

# Implementation of Standards Setting for a Algebra II Benchmark Exam Using Cognitive Diagnosis

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# Outline

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- ▶ Discuss diagnostic models and the need for standard setting.
- ▶ Talk about standard setting.
- ▶ Talk about the general procedure.
- ▶ Briefly discuss the results.

# Diagnostic Models

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- ▶ As was previously discussed diagnostic models can provide a mastery profile,  $\alpha$ .
- ▶ The probability of a correct response is determined based on the set of attributes that a person has or has not mastered and the Q-matrix.
- ▶ The models vary in their complexity and how they define the probability of a correct response.
  - ▶ I will briefly discuss the DINA, Reduced RUM, and the LCDM

# The Q-Matrix

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- ▶ To aid in our discussion, we assume that we take an item from a test intended to measure basic math.

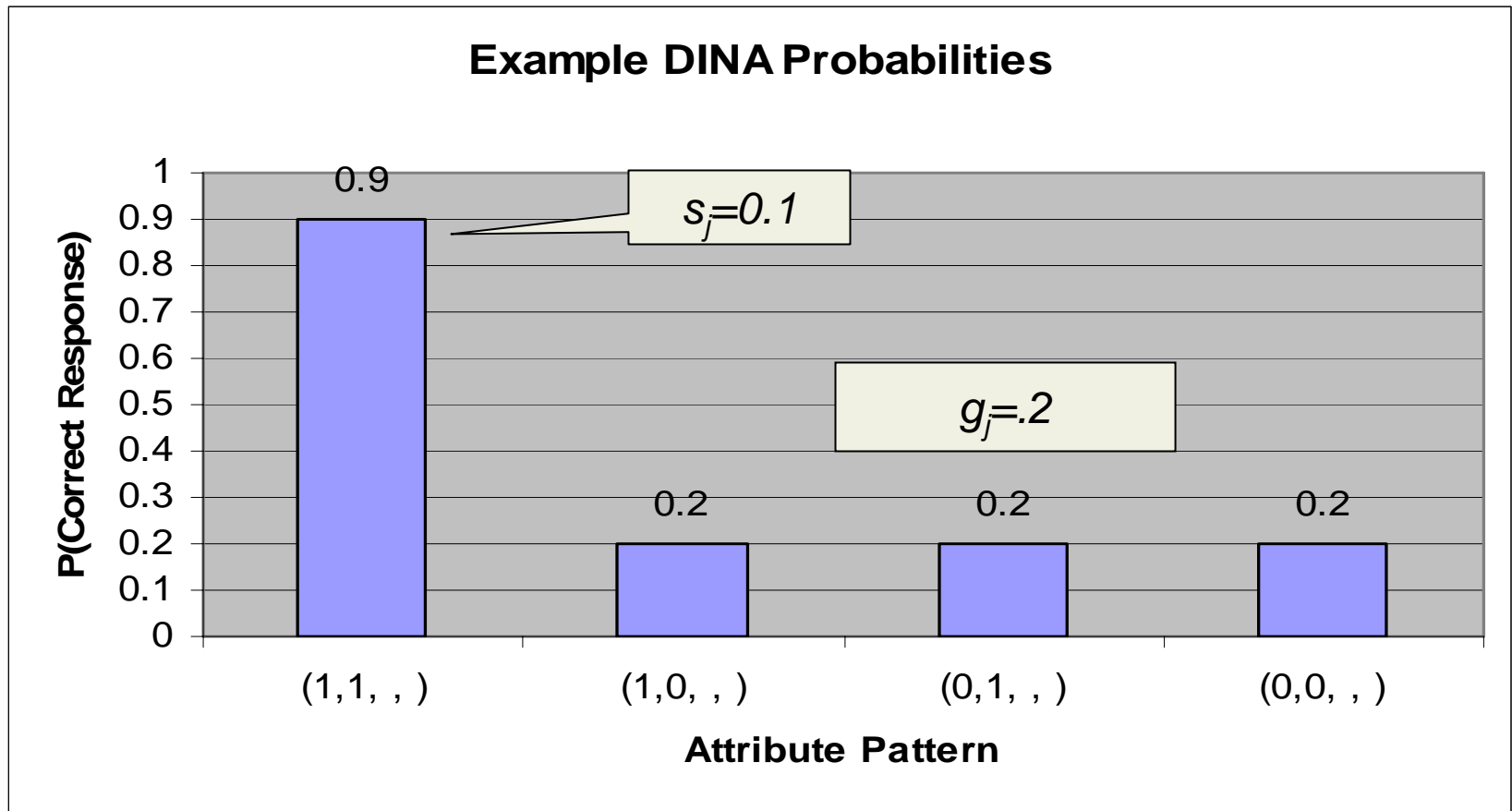
	<b>Add</b>	<b>Sub</b>	<b>Mult</b>	<b>Div</b>
<b>2+3-1</b>	1	1	0	0

