Disks and Halos in Pre-Main-Sequence Stars

Dejan Vinković, 1 Željko Ivezić, 2 Anatoly S. Miroshnichenko 3 and Moshe Elitzur 1

1 Department of Physics & Astronomy, University of Kentucky, Lexington, KY 40506, USA; dejan@pa.uky.edu, moshe@uky.edu
2 Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA; ivezic@astro.Princeton.edu
3 Department of Physics & Astronomy, University of Toledo, Toledo, OH 43606, USA; anatoly@physics.utoledo.edu

September 16, 2002

ABSTRACT

Two different geometries have been proposed to explain the dust emission from pre-main-sequence stars: flared disks and “classical” geometrically-thin optically-thick disks imbedded in optically thin halos. We show that only imaging observations can differentiate these two morphologies because flux measurements can never distinguish between them. A model constructed with one geometry implies an equivalent model with the other that produces the identical flux at every wavelength. When a flared disk is imbedded in a halo, the halo dominates the system IR flux whenever \( \tau_{\text{halo}} > \frac{1}{4} \frac{H}{R} \), where \( \tau_{\text{halo}} \) is the optical depth across the halo at visual wavelengths and \( R \) and \( H \) are, respectively, the disk radius and height at its outer edge. Even when its optical depth is much smaller, the halo can still have a significant effect on the disk temperature profile.

Key words: circumstellar matter — dust — infrared: stars — radiative transfer — stars: imaging — stars: individual: AB Aur — stars: formation — stars: pre-main-sequence

1 INTRODUCTION

Star formation is expected to produce circumstellar disks during the pre main-sequence phase. Indeed, T Tauri stars (TTS) provide good evidence for the presence of such disks (e.g. Bertout 1989; Basri & Bertout 1989; Valenti, Basri & Johns 1993). In contrast, the predominant evidence from visual and IR observations of their intermediate mass (\( \sim 2-10 \, M_{\odot} \)) counterparts, the Herbig Ae/Be stars (Haebes hereafter), is for a different morphology. For example, the 50 and 100 \( \mu \)m imaging of numerous Haebes by DiFrancesco et al (1994, 1998) reveal sizes exceeding \( 10^4 \) AU, much larger than the \( \sim 100-500 \) AU typical of accretion disks. Furthermore, the emitting regions are roughly circular and do not show the elongation that disks are expected to produce for all orientations other than face-on. Speckle interferometry in the near-IR by Leinert et al (2001) produces similar results. In 18 of 31 Haebes and related stars they find extended (\( \sim 1000 \) AU) halos with no more than slight, if any, evidence for the elongations expected in a sample of disks. These findings contrast sharply with those for T Tauri stars: using the same observing and reduction techniques and down to the same contrast level, Leinert et al (1993) found evidence for near-IR halos in only 3 of 71 TTS. Circular shapes predominate also in the near-IR IOTA interferometry of 10 Haebes by Millan-Gabet, Schloerb & Traub (2001).

While extended, roughly spherical halos seem to dominate the Haebes emission at optical and IR wavelengths, the CO line and mm-continuum observations of Mannings & Sargent (1997) give conclusive evidence for disks. Miroshnichenko et al (1999, MIVE hereafter) note that the seemingly conflicting evidence for extended halos on one hand and disks on the other can be reconciled in a simple composite model: the dust distribution in Haebes is comprised of a compact, optically thick disk embedded in an extended, optically thin halo. Detailed calculations with this model satisfactorily reproduce the spectral energy distribution (SED) of seven Haebes with good data from visual all the way to mm wavelengths. This composite morphology works because, as noted earlier by Butner, Natta & Evans (1994), the disk dominates the mm emission while the halo dominates the shorter wavelengths owing to the different temperature profiles of the two components. This role reversal gives rise to images that are extended at IR but compact at mm wavelengths, and may help explain the puzzling behavior of MWC 137 whose observed size decreases between 50 \( \mu \)m and 100 \( \mu \)m (DiFrancesco et al 1994, 1998). No single dust configuration can explain such a decrease.

Chiang and Goldreich (1997, CG hereafter) suggest that the emission from TTS can be modeled purely in terms of flared disks, and fit a number of SEDs in terms of this morphology. This proposal was extended to Haebes by Chiang at al (2001) and Natta et al (2001), who present detailed SED fits employing the CG flared disk model without a halo. One of the modeled sources is AB Aur, which was fitted also by MIVE. Dullemond et al (2001) later modified the flared disk model for AB Aur, still without any halo. The underlying premise of this approach is that halos can be ignored in the studied sources because even if they do exist, their contribution is negligible in comparison with the flared disk.

How can the SED of the same source, AB Aur, be fitted successfully both with a halo (MIVE) and without one (Dullemond et al)? Is it at all possible to distinguish between these two morphologies? What, if any, is the unique radiative signature of each of them? Obviously, imbedding a disk in a tenuous halo with a very small optical depth produces an image that is extended at IR but compact at mm wavelengths, which should be distinguishable from a purely flared disk.
When the dust is optically thin, the cavity radius can be found from

cavity of radius $R$ (equation 1). The dust temperature declines with radial distance toward its surface value $T_{out}$. The intensity at impact parameter $b$ is obtained from integration along the indicated path toward the observer.

small optical depth is not going to affect its emission appreciably. At which stage then does the halo assume a significant role? Here we address these questions. We derive general, geometry independent results for the emission from optically thin dust, and apply these results to flared disks and halo-embedded-disks. Detailed application to AB Aur will be presented in a separate paper (Vinković et al 2002).

