PROOF WRITING

Abstract. Some informal guidelines and additional readings to follow to help with proof-writting.

Guidelines for proofs

(1) Use complete sentences.
(2) Each sentence should set notation or be a true statement.
(3) Each true statement should be a conclusion that can be drawn from the previous statements using a definition, computation, or result proved in class.
(4) Do not assert the statement you wish to prove at the beginning of a proof. You should preface such statements with “We wish to prove” or similar.
(5) Oftentimes, a good first step is just unwinding the definitions.
(6) To prove “if $p$, then $q$” directly, start your proof by assuming $p$ is true. Then deduce that $q$ must be true.
(7) To prove “if $p$, then $q$” by contraposition, start your proof by assuming $q$ is false. Then deduce that $p$ must be false.
(8) To prove $p$ by contradiction, start your proof by assuming $p$ is false. Then deduce a contradiction.

Here are some examples of what is meant by (2) above.

- $ax$
  This is not a sentence.
- $ax = b$ has a solution.
  This is a sentence, but it is not true or false. We need to know more about $a$ and $b$.
- Let $a \in \mathbb{R}$, $a \neq 0$. Then $ax = b$ has a solution.
  This is a bit better. The first sentence sets notation, but the second sentence is still neither true nor false since we have not specified the universe for $b$.
- Let $a \in \mathbb{R}$, $a \neq 0$. Then $ax = b$ has a solution for every $b \in \mathbb{R}$.
  The first sentence sets notation. All of the notation is defined. The second sentence is true.

Supplemental readings