1. (5 points) Let \( f(x) = \frac{x^2}{x^2 + x - 2} \).
(a) What is the domain of \( f \)?

(b) Find the intercepts of \( f \).

(c) Find all the vertical asymptotes (if any) of \( f \). (Recall that a vertical asymptote is a line, not a number.)

(d) Find all the horizontal asymptotes (if any) of \( f \). (Recall that a horizontal asymptote is a line, not a number.)

(e) Find all the oblique asymptotes (if any) of \( f \). (Recall that an oblique asymptote is a line, not a number.)
2. A company that produces snowmobiles has a cost ($ function $C(x) = 3500x + 150000$. What is the average cost ($ per snowmobile) of producing 100 snowmobiles? Recall that the average cost is $\bar{C}(x) = \frac{C(x)}{x}$.

3. The concentration $C$ (mcg/mL) of a certain drug in a patient’s bloodstream $t$ hours after injection is given by $C(t) = \frac{t^2}{9t^2 + 6}$. As $t$ increases, what value will $C(t)$ approach?
Solutions

1. (a) Recall that the domain is the set of real numbers so that the denominator (bottom) does not vanish. We compute

\[ x^2 + x - 2 = (x + 2)(x - 1), \]

which vanishes at 2 and −1. It follows that

\[ \text{dom}(f) = \{ x \in \mathbb{R} \mid x \neq 2, -1 \}. \]

(b) Recall that in order to compute the vertical asymptotes, we first write the rational function in lowest terms. Then the zeros of the denominator correspond to vertical asymptotes. Finally remember that vertical asymptotes are LINES.

From the work above, we see that the vertical asymptotes are \( x = 2 \) and \( x = -1 \).

(c) Since the numerator and denominator have the same degree, \( f \) has a horizontal asymptote. The ratio of leading coefficients is 1, and so the horizontal asymptote is \( y = 1 \).

(d) Since the degree of the numerator is equal to the degree of the denominator, there is no oblique asymptote.

2. Recall that the average cost is

\[ \bar{C}(x) = \frac{C(x)}{x} = 3500 + \frac{150000}{x}. \]

The average cost of producing 100 snowmobiles is

\[ \bar{C}(100) = 3500 + \frac{150000}{100} = 5000. \]

The average cost of producing 100 snowmobiles is $5000.

3. As \( t \) gets large, \( C(t) \) behaves like

\[ \frac{t^2}{9t^2} = \frac{1}{9}, \]

which goes to \( \frac{1}{9} \). It follows that the concentration approaches \( \frac{1}{9} \) mcg/mL.