Mini-Lecture 2.1
Functions

Learning Objectives:

1. Determine whether a relation represents a function
2. Find the value of a function
3. Find the domain of a function defined by an equation
4. Form the sum, difference, product, and quotient of two functions

Examples:

1. Determine whether the equation defines y as function of x.

$$(a)\ y = x^2 - 2x \quad (b)\ y^2 = 3x - 4 \quad (c)\ 5x + 7y = 10 \quad (d)\ y = \frac{2}{x-3}$$

2. For $f(x) = -x^2 + 2x - 3$ find $(a)\ f(0) \quad (b)\ f(-1) \quad (c)\ f(3)$
3. Find the domain of each function.

$$(a)\ f(x) = 2x + 3 \quad (b)\ f(x) = \frac{2}{x^2} \quad (c)\ f(x) = \frac{2x}{x^2+1} \quad (d)\ f(x) = \frac{5}{\sqrt{x+4}}$$

4. For $f(x) = 2x - 3$ and $g(x) = 2x^2$, find

$$(a)\ (f+g)(x) \quad (b)\ (f-g)(x) \quad (c)(f \cdot g)(2) \quad (d)\left(\frac{f}{g}\right)(3)$$

Teaching Notes:

- This is a critical section. If the students do not understand the concept of a function, they will struggle throughout the course.
- Spend time on the correspondence aspect of a function. You may use the birthday example. Every student has only one birthday, but other students can have that same birthday. Emphasize that no student has two birthdays.
- Demonstrate the difference between a relation and a function. A circle and a line are good geometric examples of this. A function is a special type of relation.
- The input – output machine in figure 10 is a good one to use extensively. Just keep emphasizing "input=domain, output=range".
- If time permits, introduce the difference quotient as a precursor to calculus limits.

Answers: 1. (a) yes (b) no (c) yes (d) yes 2. (a) -3 (b) -6 (c) -6

3. $(a)\ (-\infty, \infty) \quad (b)\ (-\infty, 0) \cup (0, \infty) \quad (c)(-\infty, \infty) \quad (d)\ (-4, \infty)$

4. $(a)\ 2x^2 + 2x - 3 \quad (b) -2x^2 + 2x - 3 \quad (c)\ 8 \quad (d)\ \frac{1}{6}$