Read all of the following information before starting the exam:

• It is to your advantage to answer ALL of the questions.

• Clearly mark your answers on the Answer Sheet. The Answer Sheet is the ONLY page that will be graded.

• There are 15 multiple choice, 5 true/false, and 4 short answer problems on this test. It is your responsibility to make sure that you have all of the problems. There are some BONUS questions at the end.

• Each multiple choice and short answer problem is worth 5 points. Each true/false is 1 point each.

• There is no need to complete the test in order. The problems are independent.

• *Budget your time!*

• If you have read all of these instructions, remember that π is an irrational number that is about 3.

• Tear off the Answer Sheet before beginning your test.
Answer Sheet

Section 1. Multiple Choice

1. 
4. 
7. 
10. 
13. 

2. 
5. 
8. 
11. 
14. 

3. 
6. 
9. 
12. 
15. 

Section 2. True or false

| True | False | True | False | True | False | True | False | True | False |

Section 3. Short answer

1. The graph of \( f \) [ ] the x-axis at \( -4 \linik 6 \linik 3 \) since \( -4 \ 6 \ 3 \), since \( -4 \ 6 \ 3 \)

2. (a) \( \text{Polynomial} \) \( \text{Not polynomial} \) degree = 
(b) \( \text{Polynomial} \) \( \text{Not polynomial} \) degree = 
(c) \( \text{Polynomial} \) \( \text{Not polynomial} \) degree = 
(d) \( \text{Polynomial} \) \( \text{Not polynomial} \) degree = 
(e) \( \text{Polynomial} \) \( \text{Not polynomial} \) degree = 

3. 

4. The solutions are \( \frac{2}{3} \). 

5. Bonus (1 point each):

(a) 
(b) 
(c) 
(d) \( (a) \) \( (b) \) \( (c) \)
Section 1. Multiple choice

1. Suppose the daily cost $C$ of manufacturing bicycles is given by $C(x) = 80x + 5000$. Then the average daily cost $\bar{C}$ is given by $\bar{C}(x) = \frac{80x + 5000}{x}$. How many bicycles must be produced each day for the average daily cost to be no more than $100$?

(a) At least 150 bicycles.
(b) At least 250 bicycles.
(c) At least 350 bicycles.
(d) At least 450 bicycles.
(e) None of the above.

2. The graph of a rational function $g$ is shown here. The dotted lines indicate asymptotes. Which of the following must be true?

(a) The numerator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is odd.
(b) The numerator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is even.
(c) The denominator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is odd.
(d) The denominator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is even.
(e) None of the above.

3. Using the Rational Zeros Theorem, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

(a) 0.93
(b) $\pm 1, \pm 17, \pm \frac{1}{3}, \pm \frac{17}{3}$
(c) $\pm 1, \pm 3, \pm \frac{1}{17}, \pm \frac{3}{17}$
(d) $\pm 1, \pm 17$
(e) None of the above.
4. Solve \( \frac{4x + 5}{x + 2} \geq 3 \).

(a) \((-\infty, -2] \cup [1, \infty)\)

(b) \((-2, 1]\)

(c) \([-\frac{1}{2}, \infty)\)

(d) \((-\infty, -2) \cup [1, \infty)\)

(e) None of the above.

5. Which polynomial has ALL of the following properties?

- degree 4 polynomial
- the \(y\)-intercept is 24
- \(-2\) is a root of multiplicity 3
- \(3\) is a root of multiplicity 1

(a) \((x - 24)(x + 2)^3(x - 3)\)

(b) \(- (x + 2)^3(x - 3)\)

(c) \((x + 2)^3(x - 3) + 24\)

(d) \(x^4(x + 2)^3(x - 3) + 24\)

(e) None of the above.

6. My happiness level \(H\) (on a scale of 1 to 20) \(t\) hours after starting to grade exams is modelled by \(H(t) = \frac{t^2 - 2t + 17}{t^2 + 1}\). (I give good grades as long as my happiness is above a 2). Which of the following best describes the time when I am happy enough to grade?

(a) I am never happy enough to grade.

(b) Anytime in the first 3 hours.

(c) Anytime after the first 3 hours.

(d) Only between hours 2 and 5.

(e) None of the above.
7. Use the **Bounds on Zeros Theorem** to find a bound on the real zeros of
\[ g(x) = 4x^5 - 2x^3 + 2x^2 + 14. \]

(a) Every real zero of \( g \) lies between \( -\frac{5}{4} \) and \( \frac{5}{4} \).
(b) Every real zero of \( g \) lies between \( -5 \) and \( 5 \).
(c) Every real zero of \( g \) lies between \( -\frac{3}{2} \) and \( \frac{3}{2} \).
(d) Every real zero of \( g \) lies between \( -\frac{1}{2} \) and \( \frac{1}{2} \).
(e) None of the above.

