Prerequisites. None.

Possible advisors. Rychtář.

Goals. Write a computer program that will solve a very large and sparse system of linear equations. The system will be describing an evolutionary dynamics on a graph. The program will take a graph $G = (V, E)$ as an input and, by solving the corresponding system of equations, it will calculate an outcome of the evolutionary dynamics on that graph.

Project description. The following evolutionary dynamics is described in Lieberman et al. (2005); see also Nowak (2006).

Let $G = (V, E)$ be a finite (but generally very large), undirected and connected graph; $V$ is the set of vertices and $E$ is the set of edges. The vertices of the graph are “inhabited” by individuals; there are two types of individuals that are called residents and mutants. The two types differ in fitness - a number that represents an individual’s willingness to produce an offspring. If fitness of $I_1$ is 2 and fitness of $I_2$ is 8, the individual $I_2$ is four times as likely to produce an offspring than individual $I_1$. Residents are assumed to have fitness 1, mutants to have fitness $r$.

Once an offspring is produced, it replaces one of the neighbors of its parent; the neighbor is chosen at random. The state is described the the set $C \subset V$ of vertices inhabited by mutants. Once mutants inhabit the vertices in the set $C \subset V$ at some point of time, then in the next step mutants will inhabit vertices in either

(i) a set $C \cup \{j\}, j \notin C$, provided (a) a vertex $i \in C$ is chosen for reproduction and (b) it places its offspring into vertex $j \notin C$, or
(ii) a set $C \setminus \{i\}, i \in C$, provided (a) a vertex $j \notin C$ is selected for reproduction and (b) it places its offspring into $i \in C$, or
(iii) the set $C$, provided an individual from $C \setminus C$ replaces another individual from $C \setminus C$.

The rules of the dynamics yield the following system of linear equations

$$P_C = \frac{\sum_{i \in C} \sum_{j \notin C} \left( rw_{ij} P_{C \cup \{j\}} + w_{ji} P_{C \setminus \{i\}} \right)}{\sum_{i \in C} \sum_{j \notin C} \left( rw_{ij} + w_{ji} \right)}$$

with $P_\emptyset = 0$ and $P_V = 1$ where $P_C$ denote the probability of mutant fixation given mutants currently inhabit a set $C$ and the the graph structure is represented by a matrix $W = (w_{ij})$, where $w_{ij}$ is the probability of replacing a vertex $j$ by a copy of a vertex $i$, provided vertex
The system (1) is a very large and sparse system of linear equations (even when symmetries of the graphs are considered, it still has of the order of $2^{|V|}$ equations but each line has only $|V|$ nonzero entries).

The goal of the project would be to write an efficient computer program that would take an adjacency matrix of the graph as an input and produce the solution of (1) as an output. Because there are many variations of the dynamics, the program should then be easily updated to account for different dynamics and updating rules.

In the long run, the student could continue in this direction for the Ph.D. thesis and use this program to find out connections between some of graph characteristics and the solution of (1) (in an ideal case, the solution of (1) should be approximated using some graph characteristics and without solving (1) at all).

REFERENCES