# The DINA Model

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- ▶ One of the simpler models
- ▶ Divides people into two groups ( $\xi_{ij}$ )
  - ▶ Have mastered all required attributes
  - ▶ Have not mastered all required attributes
- ▶ Defines probability for those who have mastered all required attributes ( $1-s_j$ ).
- ▶ Defines probability for these who have not mastered all required attributes ( $g_j$ ).

# The DINA Model

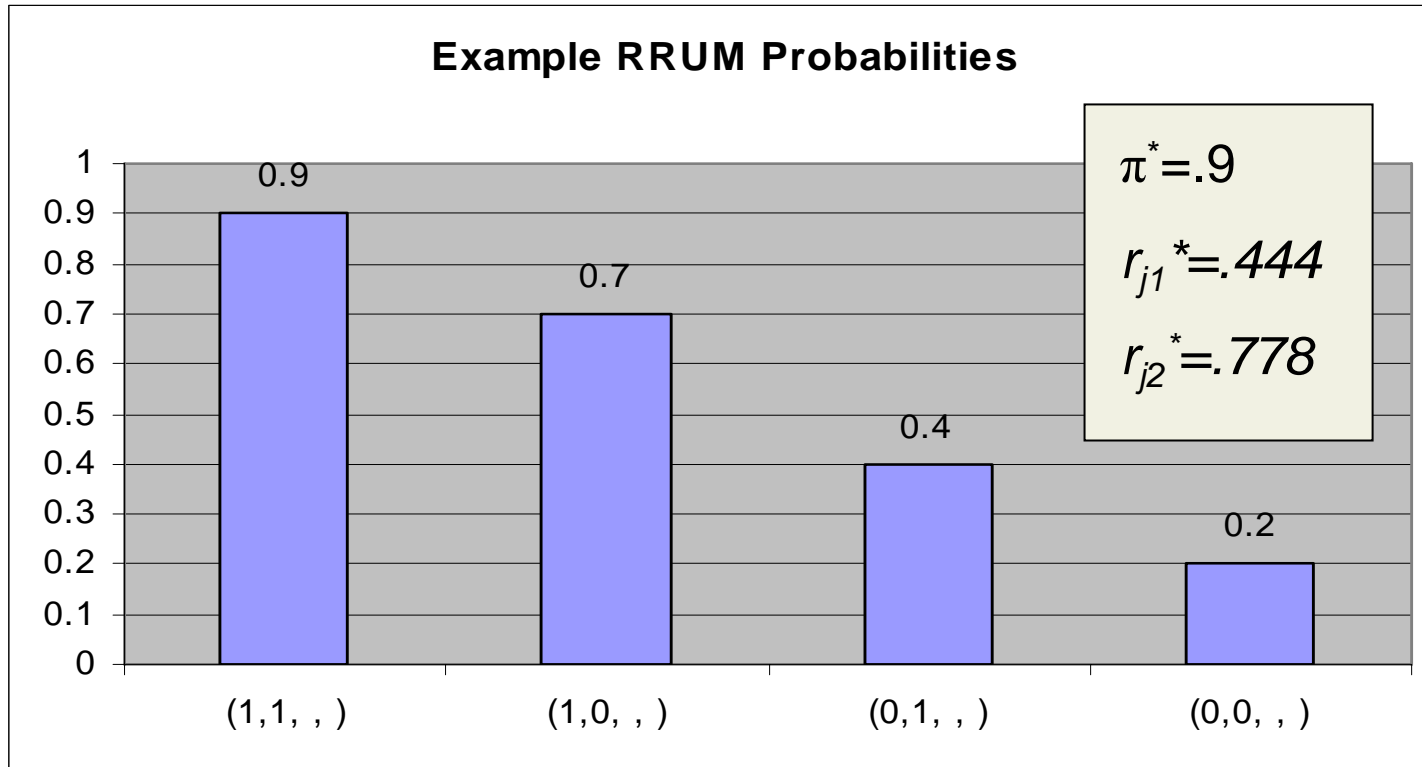


# Reduced Reparameterized Unified Model

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- ▶ As an alternative, the Reduced Reparameterized Unified Model (R-RUM) allows each attribute to contribute differently to the probability of a correct response
- ▶ Uses  $\pi^*$  to indicate the probability of a correct response if all attributes are mastered.
- ▶ The value  $r_{jk}^*$  is used to define the penalty of not mastering the  $k^{th}$  attribute