8. Which of the following best describes the graph of \( y = \frac{2x^2 - 5x + 2}{x^2 - 4} \) near \( x = 2 \).

(a) There is a vertical asymptote \( x = 2 \).
(b) There is a hole in the graph at \( x = 2 \).
(c) The graph is above the \( x \)-axis when \( x = 2 \).
(d) The graph crosses the \( x \)-axis at \( 2 \).
(e) None of the above.

For the following three questions, let
\[ G(x) = \frac{3x^2 + x}{x^2 + 2x - 8}. \]

9. Find the domain of \( G \).

(a) \( \{x \in \mathbb{R} \mid x \neq 2, -4\} \)
(b) \( \{x \in \mathbb{R} \mid x \neq 0, -\frac{1}{3}\} \)
(c) all real numbers
(d) \( \{x \in \mathbb{R} \mid x \neq 0\} \)
(e) None of the above.

10. Find the horizontal or oblique asymptotes of the graph of \( G \), if any exist.

(a) \( y = 3 \)
(b) \( y = 3x + 1 \)
(c) \( x = 2 \) and \( x = -4 \)
(d) No horizontal or oblique asymptotes.
(e) None of the above.
11. Find the vertical asymptotes of the graph of $G$, if any exist.

(a) $y = 3$
(b) $x = -2$ and $x = 4$
(c) $x = 2$ and $x = -4$
(d) No vertical asymptotes.
(e) None of the above.

12. Suppose the leading term of a polynomial $f$ is $ax^n$. The graph of $f$ is shown here. Which of the following statements MUST be true?

(a) $a > 0$ and $n$ is even
(b) $a > 0$ and $n$ is odd
(c) $a < 0$ and $n$ is even
(d) $a < 0$ and $n$ is odd
(e) Not enough information is given.

13. Find the domain of the rational function

$$R(x) = \frac{3x^2}{x(x - 1)(x^2 - 4)(x^2 + 1)}.$$ 

(a) all real numbers
(b) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, 2, -2\}$
(c) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0\}$
(d) $\text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, -1, 2, -2\}$
(e) None of the above.
14. Find all the rational zeros of the function \( f(x) = 2x^3 + 11x^2 - 7x - 6 \).

(a) \( 1 \)
(b) \( -6 \)
(c) \( -3, 2, 3 \)
(d) \( -6, -\frac{1}{2}, 1 \)
(e) None of the above.

15. Give a factorization of \( f(x) = x^4 + 2x^3 + 6x^2 - 2x - 7 \). Note that 1 and -1 are roots of \( f \).

(a) \((x - 1)(x + 1)(x^2 - x + 3)\)
(b) \((x - 1)(x + 1)(x^2 + 2x + 2)\)
(c) \((x - 1)(x + 1)(x^2 + 2x + 7)\)
(d) \((x^2 + 1)(x^2 - 2x + 11)\)
(e) None of the above.

Section 2. True or False

\[ \sqrt{A^2 + B^2} = A + B \]
\[ 3a + 2 = 5a \]
\[ \frac{n^2}{2n^2 + 1} = \frac{1}{2 + 1} \]
\[ \frac{1}{2 + t^{-1}} = \frac{1 + t}{2} \]
\[ \frac{5c^3}{c} = 4c^2 \]

Section 3. Short answer

1. Suppose \( f \) is a polynomial of degree 6, and \( -4 \) is a root of \( f \) of multiplicity 3. Choose the words that completes the statement most correctly.

The graph of \( f \) \( \square \) the \( x \)-axis at \( \begin{array}{ccc} -4 & 6 & 3 \end{array} \) since \( \begin{array}{ccc} -4 & 6 & 3 \end{array} \)

\( \square \) \( \text{touche} \) \( \square \) \( \text{crosses} \)

\( \square \) \( \text{even} \) \( \square \) \( \text{odd} \) \( \square \) \( \text{positive} \) \( \square \) \( \text{negative} \)

2. For each of the following, determine if the given function is a polynomial. If it is a polynomial, give its degree.

(a) \( f(x) = 3x^4 + 5x^2 + 1 \)
(b) \( f(x) = \frac{x^2 + x + 1}{x - 1} \)
(c) \( f(x) = \sqrt{x^2 + x - 3} \)
(d) \( f(x) = x^3 - 2x + \sqrt{3} \)
(e) \( f(x) = 2x^4 - 7x^{1/2} + 4 \)

3. Let \( f(x) = 4(x^2 + 1)(x - 2)^3 \). Find the power function that the graph of \( f \) will most resemble for large values of \( |x| \).