2 OPTICALLY THIN DUST—GENERALITIES

Consider a cloud heated from inside by a star of radius $R_*$ and effective temperature $T_*$ (figure 1). The star clears out a dust-free cavity of radius $R_0$, determined by dust sublimation $T(R_0) = T_\nu$. When the dust is optically thin, the cavity radius can be found from

$$\frac{R_0}{R_*} = \frac{1}{2} \left( \frac{\bar{\sigma}(T_\nu)}{\bar{\sigma}(T_*)} \right)^{1/2} \left( \frac{T_\nu}{T_*} \right)^{2}$$

(1)

where $\bar{\sigma}(T)$ is the Planck average at temperature $T$ of the absorption cross section $\sigma_\nu$ (Ivezić & Elitzur 1997; IE hereafter). With standard interstellar dust and $T_* = 1500$ K, the cavity radius obeys $R_0/R_* \approx 100$ at a typical Haebs temperature $T_\nu = 10,000$ K. In TTS, the dust is much closer to the star: $R_0/R_*$ is only 15 at $T_* = 5,000$ K and as small as 3 at $T_* = 3,000$ K. The intensity at frequency $\nu$ and impact parameter $b$ is

$$I_\nu(b) = \sigma_\nu \int \left[ (1 - \varepsilon_\nu) B_\nu + \varepsilon_\nu J_\nu \right] n_d dz,$$

(2)

assuming isotropic scattering. Here $n_d$ is the dust density, $z$ is distance along the path to the observer, $\varepsilon_\nu$ is the albedo at frequency $\nu$ and $J_\nu = \int I_\nu d\Omega/4\pi$. This expression neglects self-absorption by the dust; the error in this approximation is of order $1 - \exp(-\int \sigma_d n_d dz)$.

2.1 Scattering Wavelengths

Since the dust temperature cannot exceed the sublimation temperature $T_\nu$, there is no dust emission at $\lambda \lesssim 4\mu m \times (1000K)/T_\nu$, only scattering. Diffuse radiation and attenuation between the star and the scattering point can be neglected since our discussion is centered on optically thin dust. Then the only source of scattering is the stellar radiation with energy density $J_\nu = L_\nu/4\pi c^2$, where $L_\nu$ is the stellar luminosity at frequency $\nu$. From equation 2, the scattered brightness is

$$I_\nu(\theta) = \frac{L_\nu}{4\pi} \varepsilon_\nu \sigma_\nu \int n_d dz,$$

(3)

where $\theta = b/D$ and $D$ is the distance to the observer. Since the frequency- and geometry-dependence separate out, all scattering wavelengths share a common image. Only the brightness level varies with $\nu$, and because of the wavelength decline of $\varepsilon_\nu \sigma_\nu$, the observed size generally decreases with wavelength when traced to the same brightness level. In any geometry the scattering image always traces directly the variation of column density along the line of sight, the dust temperature profile is irrelevant.

2.2 Emission Wavelengths

At wavelengths longer than $\sim 3 \mu m$, $\varepsilon_\nu < 10^{-2}$ and scattering can be neglected. The Planckian enters in equation 2 as a function of $T$ at fixed $\nu$, which can be well approximated by its Rayleigh-Jeans limit at $T > T_\nu = 0.56h\nu/k$ and a sharp cutoff at $T_\nu$. With this approximation the integration is limited to locations along the path where $T > T_\nu$; regions with $T < T_\nu$ are too cold to emit appreciably at frequency $\nu$. Excluding highly patchy geometries, the highest temperature on the path occurs at $r = b$ (i.e., $z = 0$), the closest distance to the star, and only paths with $T(b) > T_\nu$ contribute to the brightness. As $z$ increases in either direction, $T$ decreases. The integration is truncated either because the temperature becomes too low, in which case the emission is temperature bounded, or because the edge of the source is reached and the emission is matter bounded. Denote the resulting integration limits $Z_i$ ($i = 1, 2$), then

$$I_\nu(\theta) = \frac{2}{c^2} \nu^2 \sigma_\nu \int_{Z_i}^{Z_2} kT n_d dz.$$

(4)

In the matter bounded case $Z_i$ is the edge of the source, the integral is independent of $\nu$ and the frequency dependence of the intensity follows $\nu^2 \sigma_\nu$. In the case of temperature bounded emission the integration limits introduce additional $\nu$-dependencies that modify this behavior. However, the integration can be extended to $\infty$ whenever (1) $Z_i \gg b$ and (2) the product $n_d T$ of dust density and temperature declines along the path faster than $1/z$. Therefore, when these two conditions are met, the frequency variation of optically thin emission is $I_\nu \propto \nu^2 \sigma_\nu$ even when it is temperature bounded. Independent of geometry, all frequencies that obey these conditions produce a common image, similar to the scattering case; only the scale of brightness varies with $\nu$. This result makes it possible to determine the wavelength dependence of the dust cross section directly from imaging observations.

