MATH 150: QUIZ 10 (4.1)

1. (4 points) For each of the functions below, determine if the function is a polynomial. If it is a polynomial, state the degree.
   (a) \( f(x) = 5x^3 - 3x + 4 \)
      Polynomial  Not a polynomial  degree = 
      
   (b) \( f(x) = \sqrt[3]{x^2} \)
      Polynomial  Not a polynomial  degree = 
      
   (c) \( f(x) = \frac{3 + x}{x - 3} \)
      Polynomial  Not a polynomial  degree = 
      
   (d) \( f(x) = \sqrt{x - 3} \)
      Polynomial  Not a polynomial  degree = 
      
2. (4 points) Form a polynomial with the following properties. Do not expand.
   • Degree 4
   • \(-2\) is a root of multiplicity 3
   • \(3\) is a root of multiplicity 1
   • \(y\)-intercept is 1
3. (4 points) Sketch a graph of the function \( f(x) = x^3(x - 1)^2(x - 2) \) using end-behavior and multiplicity of zeros.

4. (4 points) Let \( f(x) = x^4 - 5x^3 + 7x^2 - 4x + 1 \).
   
   (a) Verify that 1 is a root of \( f \).

   (b) Use (a) to factor \( f \) into a product of a degree 1 polynomial and a degree 3 polynomial.
Solutions

1. (a) Polynomial of degree 3.
   (b) Polynomial of degree 2.
   (c) Not a polynomial.
   (d) Not a polynomial.

2. The information about the roots and total degree tell us that the polynomial has the form

\[ f(x) = a(x + 2)^3(x - 3) \]

for some value \( a \). The \( y \)-intercept tells us that

\[ 1 = f(0) = a(0 + 2)^3(0 - 3) = -24a. \]

It follows that \( a = -1/24 \) so that

\[ f(x) = -\frac{1}{24}(x + 2)^3(x - 3). \]

3. The key points are listed below.
   - As \( x \to \infty \), \( f(x) \to \infty \).
   - As \( x \to -\infty \), \( f(x) \to \infty \).
   - There is a root at 0 and the graph should cross the \( x \)-axis there.
   - There is a root at 1 and the graph should not cross the \( x \)-axis there.
   - There is a root at 2 and the graph should cross the \( x \)-axis there.

4. (a) This can be checked by showing \( f(1) = 0 \). Alternatively, you can use synthetic division to quickly check that \( x - 1 \) is a factor of \( f(x) \).
   (b) By long division (or synthetic division), we get \( (x - 1)(x^3 - 4x^2 + 3x - 1) \).