4. Find all the real solutions of the equation
\[
3x^3 + 4x^2 - 7x + 2 = 0.
\]
Credit will only be given for exact solutions. (Hint: One of the solutions is \( \frac{2}{3} \).)

5. BONUS (1 extra point each):
   (a) Compute exactly
   \[
   1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ldots}}}}
   \]
   (b) Compute exactly
   \[
   3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \ldots}}}}
   \]
   (c) Compute \( \frac{355}{113} \) to 5 decimal places.
   (d) Which of the numbers above is \( \pi \)?
\[\begin{array}{ccc}
(a) & (b) & (c)
\end{array}\]
Answer Key for Exam A

Section 1. Multiple choice

1. (b) 4. (d) 7. (e) 10. (a) 13. (b)
2. (d) 5. (b) 8. (b) 11. (c) 14. (d)
3. (b) 6. (b) 9. (a) 12. (c) 15. (c)

Section 2. True or False

False  \( \sqrt{A^2 + B^2} = A + B \)
False  \( 3a + 2 = 5a \)
False  \( \frac{n^2}{2n^2 + 1} = \frac{1}{2 + 1} \)
False  \( \frac{1}{2 + t^{-1}} = \frac{1 + t}{2} \)
False  \( \frac{5c^3}{c} = 4c^2 \)

Section 3. Short answer

1. The graph of \( f \) crosses the x-axis at \( -4 \) since \( 3 \) is odd.

2. (a) Polynomial of degree 4.
   (b) Not a polynomial.
   (c) Not a polynomial.
   (d) Polynomial of degree 3.
   (e) Not a polynomial.

3. \( y = 4x^5 \)

4. Long division (or synthetic division) shows that

\[
3x^3 + 4x^2 - 7x + 2 = (x - \frac{2}{3})(3x^2 + 6x - 3) = 3\left(x - \frac{2}{3}\right)(x^2 + 2x - 1).
\]

Using quadratic formula or writing the quadratic in vertex form and solving, we see that two additional roots are \(-1 + \sqrt{2}\) and \(-1 - \sqrt{2}\). The solutions are

\[
\frac{2}{3}, -1 + \sqrt{2}, -1 - \sqrt{2}.
\]
5. BONUS (1 extra point each):

(a) \( \frac{1 + \sqrt{5}}{2} \)

(b) \( \pi \)

(c) 3.14159

(d) (b)
Read all of the following information before starting the exam:

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- If you have read all of these instructions, remember that $\pi$ is an irrational number that is about 3.

- Tear off the Answer Sheet before beginning your test.
Section 1. Multiple Choice

1.  
2.  
3.  
4.  
5.  
6.  
7.  
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9.  
10.  
11.  
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14.  
15.  

Section 2. True or false

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<td>False</td>
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<tr>
<td>True</td>
<td>False</td>
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Section 3. Short answer

1. The graph of \( f \) touches the \( x \)-axis at \(-4\), \(6\), \(3\) since \(-4\), \(6\), \(3\) is even, odd, positive, negative.

2. (a) Polynomial Not polynomial degree =  
(b) Polynomial Not polynomial degree =  
(c) Polynomial Not polynomial degree =  
(d) Polynomial Not polynomial degree =  
(e) Polynomial Not polynomial degree =  

3.  

4. The solutions are \( \frac{2}{3} \).

5. Bonus (1 point each):

   (a)  
   (b)  
   (c)  
   (d) (a) (b) (c)
Section 1. Multiple choice

1. The graph of a rational function $g$ is shown here. The dotted lines indicate asymptotes. Which of the following must be true?

(a) The numerator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is odd.
(b) The numerator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is even.
(c) The denominator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is odd.
(d) The denominator of $g$ has a factor of the form $(x - 2)^m$, where $m$ is even.
(e) None of the above.

2. Give a factorization of $f(x) = x^4 + 2x^3 + 6x^2 - 2x - 7$. Note that 1 and $-1$ are roots of $f$.

(a) $(x - 1)(x + 1)(x^2 - x + 3)$
(b) $(x - 1)(x + 1)(x^2 + 2x + 2)$
(c) $(x - 1)(x + 1)(x^2 + 2x + 7)$
(d) $(x^2 + 1)(x^2 - 2x + 11)$
(e) None of the above.