Similar to the variation along the line of sight, when $b$ increases (moving away from the star) the emission again is truncated by either the matter or temperature distribution. Denote by $T_{out}$ the temperature at the source outer edge. The corresponding emission cutoff wavelength is

$$\lambda_{out} = 100 \mu m \times \frac{40K}{T_{out}}.$$  

(5)

When $\lambda > \lambda_{out}$, the dust is sufficiently warm everywhere that the emission is truncated only by the matter distribution. The observed size is then $\Theta$, the angular displacement of the source edge from the star, for all wavelengths. However, when $\lambda < \lambda_{out}$
the brightness is truncated when $T(b) \leq T_v$ before the edge of the source is reached, resulting in a wavelength-dependent angular size $\theta_s < \Theta$. The observed size of optically thin emission increases with wavelength so long as $\lambda < \lambda_{\text{out}}$, the opposite of the trend at scattering wavelengths.

The frequency variation of the dust cross section is well described by $\sigma_{\nu} \propto \nu^n$ with $n = 1-2$. Then to a good degree of approximation, the temperature variation of optically thin dust is $T \propto 1/\nu^t$, where $t = 2/(4+n)$, producing the wavelength-dependent observed angular size

$$\theta_\lambda = \Theta \times \begin{cases} \left(\frac{\lambda}{\lambda_{\text{out}}}\right)^{1/t} & \lambda < \lambda_{\text{out}} \\ 1 & \lambda \geq \lambda_{\text{out}} \end{cases} \quad (6)$$

Since $t < \frac{1}{2}$, $\theta_\lambda$ increases faster than $\lambda^2$, a fairly steep rise.

### 2.3 Flux—the SED

The flux can be obtained from equation 2 by integration over the observed area. Since scattering does not modify the photon frequency it has no effect on the SED. At emission wavelengths, the flux at distance $D$ is

$$F_\nu = \frac{\sigma_v}{\pi D^2} \int B_\nu(T) n_\lambda dV.$$ \quad (7)

Since the temperature profile of optically thin dust depends only on distance from the star, the dependence on the source geometry enters only from $n_\lambda$.

As before, the integration is truncated by either the temperature or the matter distribution. Whenever $\lambda > \lambda_{\text{out}}$ at every point on the surface, the emission is matter bounded everywhere and the integration encompasses the entire source. Under this circumstances $F_\nu \propto \nu^2 \sigma_v$, a universal SED that depends only on the dust properties irrespective of geometry. In particular, the spectral variation $\sigma_v \propto \nu^n$ gives $F_\nu \propto \nu^{2-n}$, therefore the signature of matter bounded emission is this SED accompanied by wavelength independent images; this is the expected behavior in any geometry at sufficiently long wavelengths. At $\lambda < \lambda_{\text{out}}$ the integration volume is truncated by the temperature, and since the emission volume decreases as the frequency increases, the rise of $F_\nu$ with $\nu$ becomes less steep than in the matter dominated regime. Therefore, the SED changes from $F_\nu \propto \nu^{2-n}$ at $\lambda > \lambda_{\text{out}}$ to a flatter slope at $\lambda < \lambda_{\text{out}}$.

The break in the slope at $\lambda_{\text{out}}$ can be used to determine the surface temperature $T_{\text{out}}$. Flux spectral variation shallower than $\nu^3$ is a clear indication of temperature-bounded emission and should be accompanied by an image size that increases with wavelength.

### 2.4 Spherical Geometry

Some explicit results are easily derived in the case of spherical symmetry. Thanks to the scaling properties of dust radiative transfer (IE), only two properties are required to specify the geometry. First is the radial optical depth at one wavelength, say $\tau_v = \sigma_v \int n_\lambda d\tau$ where $\sigma_v$ is the cross-section at visual; at every other wavelength, $\tau_v = q_\nu \tau_v$ where $q_\nu = \sigma_v/\sigma_v$. The second is the dimensionless, normalized profile of the dust density distribution

$$\eta(y) = \frac{n_\lambda(y)}{n_\lambda dy} \quad (8)$$

where $y = r/R_\nu$; note that $\int \eta dy = 1$. Explicit results follow immediately for all power-law density profiles where

$$\eta = \frac{N}{y^p} \quad N = \begin{cases} \frac{(p-1)/(1-Y^{1-p})}{(\ln Y)^{-1}} & p \neq 1 \\ \frac{1}{p} & p = 1 \end{cases} \quad (9)$$

The shell extends to the outer radius $Y R_\nu$, subtending the angular region $\theta_s < \theta < \Theta$, where $\theta_s = R_\nu/D$ and $\Theta = Y \theta_s$. At scattering wavelengths

$$I_{\nu,\text{sca}}(\theta) = \frac{N}{2\pi} \int_0^{\mu_{\nu,\text{sca}}} \left(\frac{\theta}{{\theta_s}}\right)^{p+1} \frac{du}{(1+u^2)^{(p+2)/2}} \quad (10)$$

Whenever $\theta < \theta_s$ the integration can be extended to $\infty$, yielding $I(\theta) \propto 1/\theta^{p+1}$; the brightness decreases as a power law so long as the observation direction is not too close to the halo edge. At emission wavelengths, on the other hand,

$$I_{\nu,\text{em}}(\theta) = \frac{4N}{c^2} kT_s \tau_v \nu^2 q_\nu \left(\frac{\theta}{\theta_s}\right)^{p+t-1} \int_0^{\mu_{\nu,\text{em}}} \frac{du}{(1+u^2)^{(p+t)/2}} \quad (11)$$

where the observed size $\theta_\lambda$ is smaller than $\theta$ when $\lambda < \lambda_{\text{out}}$ (equation 6). As long as $\theta < \theta_s$, the integration can be extended to $\infty$ and the brightness then decreases along any radial direction in proportion to $1/\theta^{p+1}$.