# Reduced Reparameterized Unified Model



This implies that knowing addition is more important than knowing subtraction for the problem  $2+3-1=?$

# The LCDM

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- ▶ In this particular case we use the Log-linear Cognitive Diagnosis Model (LCDM, Henson, Templin, and Willse, 2007).
- ▶ The LCDM is a special case of a log-linear model with latent classes (Hagenaars, 1993) and thus is also a special case of the General Diagnostic Model (GDM, von Davier, 2005).
- ▶ The LCDM defines the Logit of the probability of a correct response as a linear function of the attributes that have been mastered.

# The LCDM

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- ▶ The Logit of a probability is used because the items are dichotomous (correct or incorrect).
- ▶ A linear model is not reasonable for probabilities because of the bounds, so we transform the probability.

$$\text{Logit} = \ln\left(\frac{P(X_{ij} = 1)}{1 - P(X_{ij} = 1)}\right)$$

- ▶ The Logit ranges from a negative infinity to a positive infinity when using probabilities and so can be modeled as a linear function.

# The LCDM

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- ▶ Given our item, we model the logit of the probability of a correct response as a function of mastery or nonmastery of the two attributes.
- ▶ Specifically,

$$\ln\left(\frac{P(X_{ij} = 1)}{1 - P(X_{ij} = 1)}\right) = \lambda_{add}\alpha_{add} + \lambda_{sub}\alpha_{sub} + \lambda_{add*sub}\alpha_{add}\alpha_{sub} - \eta$$

# Diagnostic Models

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- ▶ The LCDM is a general model that allows all possible combinations of the required attributes to have a different probability of a correct response.
- ▶ However, the solution may or may not be consistent with other criteria.
- ▶ Specifically, examinees are classified based on their responses in a way that best divides examinees.
- ▶ But, this classification may not be optimal based on an alternative set of standards, such as the performance on End of Course exams

# Standard Setting

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- ▶ Here we discuss a pilot study designed to set the standards for an Algebra II benchmark exam.
- ▶ The purpose is to provide a general procedure for standard setting when using cognitive diagnosis models based on a modified Angoff procedure.
- ▶ Also, to provide a general description of possible concerns and limitations that must be considered.

# Standard Setting

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- ▶ Instead, we should set the probability of a correct response for each possible set of required attributes.
- ▶ In getting these probabilities the standard is set for all possible combinations of mastery.
- ▶ Thus we define how a student will be classified in to each possible category.

# Procedure

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- ▶ Given that we are interested in defining standards for an Algebra II Benchmark Exam, we discuss the general procedure.
- ▶ Provide a description of the method used to request the information needed.
- ▶ Give an example of the form.
- ▶ Discuss the follow-up process of validating the standards.

# Standard Setting Procedure

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- ▶ A total of four Algebra II teachers are recruited.
  - ▶ Varied in type of students taught (e.g. advanced students versus students performing at an expected level) and experience.
- ▶ Given the test previously explained and a Q-matrix that was previously determined for each item, teachers are asked a set of questions based on the probabilities to be set by the LCDM.
- ▶ Questions were asked in a way that did not require knowledge of diagnostic models.

# Standard Setting Procedure

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- ▶ An example item in the benchmark test was:

2. If one factor of  $f(x) = 12x^2 - 14x - 6$  is  $(2x - 3)$  what is the other factor of  $f(x)$  if the polynomial is factored completely.
- a.  $(6x - 2)$
  - b.  $(6x + 2)$
  - c.  $(6x + 3)$
  - d.  $(6x - 3)$

- ▶ To correctly answer this item students must be able to:
  - ▶ Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.
  - ▶ Use quadratic functions and inequalities to model and solve problems; justify results.

# Standard Setting Procedure

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- ▶ Teachers are asked to:

...think of a student of yours, or imagine a student, that has mastered a certain set of skills. By “mastered” we mean a student who would not need to spend any additional time learning this material. We will then ask you, “If this person were to take a test with 100 items similar to a particular item, how many items would they get right?”

- ▶ Notice that by doing this we are specifically defining *mastery*.
- ▶ In addition, we have reframed the question based on a 100 item test.
  - ▶ Framing the task in this way, makes it easier to think about.