3. Use the Bounds on Zeros Theorem to find a bound on the real zeros of $g(x) = 4x^5 - 2x^3 + 2x^2 + 14$.

(a) Every real zero of $g$ lies between $-\frac{5}{4}$ and $\frac{5}{4}$.
(b) Every real zero of $g$ lies between $-5$ and 5.
(c) Every real zero of $g$ lies between $-\frac{3}{2}$ and $\frac{3}{2}$.
(d) Every real zero of $g$ lies between $-\frac{1}{2}$ and $\frac{1}{2}$.
(e) None of the above.
4. Find the domain of the rational function

\[ R(x) = \frac{3x^2}{x(x - 1)(x^2 - 4)(x^2 + 1)}. \]

(a) all real numbers
(b) \( \text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, 2, -2\} \)
(c) \( \text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0\} \)
(d) \( \text{dom}(R) = \{x \in \mathbb{R} \mid x \neq 0, 1, -1, 2, -2\} \)
(e) None of the above.

5. Suppose the daily cost \( C \) of manufacturing bicycles is given by \( C(x) = 80x + 5000 \). Then the average daily cost \( \bar{C} \) is given by \( \bar{C}(x) = \frac{80x + 5000}{x} \). How many bicycles must be produced each day for the average daily cost to be no more than $100.

(a) At least 150 bicycles.
(b) At least 250 bicycles.
(c) At least 350 bicycles.
(d) At least 450 bicycles.
(e) None of the above.

6. Suppose the leading term of a polynomial \( f \) is \( ax^n \). The graph of \( f \) is shown here. Which of the following statements MUST be true?

\[ \text{Graph of } f \]

(a) \( a > 0 \) and \( n \) is even
(b) \( a > 0 \) and \( n \) is odd
(c) \( a < 0 \) and \( n \) is even
(d) \( a < 0 \) and \( n \) is odd
(e) Not enough information is given.
7. Find all the rational zeros of the function \( f(x) = 2x^3 + 11x^2 - 7x - 6 \).

(a) 1 
(b) -6 
(c) -3, 2, 3 
(d) -6, -\frac{1}{2}, 1 
(e) None of the above.

8. My happiness level \( H \) (on a scale of 1 to 20) \( t \) hours after starting to grade exams is modelled by \( H(t) = \frac{t^2 - 2t + 17}{t^2 + 1} \). (I give good grades as long as my happiness is above a 2). Which of the following best describes the time when I am happy enough to grade?

(a) I am never happy enough to grade. 
(b) Anytime in the first 3 hours. 
(c) Anytime after the first 3 hours. 
(d) Only between hours 2 and 5. 
(e) None of the above.

9. Which polynomial has ALL of the following properties?

- degree 4 polynomial
- the \( y \)-intercept is 24
- \(-2\) is a root of multiplicity 3
- \(3\) is a root of multiplicity 1

(a) \((x - 24)(x + 2)^3(x - 3)\) 
(b) \(-(x + 2)^3(x - 3)\) 
(c) \((x + 2)^3(x - 3) + 24\) 
(d) \(x^4(x + 2)^3(x - 3) + 24\) 
(e) None of the above.

10. Solve \( \frac{4x + 5}{x + 2} \geq 3 \).

(a) \((-\infty, -2] \cup [1, \infty)\) 
(b) \((-2, 1]\) 
(c) \([-\frac{1}{2}, \infty)\) 
(d) \((-\infty, -2) \cup [1, \infty)\) 
(e) None of the above.
11. Using the *Rational Zeros Theorem*, what are the potential rational zeros of the polynomial $3x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 17$?

(a) 0.93  
(b) $\pm 1, \pm 17, \pm \frac{1}{3}, \pm \frac{17}{3}$  
(c) $\pm 1, \pm 3, \pm \frac{1}{17}, \pm \frac{3}{17}$  
(d) $\pm 1, \pm 17$  
(e) None of the above.

12. Which of the following best describes the graph of $y = \frac{2x^2 - 5x + 2}{x^2 - 4}$ near $x = 2$.

(a) There is a vertical asymptote $x = 2$.  
(b) There is a hole in the graph at $x = 2$.  
(c) The graph is above the $x$-axis when $x = 2$.  
(d) The graph crosses the $x$-axis at 2.  
(e) None of the above.

For the following three questions, let  

$$G(x) = \frac{3x^2 + x}{x^2 + 2x - 8}$$

13. Find the domain of $G$.

(a) $\{x \in \mathbb{R} \mid x \neq 2, -4\}$  
(b) $\left\{ x \in \mathbb{R} \mid x \neq 0, -\frac{1}{3} \right\}$  
(c) all real numbers  
(d) $\{x \in \mathbb{R} \mid x \neq 0\}$  
(e) None of the above.