The flux integration in equation 7 is similarly terminated at the observed boundary $\theta_s$, producing

$$F_\nu = \frac{8\pi N}{c^2} kT_s \theta_s^2 \nu^2 q_\nu \left(\frac{\theta_s}{\theta_s}\right)^{3-(p+t)} - 1 \quad (12)$$

where $\theta_s > \theta_s$, there are two families of SED. In the case of steep density distributions with $p > 3-t$, the first term inside the brackets can be omitted because $3-(p+t) < 0$. Such distributions produce $F_\nu \propto \nu^2 \sigma_v$ irrespective of the actual value of $p$. Since typically $t \sim 0.4$, this behavior applies to all cases of $p \gtrsim 2.6$.

On the other hand, whenever $p < 3-t$ the omitted term dominates and the SED is a broken power law. The power index switches from the universal $2 + n$ at $\lambda \geq \lambda_{\text{out}}$ to the geometry-dependent value $3 + n - (3-p)/t$ (see also Harvey et al 1991) at $\lambda < \lambda_{\text{out}}$.

These results show that the SED displays a dependence on the density profile only when $p \lesssim 2.6$, and then only in the spectral region $\lambda \lesssim \lambda_{\text{out}}$. All other regions of $p$ and $\lambda$ produce the universal behavior $F_\nu \propto \nu^2 \sigma_v$.

### 3 HALO-IMBEDDED-DISKS

In this model the star is surrounded by a disk and a spherical dusty envelope (figure 2). The envelope extends from the inner radius $R_\nu$ to some outer radius $R_h = Y R_\nu$. Thanks to scaling, instead of these radii we can specify the dust temperature on each boundary. The halo is fully characterized by its density profile $\eta(y)$ (eq. 8) and optical depth $\tau_v$; only $\tau_v \lesssim 1$ is relevant in TTS and Haebes since the star is always visible.

We assume a geometrically thin flat passive disk, i.e., negligible accretion. Because of its potentially large optical depth, the disk can extend inside the dust-free cavity where its optical depth comes from the gaseous component. The geometrically-thin disk
Figure 2. Geometry of the halo-imbedded-disk model: a flat geometrically-thin optically-thick disk extends from the stellar surface to radius \( R_d \). An optically thin spherical halo extends from the dust sublimation radius \( R_s \) to \( R_h \). The small pillbox at the disk surface serves as a Gaussian surface for flux conservation.

The main contributors to the bolometric flux, emanate from close to the star so that their pathlength is approximately isotropic. If \( L_{\text{disk}} \) denotes the disk contribution to the overall luminosity \( L \), then \( F_{\text{disk}}(D, i) = (L_{\text{disk}}/2\pi D^2) \cos i \). Similarly, the (halo + attenuated stellar) spherical component of the flux obeys \( F_{\text{sph}}(D) = L_{\text{sph}}/4\pi D^2 \). Therefore the fractional contribution of the disk to the overall bolometric flux is

\[
\rho = \frac{F_{\text{disk}}}{F_{\text{disk}} + F_{\text{sph}}} = \frac{2\pi \cos i}{1 + 2\pi \cos i} \tag{14}
\]

where \( x = L_{\text{disk}}/L_{\text{sph}} \). Face-on orientation gives the maximal \( \rho = 2\pi/(1 + 2\pi) \). The standard “bare” disk has \( L_{\text{disk}} = \frac{1}{2} L \) (Kenyon & Hartmann 1987), therefore in this case \( x = \frac{1}{2} \) and \( \rho \leq \frac{1}{2} \). Larger fractions can occur when the disk is imbedded in a halo because of the heating effect of the diffuse radiation, discussed below.

We performed detailed model calculations with the code DUSTY (Ivezić, Nenkova & Elitzur 1999) which takes into account the energy exchange between the star, halo and disk, including dust scattering, absorption and emission. Because its optical depth is typically \( \tau_\nu \lesssim 1 \), the halo is transparent to the disk emission in all the models we consider and we neglect the disk effect on the halo. In all the calculations, the spectral shapes \( q_\nu \) of the grain absorption and scattering coefficients are those of standard interstellar mix, the sublimation temperature \( T_s = 1500 \) K. The spectral shape of the stellar radiation is taken from the appropriate Kurucz (1994) model atmosphere.

3.1 Temperature Profiles

The heating rate of a thin flat disk by the stellar radiation at \( r \gg R_s \) is

\[
\mathcal{H}_* = \frac{2F_* R_s}{3\pi R_a a^3} \tag{15}
\]

where \( F_* \) is the stellar flux at \( R_s \) and \( a = r_a/R_s \), with \( r_a \) distance from the axis (Friedjung 1985). This result reflects the \( 1/a^2 \) decline of the stellar solid angle and the \( 1/a \) variation of the grazing angle, yielding disk temperature variation \( T \propto a^{-3/4} \). Natta (1993) noted that imbedding the disk in a dusty halo can significantly affect its temperature even at small halo optical depths. With a simple model for scattering at a single wavelength she found that the disk temperature law becomes \( T \propto a^{-(1+p)/4} \) if the halo density profile is \( \eta \propto y^{-p} \).

