Combinatorics, Group Theory and Topology at UNCG
in the
Department of Mathematics & Statistics

Combinatorics, Group Theory and Topology are three active areas of research in pure mathematics at UNCG.

The Combinatorics Group works with combinatorial probability, computational complexity, and discrete geometry.

Group Theory research areas include geometric group theory, representation theory, and arithmetic groups.

UNCG’s topologists work with general and set-theoretic topology, geometric topology, topological algebra, and asymptotic topology.

Above right is the permutahedron of order 4; under that is a compactification of $\mathbb{R}$ with remainder equal to an interval.

Mathematics Programs at UNCG

- Ph.D. in Computational Mathematics
- M.A. in Mathematics
- B.S. and B.A. in Mathematics

Graduate teaching assistantships and tuition waivers are available.
Group Theory

A group is the collection of symmetries of an object. In the study of group theory, there is an information exchange between a group and the object on which the group acts. A modern approach comes from considering the object to be the group itself, with perhaps additional geometric or measure-theoretic structure. The result is an interchange between the algebraic information of a group and its geometric and measure-theoretic counterparts. The class of lattices, or more generally finitely generated groups provides a rich and natural source of examples for this framework. This yields an expansive field of research that touches upon many beautiful areas of mathematics.

Dr. Talia Fernós
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Dr. Fernós earned a Ph.D. in 2006 from the University of Illinois at Chicago, and she joined the UNCG faculty in 2010. Upon completion of her PhD, she was awarded the NSF Mathematical Sciences Postdoctoral Fellowship. Dr. Fernós has organized conferences and established collaborations in several countries. Her research studies infinite groups from both geometric and analytical perspectives.

Dr. Igor Erovenko
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Dr. Erovenko earned a Ph.D. in 2002 from the University of Virginia, and he joined the UNCG faculty in 2002. His research concerns arithmetic groups, which can be roughly thought of as discrete subgroups of linear (matrix) groups. He focuses on how combinatorial properties of such a discrete subgroup considered as an abstract group affect its arithmetic and geometric properties.

Combinatorics

Combinatorics is the study of objects whose constituents are discrete, or separated, as opposed to those that are continuous. It is a vibrant field with major interactions with algebra, analysis, and probability and it has a substantive connection with almost every field in mathematics. There are dozens of overlapping areas of specialty within combinatorics, but graphs and hypergraphs, enumeration, and combinatorial geometry are particularly large and active categories of research.
Topology is the study of continuity. It is often described as a branch of geometry where two objects that can be continuously deformed to one another are considered to be the same. One goal of topology is to understand this notion of continuity in its essential form and to develop “invariants” that can help distinguish when two spaces are different. Invariants include the number of “connected” components, how many “holes” it has, or its “dimension.” While this captures some of the spirit of topology, the reality is that the picture is more complex. Indeed, part of the study is to come up with pathological examples that do not behave as one might expect them to and to develop new invariants to help distinguish these spaces. The study is then an awe inspiring safari exploring this plethora of wild examples.

Dr. Greg Bell  
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Dr. Bell received his Ph.D. from the University of Florida in 2002. He joined the UNCG faculty in 2005 after completing post-doctoral work at the Pennsylvania State University. His research is in asymptotic invariants of finitely generated groups with a focus in large-scale topological invariants of Cayley graphs of finitely generated groups and their consequences.

Dr. Jerry Vaughan  
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Dr. Vaughan received his Ph.D. from Duke University, and he joined the UNCG faculty in 1973. His research covers many areas in general and set-theoretic topology. His most recent work concerns the Stone-Čech compactifications of certain easily visualized topological spaces called ψ-spaces.

Dr. Clifford Smyth  
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Dr. Smyth earned a Ph.D. in 2001 from Rutgers University and did postdoctoral work at IAS, CMU and MIT. He joined the UNCG faculty in 2008. His research lies in combinatorial probability, with forays into computational complexity, enumerative combinatorics, and other fields.
Research Activity at UNCG

Asdim of the Farey graph

The Farey Graph is a graph whose vertices are in bijection with \( \mathbb{Q} \), with the vertices \( \frac{a}{b} \) and \( \frac{c}{d} \) joined by an arc precisely when \( |ad - bc| = 1 \). G. Bell and K. Fujiwara showed that the Farey graph has asymptotic dimension 1 and that it has a variant of property A that is defined for non-locally finite graphs.

Topological and its Applications


Comparing Cardinals

The diagram at the left is intended to display the basic relations among the ten cardinals shown there. Each cardinal is an uncountable cardinal not larger than the cardinality of the continuum c. Following the lines, if a cardinal \( \kappa \) is lower on the diagram than cardinal \( \lambda \), this indicates that there is a proof that \( \kappa \leq \lambda \), and that there is no proof that they are equal (i.e., in some models of set theory \( \kappa < \lambda \)). If \( \kappa \) and \( \lambda \) are not related by line segments, then neither can be proved less than or equal to the other.

The Solenoid and Property (T)

Property (T) studies the unitary dual of a group, in particular the topological structure near the trivial representation. An example of what a unitary dual can look like is the solenoid. Using automorphisms of this solenoid, Shalom proved directly that \( \text{SL}(n, \mathbb{Z}[S^{-1}]) \) has property (T). T. Fernós has done extensive work on Property (T).

Long Monotone Paths

Let \( L = \{l_1, ..., l_n\} \) be a set of \( n \) given lines in \( \mathbb{R}^2 \). A path in the arrangement \( A(L) \) is a simple polygonal chain joining a set of distinct vertices in \( V = \{l_i \cap l_j : i < j\} \) by segments which are on lines in \( L \). The length of a path is one plus the number of vertices in \( V \) at which the path turns. A path is monotone in direction \( (a, b) \) if its sequence of vertices is monotone when projected orthogonally along the line with equation \( ay - bx = 0 \). An interesting open question asks for the value of \( \lambda \), the maximal monotone path length \( N \) that can occur in an arrangement of \( n \) lines. Clearly \( \lambda(n) \leq \binom{n}{2} + 1 \).

In [Balogh, Regev, Smyth, Steiger, Szegedy, Discrete Comput Geom 32:167-176] the authors nearly settle the problem showing \( \lambda(n) = \Omega(n^{2-d} / \sqrt{\log n}) \). The long monotone path and the line arrangement in which it lies are recursive in nature, the template for the kth level is shown.

Contact Information

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