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MATH 150: QUIZ 11 (4.2)

1. (4 points) For each of the functions below, determine if the function is a rational function.
   (a) $f(x) = 5x^3 - 3x + 4$

   Rational function   Not a rational function

   (b) $f(x) = \dfrac{\sqrt{3}x^2}{x^3 - \pi x + \sqrt{2}}$

   Rational function   Not a rational function

   (c) $f(x) = \dfrac{3 + x}{x - 3}$

   Rational function   Not a rational function

   (d) $f(x) = \sqrt{x - 3}$

   Rational function   Not a rational function

2. (3 points) Find the domain of the rational function

$$f(x) = \frac{3x^2}{x(x-1)(x^2-4)(x^2+1)}.$$

3. (3 points) Let $f(x) = \dfrac{2x^3 + 3x}{x^2 - 1}$. In the questions that follow, remember that the asymptote is a LINE, not a number!

(a) Identify all the vertical asymptotes of f.

(b) Identify all the horizontal asymptotes of f.

(c) Identify all the oblique asymptotes of f.

4. (3 points) Recall that $g(x) = \dfrac{1}{x^2}$ has vertical asymptote $x = 0$ and horizontal asymptote $y = 0$. Use transformations of functions to deduce the vertical and horizontal asymptotes of

$$f(x) = \frac{3}{(x-2)^2} + 1.$$

Solutions

1. (a) Rational function.
   (b) Rational function.
   (c) Rational function.
   (d) Not a rational function.
2. Recall that the domain is the set of real numbers such that the denominator (bottom) does not vanish. It follows that
$$\mathrm{dom}(f) = \{x \in \mathbb{R} \mid x \neq 0, 1, 2, -2\}.$$
   Things to note here:
   - $x^2 + 1$ is never 0, and so does not affect the domain.
   - $x^2 - 4$ factors as $(x-2)(x+2)$ and so is 0 when $x = \pm 2$.
3. (a) First note that f is already in reduced form. Therefore the vertical asymptotes occur where the denominator $x^2 - 1 = (x-1)(x+1)$ vanishes. It follows that the vertical asymptotes are the lines $x = 1$ and $x = -1$.
   (b) Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote.
   (c) Since the degree of the numerator is exactly 1 more than the degree of the denominator, there is an oblique asymptote. We can find the oblique asymptote by long division. We compute that
$$\frac{2x^3 + 3x}{x^2 - 1} = 2x + \frac{5x}{x^2 - 1}$$
   so that the obliques asymptote is the line $y = 2x$.
4. Note that the graph of f is obtained from the graph of g by
   (a) Shift right 2.
   (b) Vertical stretch by factor of 3.
   (c) Shift up 1.

   It follows that the vertical asymptote of f is $x = 2$ and the horizontal asymptote is $y = 1$.