Our calculations confirm this important point. Figure 3 shows the temperature profile for a disk around a \( T_* = 10,000 \) K star when “bare” and when imbedded in a spherical halo with \( \eta \propto y^{-2} \) and \( \tau_\nu = 0.1 \) and \( 1^1 \). Even though a halo with \( \tau_\nu \) as small as 0.1 is almost transparent to the stellar radiation, it still causes a large rise in disk temperature. As is evident from the bottom panel, the halo contribution to the disk heating overtakes the stellar contribution inside the dust-free cavity and dominates completely once the dust is entered.

A dusty envelope with \( \tau_\nu = 0.1 \) intercepts only about 10% of the stellar luminosity while the disk intercepts 25% of that luminosity. So how can the halo dominate the disk heating? The reason is that direct heating of the disk by the star occurs predominantly

\[1^1 \quad \text{The addition of a halo can only raise the disk temperature, yet figure 3 shows that our calculated profile for } \tau_\nu = 1 \text{ dips slightly below that of the bare disk at } a \lesssim 0.2. \text{ This happens because we neglect the disk emission in the calculation of the halo temperature. The error introduced by this approximation is of order } 2\% \text{ when } \tau_\nu = 1, \text{ and even less at smaller } \tau_\nu. \]
at small radii. The disk absorbs more than 90% of its full stellar allotment within 10\(R_\star\) while its entire remaining area, even though so much larger, absorbs only 0.025\(L\). From equation 1, the halo starts at \(R_\text{a} \sim 100R_\star\), where \(\mathcal{H}_s\) has already declined to \(\sim 10^{-6}\) of its value at the stellar surface. In contrast, the halo emission is isotropic, therefore half of the radiation it intercepts is re-radiated toward the disk, greatly exceeding the direct stellar contribution. Further insight can be gained from the approximate solution presented in IE for radiative transfer in spherical symmetry. From equations 20 and B4 of IE it follows that disk heating by a halo with \(\tau_\nu \lesssim 1\) and \(\eta \propto y^{-p}\) is roughly

\[
\mathcal{H}_h = \frac{3F_\nu p - 1}{8 \rho + 1} \tau_\nu \times \begin{cases} 
1 & a < 1 \\
1/a^{1+p} & a > 1
\end{cases}
\]  

(16)

when \(p > 1\); when \(p = 1\) the factor \((p-1)/(p+1)\) is replaced by \(1/(2 \ln Y)\). This yields \(T \propto \nu^{-2/(1+p)}\), corroborating Natta’s result and extending its validity beyond the single-wavelength scattering approximation she employed. The temperature profile is similar to that of a bare disk when \(p = 2\) but more moderate when \(p < 2\). More importantly,

\[
\mathcal{H}_h \bigg|_{a=1} = \frac{9\pi p - 1}{16 \rho + 1} \frac{R_\star}{R_\nu} \tau_\nu.
\]  

(17)

And since \(R_\nu \sim 100R_\star\), the halo dominates the heating at \(a = 1\) for \(\tau_\nu\) as small as 0.02 when \(p = 2\). As \(\tau_\nu\) increases, the halo dominance of the heating moves inside the cavity. There the halo heating remains approximately constant while the stellar heating varies as \(a^{-3}\), therefore stellar heating dominates only at \(a \lesssim (60\tau_\nu)^{-1/3}\), at larger distances the halo takes over. This explains the results presented in the lower panel of figure 3.

The figure also shows the temperature profile of the halo. This profile is largely independent of \(\tau_\nu\), varying roughly as \(y^{-2/(4+n)}\) when the long-wavelength spectral shape of the dust absorption coefficient is \(q_\nu \propto \nu^n\). The important property evident in the figure is that the disk is much cooler than the envelope at all radii at which both exist and can also contain cooler material in spite of being much smaller, with far reaching consequences for the system radiation.

3.2 SED

Introduce the normalized SED \(f_\nu = F_\nu/\int F_\nu \, d\nu\), with similar separate definitions for the disk and spherical components of the flux. From the definition of \(\rho\) in equation 14,

\[
f_\nu = \rho f_{\nu,\text{disk}} + (1 - \rho) f_{\nu,\text{sph}}
\]  

(18)

Since the disk flux obeys \(F_{\nu,\text{disk}}(i) = F_{\nu,\text{disk}}(0) \times \cos i\) for the range of parameters considered here, \(f_{\nu,\text{disk}}\) is independent of the viewing angle \(i\), and the entire \(i\)-dependence of the SED comes from the mixing factor \(\rho\).

Figure 4 shows sample SEDs for some representative models. The halos extend from \(R_\text{a} = 1,000R_\star\) with density profiles and overall optical depths as indicated. The behavior of SEDs for spherical shells was discussed in IE and since the halo emission is unaffected by the imbedded disk, the SEDs plotted in dashed lines need no further discussion. The disk, on the other hand, is strongly affected by the halo as is evident from contrasting each disk SED, plotted in long-dashed line, with what it would have been in the absence of a halo (dot-dashed line). The two curves are identical within the first bump around 1 \(\mu\)m, caused by the stellar heating. In the absence of a halo, the disk SED drops from that peak as \(\sim \nu^{1/3}\). However, halo heating of the outer regions of the disk generates the second broad bump of disk radiation, which is almost two orders of magnitude higher than the “bare” disk emission.