# Example Response for Item 1

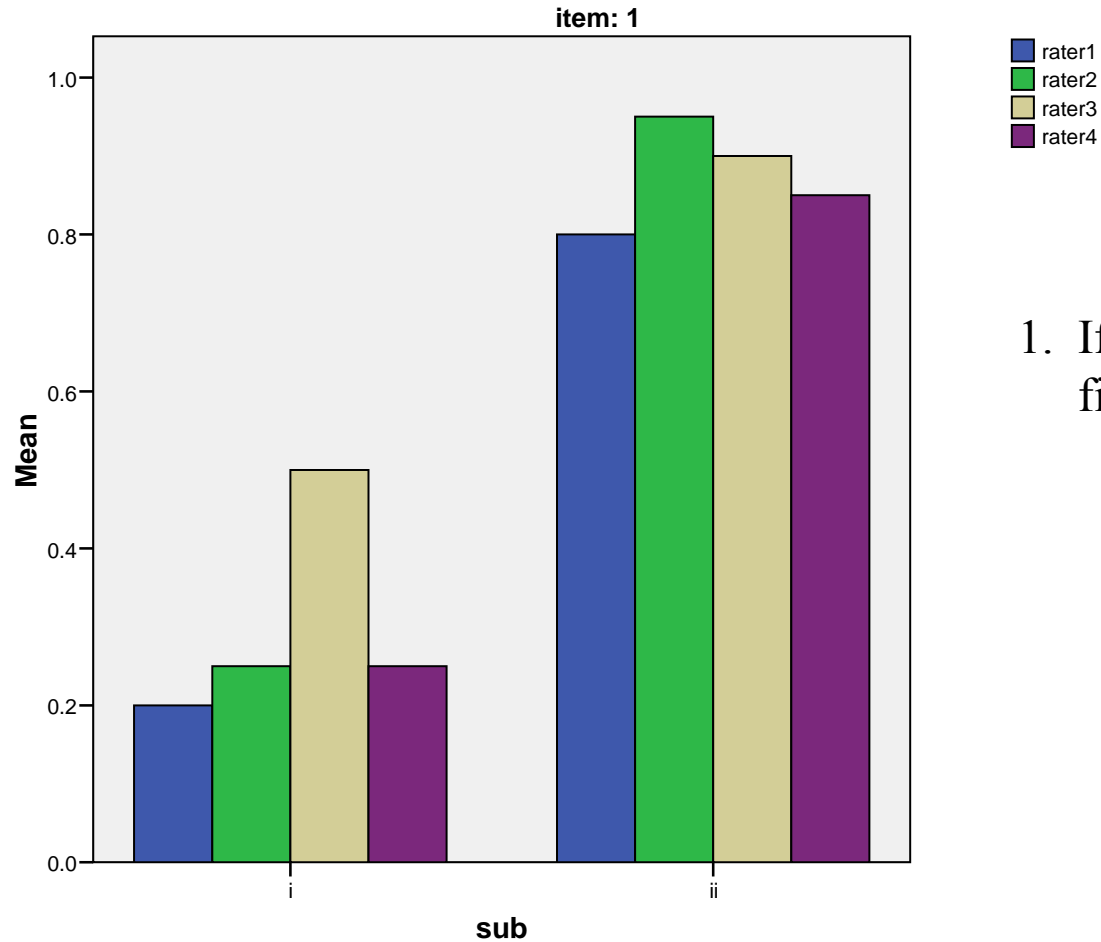
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1. If  $f(x) = x^2 + 2$  and  $g(x) = x - 3$  find  $f \circ g(x)$ .
  - a.  $x^2 - 6x + 11$
  - b.  $x^2 + 11$
  - c.  $x^2 + x - 1$
  - d.  $x^3 - 3x^2 + 2x - 6$

*Imagine a test that contains a total of 100 items just like this test (assume they are not identical)*