14. Find the horizontal or oblique asymptotes of the graph of $G$, if any exist.

(a) $y = 3$  
(b) $y = 3x + 1$  
(c) $x = 2$ and $x = -4$  
(d) No horizontal or oblique asymptotes.  
(e) None of the above.
15. Find the vertical asymptotes of the graph of \( G \), if any exist.

(a) \( y = 3 \)
(b) \( x = -2 \) and \( x = 4 \)
(c) \( x = 2 \) and \( x = -4 \)
(d) No vertical asymptotes.
(e) None of the above.

Section 2. True or False

___ \( \sqrt{A^2 + B^2} = A + B \)
___ \( \frac{5c^3}{c} = 4c^2 \)
___ \( 3a + 2 = 5a \)
___ \( \frac{1}{2 + t^{-1}} = \frac{1 + t}{2} \)
___ \( \frac{n^2}{2n^2 + 1} = \frac{1}{2 + 1} \)

Section 3. Short answer

1. Suppose \( f \) is a polynomial of degree 6, and \(-4\) is a root of \( f \) of multiplicity 3. Choose the words that completes the statement most correctly.

The graph of \( f \) touches crosses the \( x \)-axis at \(-4 \quad 6 \quad 3\) since \(-4 \quad 6 \quad 3\)

is even odd positive negative

2. For each of the following, determine if the given function is a polynomial. If it is a polynomial, give its degree.

(a) \( f(x) = 3x^4 + 5x^2 + 1 \)
(b) \( f(x) = \frac{x^2 + x + 1}{x - 1} \)
(c) \( f(x) = \sqrt{x^2 + x - 3} \)
(d) \( f(x) = x^3 - 2x + \sqrt{3} \)
(e) \( f(x) = 2x^4 - 7x^{1/2} + 4 \)

3. Let \( f(x) = 4(x^2 + 1)(x - 2)^3 \). Find the power function that the graph of \( f \) will most resemble for large values of \( |x| \).
4. Find all the real solutions of the equation

\[3x^3 + 4x^2 - 7x + 2 = 0.\]

Credit will only be given for exact solutions. (Hint: One of the solutions is \(\frac{2}{3}\)).

5. BONUS (1 extra point each):

(a) Compute exactly

\[1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\ldots}}}}\]

(b) Compute exactly

\[3 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \frac{1}{6 + \ldots}}}}\]

(c) Compute \(\frac{355}{113}\) to 5 decimal places.

(d) Which of the numbers above is \(\pi\)?

\[\boxed{\text{(a) } \text{(b) } \text{(c)}}\]
Answer Key for Exam B

Section 1. Multiple choice

1. (d)  
2. (c)  
3. (e)  
4. (b)  
5. (b)  
6. (c)  
7. (d)  
8. (b)  
9. (b)  
10. (d)  
11. (b)  
12. (b)  
13. (a)  
14. (a)  
15. (c)  

Section 2. True or False

False  \[ \sqrt{A^2 + B^2} = A + B \]
False  \[ \frac{5c^3}{c} = 4c^2 \]
False  \[ 3a + 2 = 5a \]
False  \[ \frac{1}{2 + t^{-1}} = \frac{1 + t}{2} \]
False  \[ \frac{n^2}{2n^2 + 1} = \frac{1}{2 + 1} \]

Section 3. Short answer

1. The graph of \( f \) crosses the x-axis at \(-4\) since \[3\] is odd.

2. (a) Polynomial of degree 4.
   (b) Not a polynomial.
   (c) Not a polynomial.
   (d) Polynomial of degree 3.
   (e) Not a polynomial.

3. \( y = 4x^5 \)

4. Long division (or synthetic division) shows that

\[
3x^3 + 4x^2 - 7x + 2 = (x - \frac{2}{3})(3x^2 + 6x - 3) = 3(x - \frac{2}{3})(x^2 + 2x - 1).
\]

Using quadratic formula or writing the quadratic in vertex form and solving, we see that two additional roots are \(-1 + \sqrt{2}\) and \(-1 - \sqrt{2}\). The solutions are

\[
\frac{2}{3}, -1 + \sqrt{2}, -1 - \sqrt{2}.
\]
5. BONUS (1 extra point each):

(a) $\frac{1 + \sqrt{5}}{2}$

(b) $\pi$

(c) 3.14159

(d) (b)