The halo heating effect is also evident from other disk properties. The disks in these models start at the stellar surface and extend to a radius \(R_\text{a}\) where the temperature is \(T_\text{a} = 25\,K\). In the absence of a halo, this temperature would be reached at \(R_\text{d} = R_\text{a}/R_\star = 18\). As Table 1 shows, heating by even a tenuous halo with \(\tau_\nu = 0.1\) pushes this radius out by almost factor 5 for the steep density profile \(r^{-2}\) and another factor of 2 for the flatter \(r^{-1}\) profile which spreads the heating further away from the star. The impact of halo heating increases with \(\tau_\nu\), pushing \(R_\text{d}\) further out still. Similarly, the disk luminosity, \(xL/(1 + x)\), is only 0.25\(L\) in the absence of

<table>
<thead>
<tr>
<th>(\tau_\nu)</th>
<th>(r^{-1})</th>
<th>(\tau_\nu = 0.1)</th>
<th>(\tau_\nu = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(Y_\text{d})</td>
<td>(x)</td>
<td>(Y_\text{d})</td>
</tr>
<tr>
<td>----------</td>
<td>----------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.43</td>
<td>190</td>
<td>1.33</td>
<td>400</td>
</tr>
<tr>
<td>0.47</td>
<td>85</td>
<td>1.56</td>
<td>135</td>
</tr>
</tbody>
</table>

Table 1. Derived parameters for the models whose SEDs are presented in figure 4. The luminosity ratio of the components is \(x = F_{\nu,\text{disk}}/F_{\nu,\text{sph}}\) and the disk radius is \(R_\text{d} = Y_\text{d}R_\star\), set from the requirement \(T_\text{d} = 25\,K\). A “bare” disk (\(\tau_\nu = 0\)) has \(x = \frac{1}{3}\) and \(Y_\text{d} = 18\).
Figure 4. Sample SEDs for halo-imbedded-disks around stars with $T^\star = 10,000$ K. The halo starts at $R_s$ where $T_s = 1,500$ K and extends to $1,000 R_s$, with density profile $\eta$ and optical depth $\tau_V$ as indicated. The disk starts at the stellar surface and extends to the radius $R_d$ set by the edge temperature $T_d = 25$ K (see Table 1). Top row: The SED for pole-on viewing. Full lines denote the overall flux, dashed lines the spherical (halo + attenuated stellar) component and long-dashed lines the disk component. The thick dot-dashed line shows the flux from a face-on “bare” disk in the absence of an imbedding halo. Mid row: Same models viewed at inclination angle $i = 85^\circ$. Bottom row: Variation of the overall SED with viewing angle $i$. Results for $i < 70^\circ$ are barely distinguishable from $i = 0^\circ$.

Van der Blick, Prusti & Waters (1994) find that its 60 $\mu$m size is $35'' \pm 5''$, yet 850 $\mu$m imaging by Holland et al. (1998) produced a size of only $24 \times 21'' \pm 3''$. So the dust distribution around Vega, too, could have both spherical and disk components.

A switch from envelope to disk domination provides a simple explanation for the otherwise puzzling decrease in the observed size of MWC 137 between 50 $\mu$m and 100 $\mu$m. A similar effect was recently detected also in the dust-shrouded main-sequence star Vega.
the effects of varying $\tau_{350}$ from its smallest allowed value $\tau_{350} = 1$ to a value sufficiently large that the edge is optically thick at all the displayed wavelengths.

### 3.2.1 The Disk Inner Radius

The bottom panel of each model in figure 4 shows the variation of the overall SED with viewing angle $i$. The entire variation comes from the mixing factor $\rho$ (see equations 14 and 18). Since the parameters $x$ and $i$ enter only in the product $x \cos i$ but not separately, the SED is subject to a degeneracy: systems viewed at different inclination angles will have the same SED if they have the same $x \cos i$ in addition to all other properties. Because of the rapid decline with distance of the radiation absorbed from the star (cf eq. 15), the disk luminosity, i.e., $x$, has a steep dependence on its inner radius $R_{\text{in}}$: moving the disk inner edge from $R_*$ to only $2R_*$ removes 56% of the stellar luminosity intercepted by the disk, $3R_*$ results in a 72% removal. Such central holes reduce $x$ but do not impact any other relevant property because they remove only the hottest disk material whose contribution to the overall flux is negligible in comparison with the stellar component.

Figure 7 plots contours in the $i$-$R_{\text{in}}$ plane of constant mixing factor $\rho$. It shows, for example, that the SEDs presented in figure 4 for $i = 70^\circ$ would be the same for systems viewed at $i = 35^\circ$ if the disk inner radius is increased from $1R_*$ to $2.2R_*$ in the $\tau_\nu = 0.1$ case and $6R_*$ for $\tau_\nu = 1$. Although the sizes of these holes cannot be determined from SED modeling, MIVE conclude that their existence is essential to produce a plausible distribution of inclination angles.

