faces an upward sloping labor supply curve
    b. we can write the inverse supply function as $w(L)$ where $w' > 0$
6. The first-order conditions for an interior solution are

$$p\,\varphi'(L^*) = w(L^*) + w'(L^*)\,L^*$$

7. The effective marginal cost of labor (right hand side of equation) is higher for a monopsonistic firm than for a competitive firm; given $\varphi'' < 0$, a monopsonistic firm will employ labor at a level below the competitive equilibrium

D. Two factor model

1. Let's adjust the model to include a second factor, K (e.g., capital)
2. Production now depends on L and K, such that
    a. $Q = F(L\,,K)$
    b. where F is increasing at a diminishing rate in each of its arguments $F_i > 0$, $F_{ii} < 0$ for $i = L, K$
    c. also assume that $F_{LK} > 0$ (capital adds to the productivity of labor and vice versa)
3. Each unit of K costs r
4. Profits can now be written (for convenience, we will drop the product price term)

$$\Pi = F(L, K) - w\,L - r\,K$$

5. The firm chooses both L and K to maximize profits; the first-order conditions at the interior are

$$F_L = w \quad \text{and} \quad F_K = r$$

6. These lead to the efficiency in production condition
   a. $F_L / F_K = w / r$
   b. marginal technical rate of substitution equals the ratio of the marginal costs/prices
7. Consider the elasticity of substitution between L and K to a change in their relative prices holding output constant; denote this elasticity as σ (> 0)
   a. let s denote the share of labor costs in total revenue, where $s = w\,L / Q$
   b. then the elasticity of labor demand with respect to a change in wages (its own price) holding output and the price of capital constant is $\eta_{LL} = -(1 - s)\,\sigma < 0$
   c. the elasticity of labor demand with respect to a change in the price of capital holding output constant is $\eta_{LK} = (1 - s)\,\sigma > 0$
   d. sometimes refer to these as "substitution effects"
8. Changes in the costs of inputs won't only change the mix of inputs but also the overall cost of production; this in turn will affect the product price and the amount sold
   a. we refer to the changes in capital and labor associated with changes in the amount of output as "scale effects"

b. the total own-price elasticity of labor (including scale effects) can be written $\eta'_{LL} = -(1 - s)\sigma - s\eta$, where η is the own-price elasticity of output

   1) first term is the substitution effect

   2) second term is the scale effect

c. the absolute own-price elasticity of labor demand increases with the substitutability of other factors of production (second law of derived demand)

d. total cross-price elasticity of labor can be written $\eta'_{LK} = (1 - s)\,(\sigma - \eta)$

9. The foregoing discussion assumes that the firm operates in perfectly competitive factor markets

   a. the firm may instead have some power over capital or other factor prices

   b. the absolute own-price elasticity of labor demand will increase with the factor-price elasticity of other factors (third law)

10. Finally, the first three laws imply that the absolute own-price elasticity of labor demand will (usually) increase with the share of labor expenses in total production (fourth law of derived demand)

E. Cost minimization

1. The preceding analysis has examined firms' behavior through the lens of profit maximization; however, firms' behavior can be equivalently framed through the *dual* process of cost minimization

2. In the dual problem, the firm minimizes its expenditure on factor inputs (labor and capital) subject to technological constraints and an output constraint
3. The function that solves this problem is called a *cost function*
    a. the cost function is a function of all of the factor prices and of the level of output
    b. in the two-factor case, the function would be written $C = C(w, r, Q)$
    c. the cost function is increasing in each of its arguments; it is also homogeneous of degree 1 in the factor prices
4. Each of the elasticity measures from the production approach can also be computed using elements from the cost function; for example, the elasticity of substitution can be computed as

$$\sigma = \frac{CC_{wr}}{C_w C_r}$$

5. Depending on the data available or the problem at hand, it may be easier to work with the cost function than with the production function
6. It is especially easy to derive labor and capital demand functions from the cost function

