1. Using the Rational Zeros Theorem, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

2. Use the Bounds on Zeros Theorem to find a bound on the real zeros of $g(x) = 4x^5 - 2x^3 + 2x^2 + 14$. 


3. Let $f(x) = 3x^4 - 6x^3 - 11x + 4x + 6$.
   (a) What is the remainder if $3x^4 - 6x^3 - 11x^2 + 4x + 6$ is divided by $x - 3$.

   (b) Compute $f(3)$.

4. Use the Intermediate Value Theorem to show that $f(x) = x^4 + x^3 - 4x^2 - 5x - 5$ has at least one real root between $x = 2$ and $x = 3$. 
Solutions

1. By the Rational Roots Theorem, if \( \frac{p}{q} \) is a rational root of \( f \), then \( p \) is a factor of 17 and \( q \) is a factor of 3. In other words, \( p \in \{\pm 1, \pm 17\} \) and \( q \in \{\pm 1, \pm 3\} \). That means that
\[
\frac{p}{q} \in \left\{ \pm 1, \pm 17, \pm \frac{1}{3}, \pm \frac{17}{3} \right\}.
\]

2. Finding the bounds on the roots of \( g \) is the same as finding a bound on the roots of
\[
\frac{1}{4}g(x) = x^5 - \frac{1}{2}x^3 + \frac{1}{2}x^2 + \frac{7}{2}.
\]
Since the leading coefficient is 1, we can use the Bounds on Zeros Theorem. We compute
\[
M_1 = \max \left\{ 1, \frac{1}{2} + \frac{1}{2} + \frac{7}{2} \right\} = \max \left\{ 1, \frac{9}{2} \right\} = \frac{9}{2}
\]
\[
M_2 = 1 + \max \left\{ \frac{1}{2}, \frac{1}{2}, \frac{7}{2} \right\} = 1 + \frac{7}{2} = \frac{9}{2}.
\]
\[
m = \min \left\{ \frac{9}{2}, \frac{9}{2} \right\} = \frac{9}{2}.
\]

Every real zero of \( g \) lies between \(-\frac{9}{2}\) and \(\frac{9}{2}\).

3. (a) By Synthetic Division, or computing \( f(3) \), we see that the remainder is 0.
(b) By Synthetic Division, or computing \( f(3) \), we see that \( f(3) = 0 \).

4. Since \( f(2) = -7 \) and \( f(3) = 52 \) have opposite signs, the Intermediate Value Theorem says that \( f \) must have at least one real root between 2 and 3.