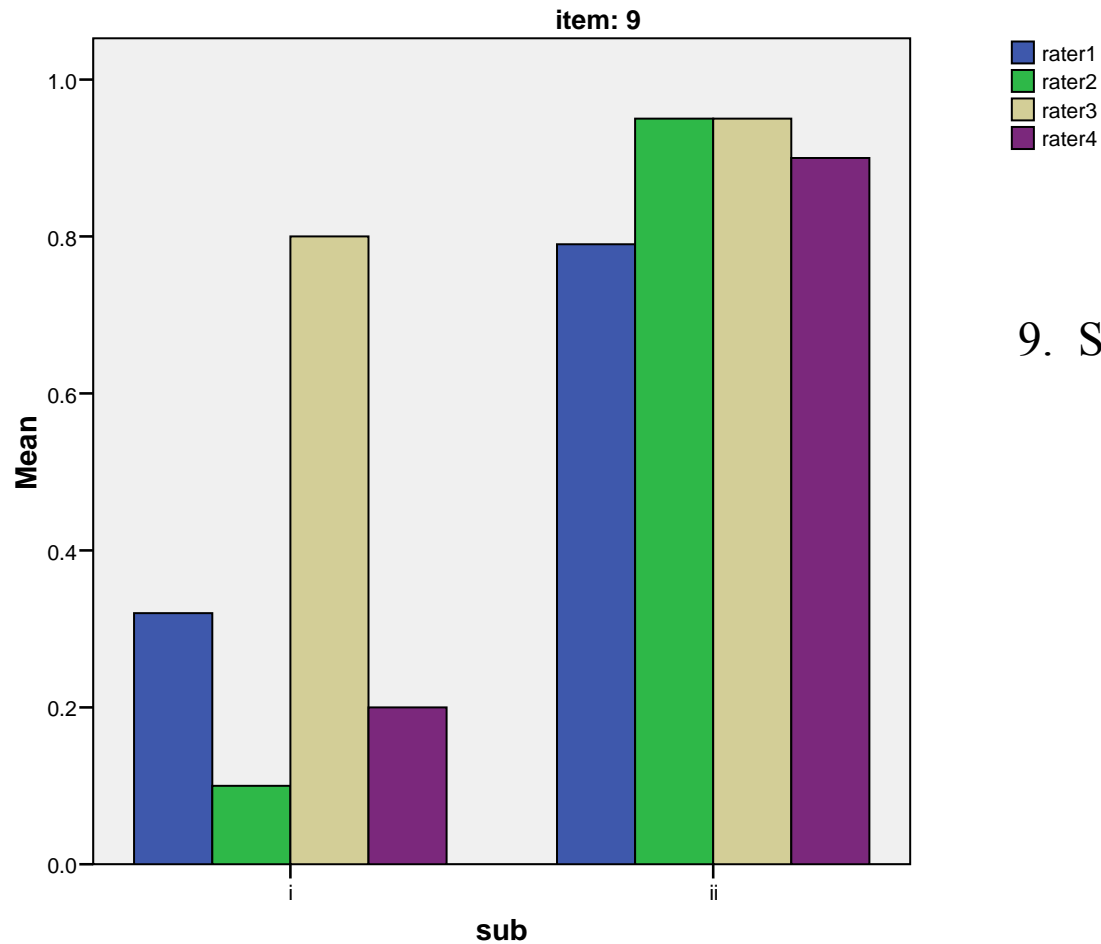
- 1i. 25 Think of one of your students that has NOT MASTERED Objective 2.01. How many of these items on the 100 item test should they get right?
- 1ii. 85 Think of one of your students that has MASTERED Objective 2.01. How many of these items on the 100 item test should they get right?

# Example Responses (Item 1)



1. If  $f(x) = x^2 + 2$  and  $g(x) = x - 3$  find .
  - a.  $x^2 - 6x + 11$
  - b.  $x^2 + 11$
  - c.  $x^2 + x - 1$
  - d.  $x^3 - 3x^2 + 2x - 6$