![Figure 5. Images at various wavelengths of a halo-imbedded-disk model that fits the observations of AB Aur (Vinković et al 2002). The viewing inclination angle is $i = 76^\circ$.](image)

![Figure 6. The effect on the SED of varying the temperature ($T_d$) and 350\,\mu m optical depth ($\tau_{350}$) of the disk outer edge. The displayed model has $\tau_\nu = 0.5$ and $\eta \propto r^{-2}$. The viewing angle is $85^\circ$. Dashed lines correspond to $\tau_{350} = 1$, full lines to $\tau_{350}$ sufficiently large that the disk edge remains optically thick at all displayed wavelengths.](image)

![Figure 7. Contours of fixed mixing coefficient $\rho$ (equation 14), as marked: The SEDs are the same as those presented in figure 4 for $i = 70^\circ$ and $85^\circ$ when the disk inclination $i$ and its inner radius $R_{\text{in}}$ vary together along each of the plotted curves. The contours are virtually the same for the $r^{-1}$ and $r^{-2}$ halo density profiles.](image)
4 FLARED DISKS

4.1 The CG Layer

The surface skin of any optically thick object is of course optically thin. Although the emission from the disk surface layer has been traditionally neglected, Chaing and Goldreich note that it could become significant under certain flaring conditions. The stellar radiation penetrates to an optical depth \( \tau = 1 \) along a direction slanted to the surface by angle \( \alpha \) (figure 8). The optical depth of the corresponding skin layer along the normal to the surface \( \hat{n} \) is \( \alpha \) at visual and \( \alpha_q \) at wavelength \( \nu \). The grazing angle of a flat thin disk whose inner radius is determined by dust sublimation is

\[
\alpha_{\text{flat}} = \frac{\alpha^*}{a}, \quad \text{where} \quad \alpha^* = \frac{4}{3\pi} \frac{R_s}{R_a}, \quad a = \frac{r_a}{R_a}.
\]

In Haebes \( \alpha^* \sim 10^{-2} \), and the optical depth of the surface layer cannot exceed this value. Flaring is defined by the radial profile of the disk height \( H (\ll r_a) \) or, equivalently,

\[
\beta = \arctan \frac{H}{r_a} \approx \frac{H}{r_a}.
\]

As is evident from figure 8, \( \alpha = \gamma - \beta \) where \( \tan \gamma = dH/dr_a \), therefore the grazing angle of a flared disk is

\[
\alpha = a \frac{d\beta}{da}.
\]

The CG surface layer serves as an effective optically thin disk atop the underlying optically thick disk core. Since the radiative heating of optically thin dust is independent of geometry, the temperature profile of the CG layer is similar to that of a halo. Furthermore, emission from this surface layer provides additional heating for the underlying optically thick core, just like the halo. Therefore the CG model can be considered a special case of the general family of models in which the disk is embedded in an optically thin halo, only in this case the “halo” is really the disk surface layer, fully determined from the flaring geometry. And since the CG layer exists in every disk, including flat, one may ask whether it alone suffices to explain all the observations and whether an additional putative halo is even necessary.

To address these questions we must find radiative signatures that can discriminate between emission from the CG layer and a spherical halo. For any geometry, the flux from optically thin dust emission is obtained by integration over the volume occupied by this dust (equation 7). Since the temperature profile of optically thin dust depends only on distance from the radiative source and is independent of geometry, the geometry dependence enters only from \( n_a \), which also determines the overall optical depth. In the case of spherical geometry, the radial optical depth is \( \tau_{\nu} = \sigma_\nu \int n_a dr \).

With equation 8 for the dimensionless density profile, the flux can be written as

\[
F_{\text{sph},\nu} = \frac{4\pi R_s^2}{D^2} q_\nu \int B_\nu(T) \eta y^2 dy,
\]

where \( T \) and \( \eta \) are given functions of radius \( y \). In the case of a face-on CG layer, the optical depth along the disk axis is \( \sigma_\nu \int n_a dz = q_\nu \alpha \) so that

\[
F_{\text{CG},\nu} = \frac{2\pi R_s^2}{D^2} q_\nu \int B_\nu(T) \alpha ada.
\]

Here \( T \) and \( \alpha \) are given functions of cylindrical radius \( a \), although strictly \( T \) is a function of \( y = a \sqrt{1 + \beta^2} \); this slight difference can be ignored because \( \beta \ll 1 \) everywhere in the disk. Since \( y \) and \( a \) enter only as integration variables, the last two expressions are mathematically identical if

\[
\eta \propto \frac{\alpha(y)}{y} \quad \text{and} \quad \tau_{\nu} = \int \frac{\alpha da}{y}.
\]

This shows that it is impossible to distinguish a CG surface layer from a spherical halo with flux measurements because in either case we can construct a model with the other geometry and the exact same flux. Specifically, in the case of a thin flat disk equations 19 and 24 show that it is equivalent to a spherical halo with \( \eta \propto 1/y^2 \) and \( \tau_{\nu} = \frac{1}{2} \alpha^* (\lesssim 10^{-2}) \) in Haebes). In the case of a flared disk, equations 21 and 24 show that its CG layer and the spherical halo with

\[
\eta \propto \frac{d\beta}{dy}, \quad \tau_{\nu} = \frac{1}{2} \left[ \beta(R_a) - \beta(R_s) \right]
\]

will produce precisely the same flux. And conversely—given a spherical halo, the flared disk with the same outer radius and

\[
\beta(\alpha) = \beta(1) + 2\tau_{\nu} \int^a_1 \eta(y) dy
\]

will produce the exact same flux from its CG-surface layer.