$$L^* = C_w \quad \text{and} \quad K^* = C_r$$

F. Multifactor models

1. Issues are similar to two-factor case, though evaluation is more complicated
2. More consideration, however, needs to be given to possible separability or non-separability of inputs in the production function
3. Models are particularly useful to labor economists because they allow us to distinguish among different types of labor (e.g., high-skill vs. low-skill, different occupations, etc.) with their own prices and markets
4. Consider a general multifactor case
   a. let the production function be $Q = F(X_1, X_2,\ldots, X_N)$
   b. let the cost function be $C = C(w_1, w_2,\ldots, w_N)$
5. Can define the *partial elasticity of substitution* between factors *i* and *j* as σ_{ij}
   a. this elasticity holds output and the prices of other factors constant
   b. the elasticity can be conveniently evaluated as

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j}$$

   c. a shortcoming with this elasticity is that its magnitude is sensitive to the level of factor prices
6. We can also define the *partial elasticity of factor demand* as η_{ij}
   a. this is defined and computed as

$$\eta_{ij} = \frac{\partial \ln X_i}{\partial \ln w_j} = \frac{w_i X_i}{Q} \sigma_{ij} = s_i \sigma_{ij}$$

b. where $\eta_{ii} < 0$ and $\sum_j \eta_{ij} = 0$

c. this implies that $\eta_{ij} > 0$ for at least one $j \neq i$

7. Can now classify factors as being
   a. *p*-substitutes if $\eta_{ij} > 0$ or
   b. *p*-complements if $\eta_{ij} < 0$
8. Hamermesh gives the following example
   a. consider a firm that produces output using skilled and unskilled workers and at least one other factor
   b. if skilled and unskilled workers are *p*-substitutes, an increase in the wage rate for unskilled workers (e.g., an increase in the minimum wage) not only leads to a decrease in unskilled employment at a firm but also an increase in skilled employment
   c. if skilled and unskilled workers are *p*-complements, then an increase in the wage rate for unskilled workers leads to a decrease in skilled employment
   d. if skilled and unskilled workers are the only factors of production, they will necessarily be *p*-substitutes

G. Distinction between workers and hours

1. The discussion so far has not been very specific about the definition of labor inputs

a. have considered "units" of labor, but those units are hours supplied by different workers

b. need to consider a time period; within that time period it may not be possible for a single worker to provide all of the hours demanded

c. if this is the case, the firm then faces a decision between the *extensive margin* of how many workers to hire, E, and the *intensive margin* of how many hours to demand from each worker, H

2. Fixed and variable costs

   a. there are likely to be specific costs associated with each decision margin

   b. fixed costs of employment—these vary with employment but not with hours; examples might include hiring costs, training costs, certain types of benefits payments

   c. variable costs of employment—these vary with hours; wages would be the primary variable cost but certain types of benefits (e.g., retirement) and tax payments (e.g., FICA) would also be included

   d. we could further assume that wages vary with hours (e.g., overtime); for simplicity, however, we will ignore this

   e. let F denote the fixed costs, and let w denote the variable costs; the total costs of employment would be $E\,F + E\,H\,w$

3. Production function

a. suppose that the production function depends on capital and labor, $F^*(E, H, K)$

b. for simplicity, suppose further that capital and labor are separable so that $F^*(E, H, K) = F(L, K)$ where $L = \varphi_1(E)\, \varphi_2(H)$; note this restricts our ability to consider whether employment or hours are complements or substitutes

4. Firm's problem is to choose E, H, and K to maximize profits, which are defined as

$$\Pi = F(\varphi_1(E)\, \varphi_2(H), K) - E\, F - E\, H\, w - r\, K$$

5. The interior first order conditions are

a. $F_L\, \varphi_1' = F + H\, w$

b. $F_L\, \varphi_2' = E\, w$

c. $F_K = r$

6. The optimal allocation of workers and hours satisfies

$$\frac{\varphi_1'}{\varphi_2'} = \frac{F + Hw}{Ew}$$

a. increases in F increase the costs of employment

b. increases in w increase the costs of both employment and hours

References

Hamermesh, Daniel. *Labor Demand.* Princeton, NJ: Princeton University Press, 1993.