# Example Responses (Item 9)



9. Solve  $3(ax + b) = 2(cx - d)$  for  $x$ .

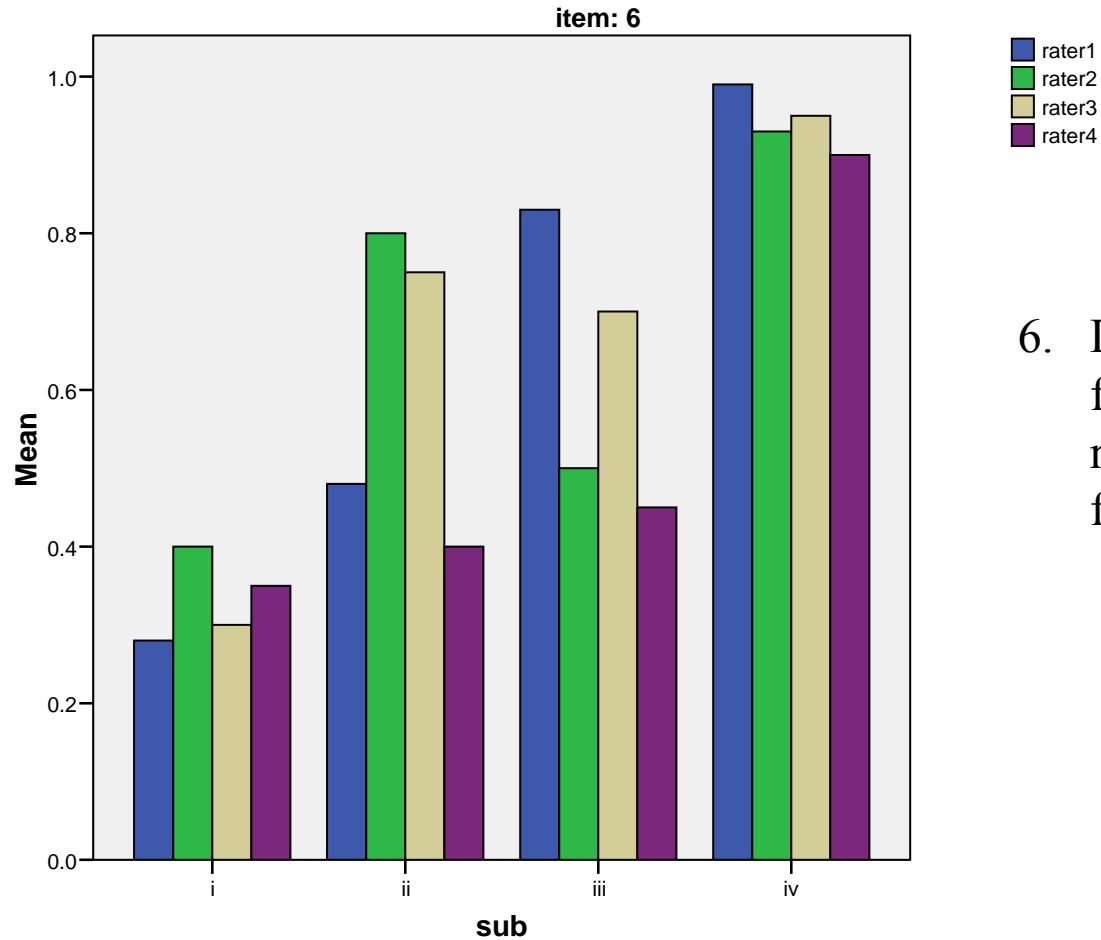
a.  $x = \frac{b - d}{a - c}$

b.  $x = \frac{bd}{ac}$

c.  $x = \frac{-3b - 2d}{3a - 2c}$

d.  $x = \frac{2b - 2d - 3ax}{2c}$

# Example Responses (Item 6)



6. Determine which of the following graphs does not represent  $Y$  as a linear function of  $X$ .

# Analyses

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- ▶ Based on the teachers responses, the average probability of a correct response is calculated.
- ▶ These averages are used to compute model parameters.
  - ▶ Specifically, if we know the probabilities (based on the teachers' responses) then we can compute the logit.
  - ▶ Also, the questions where phrased in a way that specified the attribute pattern 
$$\ln\left(\frac{P(X_{ij} = 1)}{1 - P(X_{ij} = 1)}\right) = \lambda_{add}\alpha_{add} + \lambda_{sub}\alpha_{sub} + \lambda_{add*sub}\alpha_{add}\alpha_{sub} - \eta$$
  - ▶ Therefore we can directly compute the parameters of the model

# Analyses

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- ▶ Using these fixed parameters as truth, MCMC estimation is used to obtain estimates of the posterior probability that each skill has been mastered.
- ▶ An example could be:

<b>Student ID</b>	<b>1.03</b>	<b>2.01</b>	<b>2.02</b>	<b>2.04</b>	<b>2.08</b>
24	0.25	0.87	0.99	0.44	0.05

- ▶ These can be converted to student mastery or nonmastery based on the mostly likely value.
  - ▶ Greater than 0.50 equals a master.
  - ▶ Less than 0.50 equals a nonmaster.

