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MATH 150: QUIZ 14 (4.5)

1. Using the *Rational Zeros Theorem*, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

2. Use the *Bounds on Zeros Theorem* to find a bound on the real zeros of
$$g(x) = 4x^5 - 2x^3 + 2x^2 + 14.$$

3. Let $f(x) = 3x^4 - 6x^3 - 11x + 4x + 6$.
   (a) What is the remainder if $3x^4 - 6x^3 - 11x^2 + 4x + 6$ is divided by $x - 3$.

   (b) Compute $f(3)$.

4. Use the *Intermediate Value Theorem* to show that $f(x) = x^4 + x^3 - 4x^2 - 5x - 5$ has at least one real root between $x = 2$ and $x = 3$.

Solutions

1. By the Rational Roots Theorem, if $\dfrac{p}{q}$ is a rational root of f, then p is a factor of 17 and q is a factor of 3. In other words, $p \in \{\pm 1, \pm 17\}$ and $q \in \{\pm 1, \pm 3\}$. That means that
$$\frac{p}{q} \in \left\{\pm 1,\ \pm 17,\ \pm\frac{1}{3},\ \pm\frac{17}{3}\right\}.$$
2. Finding the bounds on the roots of g is the same as finding a bound on the roots of
$$\frac{1}{4}g(x) = x^5 - \frac{1}{2}x^3 + \frac{1}{2}x^2 + \frac{7}{2}.$$
Since the leading coefficient is 1, we can use the Bounds on Zeros Theorem. We compute
$$M_1 = \max\left\{1, \frac{1}{2} + \frac{1}{2} + \frac{7}{2}\right\} = \max\left\{1, \frac{9}{2}\right\} = \frac{9}{2}$$
$$M_2 = 1 + \max\left\{\frac{1}{2}, \frac{1}{2}, \frac{7}{2}\right\} = 1 + \frac{7}{2} = \frac{9}{2}.$$
$$m = \min\left\{\frac{9}{2}, \frac{9}{2}\right\} = \frac{9}{2}.$$
Every real zero of g lies between $-\dfrac{9}{2}$ and $\dfrac{9}{2}$.
3. (a) By Synthetic Division, or computing $f(3)$, we see that the remainder is 0.
   (b) By Synthetic Division, or computing $f(3)$, we see that the $f(3) = 0$.
4. Since $f(2) = -7$ and $f(3) = 52$ have opposite signs, the Intermediate Value Theorem says that f must have at least one real root between 2 and 3.