2. The function has a global maximum at \( c \) and a global minimum at \( b \). This is consistent with the Extreme Value Theorem since the function is continuous on a closed interval and attains its global extrema.

4. The function does not have a global maximum or global minimum. Since the function is not continuous, the Extreme Value Theorem does not say anything about the function.

6. The function has a global maximum at \( a \) and a global minimum at \( c \). Since the function is not continuous, the Extreme Value Theorem does not say anything about the function.

9. We have a global maximum of 5 at 0 and no global minimum.

12. (b) We have horizontal tangent lines at \( a \) and \( b \), and a negative slope at \( c \).

14. (a) We have corners at \( a \) and \( b \) and a negative slope at \( c \).

16. See graph of \( y = \frac{6}{x^2+2} \) on \((-1, 1)\) below. The function has absolute maximum of 3 at 0 and no absolute minimum. The domain is not a closed interval, so the Extreme Value Theorem does not say anything about this function.

20. The function has an absolute maximum of 2 at 4 and no absolute minimum. The function is not continuous, so the Extreme Value Theorem does not say anything about this function.
22. The global maximum is 0 and global minimum is $-3$.

24. The global maximum is 4 and global minimum is $-5$.

26. The global maximum is 1 and global minimum is $\frac{1}{2}$. 
4.1 SOLUTIONS

30. The global maximum is 0 and global minimum is $-\sqrt{5}$.

37. We compute

$$g'(x) = e^{-x} - xe^{-x} = e^{-x}(1 - x).$$

This is never undefined, and equal to 0 when $x = 1$. Since 1 is also an endpoint, we just evaluate $g$ at $x = 1$ and $x = -1$ to see that the global maximum is $e^{-1}$ and global minimum is $-e^{-1}$. 
42. We want to find the absolute extrema of \( f(x) = x^{5/3} \) on \([-1, 8]\). We compute
\[
f'(x) = \frac{5}{3} x^{2/3}.
\]
This is never undefined and equal to zero at 0. We plug in this critical point and endpoints into \( f \) and compare values
\[
f(0) = 0, \quad f(-1) = -1, \quad f(8) = 2^5 = 32.
\]
It follows that the absolute maximum of 32 is attained at 8 and absolute minimum of -1 is attained at -1.

44. We want to find the absolute extrema of \( h(\theta) = 3\theta^{2/3} \) on \([-27, 8]\). We compute
\[
h'(\theta) = 2\theta^{-1/3}.
\]
This is undefined when \( \theta = 0 \) and never 0. We plug in this critical point and the endpoints into \( h \) and compare values
\[
h(-27) = 27, \quad h(8) = 12, \quad h(0) = 0.
\]
It follows that \( h \) attains an absolute maximum of 27 at -27 and absolute minimum of 0 at 0.

54. We have \( y = x^3 - 2x + 4 \), so
\[
\frac{dy}{dx} = 3x^2 - 2 = 3(x - \sqrt{2/3})(x + \sqrt{2/3}).
\]
This is never undefined and is equal to zero when \( x = \pm \sqrt{2/3} \). Making a sign chart, we see that \( \frac{dy}{dx} > 0 \) on \((-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty) \) and \( \frac{dy}{dx} < 0 \) on \((-\sqrt{2/3}, \sqrt{2/3}) \).
Compute that
\[
y(\sqrt{2/3}) = \frac{2}{3} \sqrt{\frac{2}{3}} - 2\sqrt{\frac{2}{3}} + 4 = -\frac{4}{3} \sqrt{\frac{2}{3}} + 4
\]
\[
y(-\sqrt{2/3}) = -\frac{2}{3} \sqrt{\frac{2}{3}} + 2\sqrt{\frac{2}{3}} + 4 = \frac{4}{3} \sqrt{\frac{2}{3}} + 4.
\]
Thus we have an absolute and local maximum of \( \frac{4}{3} \sqrt{\frac{2}{3}} + 4 \) at \(-\sqrt{2/3}\) and an absolute and local minimum of \(-\frac{4}{3} \sqrt{\frac{2}{3}} + 4 \) at \( \sqrt{2/3}\).