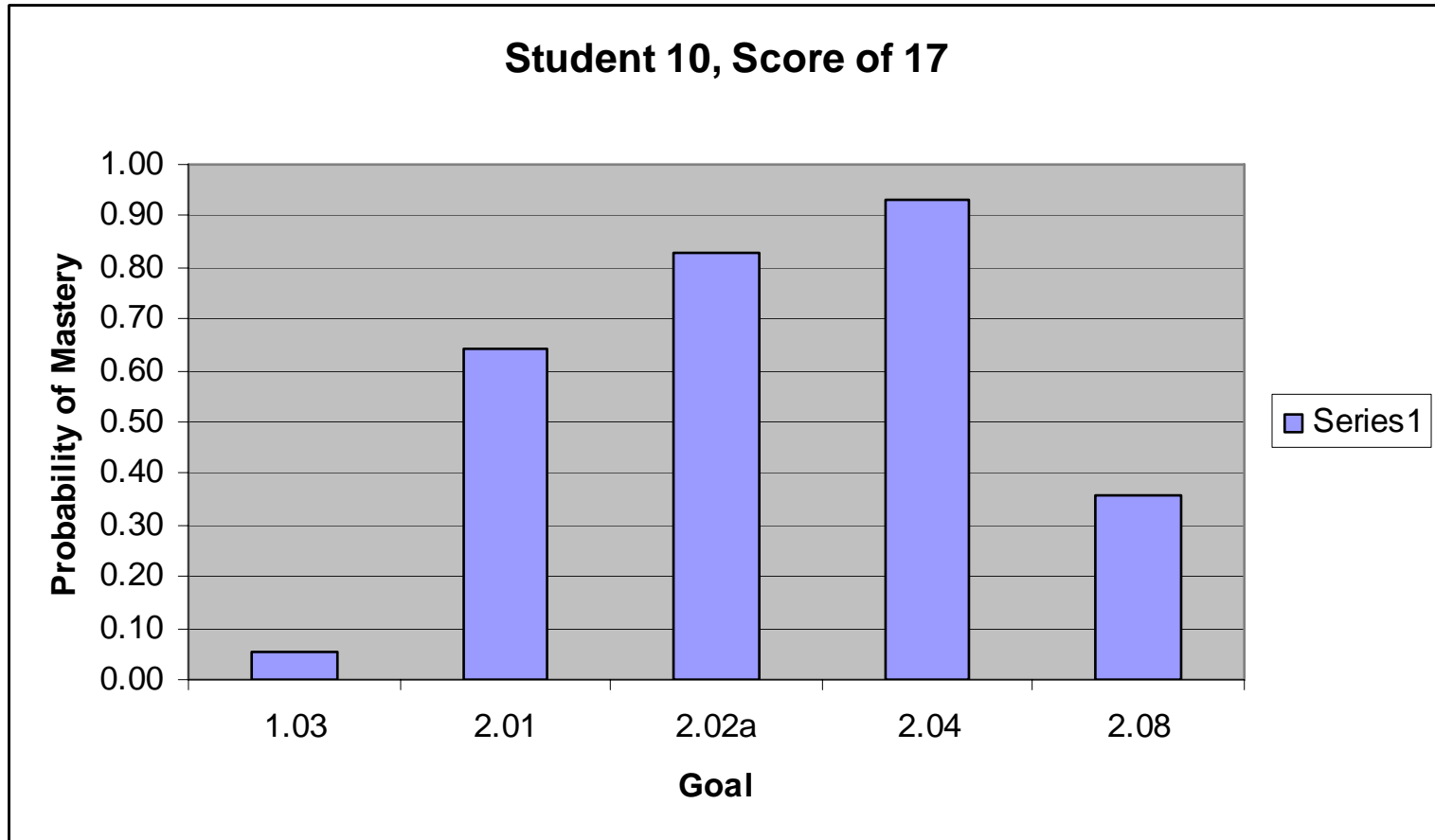
# Analyses

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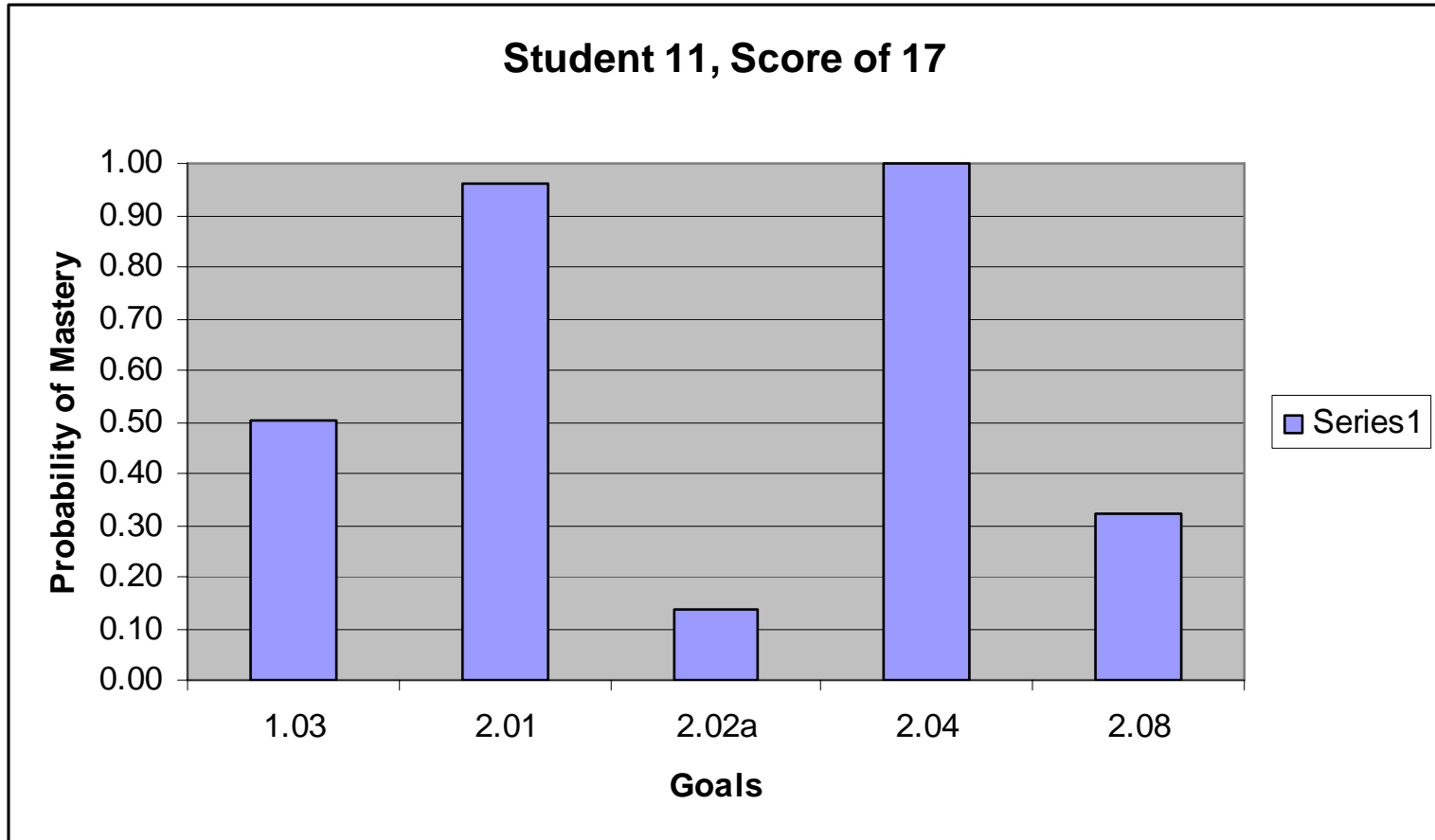
- ▶ A final meeting is held with the teachers for validation.
- ▶ Of particular interest is a teacher with 31 students who completed the benchmark exam.
- ▶ She was able to directly comment on whether the classifications were reasonable.

# Example Feedback

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# Example Feedback



# Additional Feedback

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- ▶ In addition to talking specifically about each student more general statements related to the class were discussed.
  - ▶ For example, those skills that most student seemed to know and those skills that needed to be improved.
- ▶ As a preliminary validation, those skills that we identified as needing additional work were confirmed by the teacher.
- ▶ For example, of 31 students, we estimated only 2 students had mastered goal 2.08 (Use equations and inequalities with absolute value to model and solve problems: justify results.).
  - ▶ This skills had not been covered by the time the benchmark was administered.

# Additional Feedback

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- ▶ Finally, expected benchmark performance for specific students was discussed.
- ▶ For example:
  - ▶ A student who had not mastered any skills was expected to get between 8 and 9 of these items right.
  - ▶ A student who had mastered all five attributes is expected to get between 22 and 23 item correct.

# Conclusions

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- ▶ In general, the procedure used here seems promising.
  - ▶ Teachers comments of the format of the questions were encouraging.
  - ▶ Responses were generally consistent across teachers.
  - ▶ Further refinement is planned.
- ▶ However, in our follow up interviews a few issues specific to this benchmark were mentioned.
  - ▶ Use of calculators versus actual mastery.
  - ▶ Expectation of teachers based on the level of the students.

# Acknowledgements

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- ▶ Jon Templin
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- ▶ Deb Bartz
- ▶ John Willse