58. We want the local and global extrema of \( y = x - 4\sqrt{x} \). Notice that the domain of the function in \([0, \infty)\). We compute \( \frac{dy}{dx} = 1 - 2x^{-1/2} \). This is undefined at 0 and equal to zero when \( x = 4 \). We compute
\[
y(0) = 0 - 0 = 0
\]
\[
y(4) = 4 - 4\sqrt{4} = -4.
\]
Making a sign chart, we see that \( \frac{dy}{dx} > 0 \) on \((4, \infty)\) and \( \frac{dy}{dx} < 0 \) on \((0, 4)\). It follows that we have a local and absolute minimum of -4 at 4 and no absolute maximum. We have a local maximum of 0 at 0.
66. We want the local and global extrema of \( y = x^2 \ln x \). Note that the domain of the function is \((0, \infty)\). We compute
\[
\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x.
\]
This is never undefined and is equal to zero when
\[
2x \ln x + x = 0
\]
\[
x(2 \ln x + 1) = 0.
\]
This is zero when \( x = 0 \), but that is not in the domain. It is also equal to zero when \( \ln x = -1/2 \). This is when \( x = e^{-1/2} \). We compute \( y(e^{-1/2}) = -\frac{1}{2} e^{-1} \). Making a sign chart, we see that \( \frac{dy}{dx} < 0 \) on \((0, e^{-1/2})\) and \( \frac{dy}{dx} > 0 \) on \((e^{1/2}, \infty)\). It follows that we have a local and global minimum of \(-\frac{1}{2} e^{-1}\) at \( e^{-1/2} \) and no local or global maximum.

72. Consider \( y = x^{2/3}(x^2 - 4) = x^{8/3} - 4x^{2/3} \). The domain is \( \mathbb{R} \). We compute
\[
\frac{dy}{dx} = \frac{8}{3} x^{5/3} - \frac{8}{3} x^{-1/3} = \frac{8}{3} \left( \frac{x^2 - 1}{x^{1/3}} \right).
\]
This is undefined at 0 and is equal to zero at \( \pm 1 \). We compute
\[
y(0) = 0 - 0 = 0
\]
\[
y(1) = 1 - 4 = -3
\]
\[
y(-1) = 1 - 4 = -3
\]
Making a sign chart, we see that \( \frac{dy}{dx} < 0 \) on \((\infty, -1) \cup (0, 1)\) and \( \frac{dy}{dx} > 0 \) on \((-1, 0) \cup (1, \infty)\). It follows that we have a local maximum of 0 at 0. We have no absolute maximum. We have local and global maximum of -3 at 1 and -1.

78. Recall that
\[
|z| = \begin{cases} 
z & \text{if } z \geq 0, \
-z & \text{if } z < 0. 
\end{cases}
\]
It follows that to understand \( f(x) = |x^3 - 9x| \), we first need to understand where \( x^3 - 9x \) is positive and where it is negative. We factor
\[
x^3 - 9x = x(x+3)(x-3)
\]
and see that it is zero at 0, 3, -3 and never undefined. We make a sign chart and see that \( x^3 - 9x < 0 \) on \(( -\infty, -3) \cup (0, 3)\) and \( x^3 - 9x > 0 \) on \((-3, 0) \cup (3, \infty)\). It follows that
\[
f(x) = \begin{cases} 
x^3 - 9x & \text{if } x \in (-3, 0) \cup (3, \infty), 
-x^3 + 9x & \text{if } x \in (-\infty, -3) \cup (0, 3).
\end{cases}
\]
From this, we see that
\[
f'(x) = \begin{cases} 
3x^2 - 9 & \text{if } x \in (-3, 0) \cup (3, \infty), 
-3x^2 + 9 & \text{if } x \in (-\infty, -3) \cup (0, 3).
\end{cases}
\]
The points 0, 3, -3 require more work.
a. As we approach 0 from the left, the slope of the tangent line is $-9$. As we approach 0 from the right, the slope of the tangent line is 9. It follows that we have a corner there, and so $f'(0)$ does not exist.

b. As we approach 3 from the left, the slope of the tangent line is $-27 + 9 = -18$. As we approach 3 from the right, the slope of the tangent line is $27 - 9 = 18$. As above, we have a corner and so $f'(3)$ does not exist.

c. As we approach $-3$ from the left, the slope of the tangent line is $-27 + 9 = -18$. As we approach from the right, the slope of the tangent line is $27 - 9 = 18$. As above, we have a corner and so $f'(-3)$ does not exist.