4.2 SED Equivalence of Flaring and Halos

There is a complete equivalence between the CG surface layers of flared disks and spherical halos as long as observations are restricted to flux measurements (in particular, the equivalent halo of a flared disk with grazing angle \( \alpha \propto 1/a^2 \) has density distribution \( \eta \propto 1/y^{p+1} \)). Although this equivalence was derived only for face-on viewing of the disk, it carries to most inclinations angles since the flux from optically thick dust involves a volume integration (equation 7) and the observed fraction of the disk surface layer remains largely intact as long as internal occultation is not too significant. In addition, the flux from the optically thick core of a CG flared disk and from a flat disk imbedded in the equivalent halo are identical because they both have the same temperature structure and that is the only relevant ingredient in optically thick emission. Furthermore, the halo equivalent to the CG-layer of a flat disk has only \( \tau_{\nu} \lesssim 10^{-2} \) in Haebes and we are justified in neglecting it in comparison to the imbedding halo. Therefore, the overall flux from a flared disk and from its equivalent halo-imbedded disk are identical at all wavelengths. In particular, the SEDs presented in the previous section cover fully the cases of flared disks with \( \beta \sim 1/a \) (\( \eta \propto 1/y^2 \)) and \( \beta \sim 1/a \) (\( \eta \propto 1/y \)).
It is impossible to distinguish between flared disks and halo-imbedded-disks with flux measurements. Only imaging can differentiate between these two geometries.

4.3 Flared Disk Images

The brightness contours of face-on flared disks are concentric circles centered on the star, same as in spherical halos. Inclined viewing changes the contours substantially. Consider the intensity of radiation scattered from the CG surface layer. It obeys

\[ I \propto \frac{\tau(\phi)}{r^2} \]

where \( r \) is the distance to the star and \( \tau(\phi) \) the optical depth toward the observer at the scattering point (equation 2). Both factors introduce distinct image asymmetries.

The fundamental reason for image distortion by inclination is that the same projected distance from the star corresponds to widely different locations on the surface of the disk. On that surface, contours of equal distance from the star are circles of radius \( r_a \). When viewed face-on from distance \( D \), each contour appears as a concentric circle of radius \( \theta_a = r_a/D \), as seen in the top image in figure 9. At inclination viewing angle \( i \) to the disk axis, the contour is no longer circular. Absent flaring, the contour becomes an ellipse centered on the star with major axis \( 2\theta_a \) and minor axis \( 2\theta_a \cos i \), aligned with the projection of the disk axis on the plane of the sky. Flaring raises the contour to height \( H = r_a \tan \beta \) above the equatorial plane (figure 8), and the star is shifted toward the observer along the minor axis by \( \theta_a \tan \beta \sin i \). A point on the contour at position angle \( \phi \) from the near side of the minor axis is observed at displacement \( \theta = \theta_a g(\phi) \) from the star, where

\[ g(\phi) = \left[ (\tan \beta \sin i - \cos i \cos \phi)^2 + \sin^2 \phi \right]^{1/2}. \]

These contours are shown in the bottom image of figure 9. The off-center position of the star on the minor axis creates an asymmetry such that the far and near portions of this axis obey \( \theta_{\text{far}}/\theta_{\text{near}} = \cos(i - \beta)/\cos(i + \beta) \). Because \( \beta \) increases with \( \theta_a \), this asymmetry increases with distance from the star.

At observed displacement \((\theta, \phi)\) from the star, a point on the surface of the disk is located at \( r \simeq r_a = D\theta/g(\phi) \). At that point the optical depth of the CG layer toward the observer is \( \tau(\phi) = q_\gamma \alpha/o(\phi) \), where

\[ o(\phi) = \hat{n} \cdot \hat{\alpha} = \cos \gamma - \sin i \sin \gamma \cos \phi. \]

Therefore the scattering image obeys

\[ I(\theta, \phi) \propto \left( \frac{g(\phi)}{\theta} \right)^2 \frac{\alpha}{o(\phi)}. \]

In this expression, both \( \alpha \) and \( \gamma \) are determined at the location \( r_a = D\theta/g(\phi) \) on the disk surface. A power-law grazing angle \( \alpha \propto 1/r_a^{p+2} \) produces the image \( I(\theta, \phi) \propto [g(\phi)/\theta]^{2+p}/o(\phi) \). This expression and the resulting brightness contours explain the results of Monte Carlo scattering calculations for flared disks (Whitney & Hartmann 1992, Wood et al 1998). Figure 10 shows the scattering images at three viewing angles of a flared disk with the parameters suggested for AB Aur by Dullemomd et al (2001).

The images produced at emission wavelengths are handled in complete analogy. The only change is the replacement of \( r^{-2} \) by the temperature \( T \), i.e., another function of \( r \), modifying the dependence of brightness on \( g(\phi)/\theta \).
4.3.1 Image Asymmetry

Brightness contours not subject to rim occultation are ellipses with eccentricity $e = \cos \theta_i$ that directly determines the inclination angle irrespective of the flaring profile. The images shown in figure 10 possess an additional deviation from circular symmetry, unique to flaring and conveniently measured by the ratio of brightness at diametric locations across an axis through the star

$$A(\theta, \phi) = \frac{I(\theta, \phi + \pi)}{I(\theta, \phi)} - 1.$$ (31)

This asymmetry parameter vanishes for flat disks at all inclination angles and for pole-on viewing irrespective of the flaring. However, flaring introduces substantial asymmetry even at modest inclination angles, as is evident from the right panels in figure 10; $A$ exceeds 30% on the minor axis already at $i = 25^\circ$ before blowing up altogether when the edge of the source is reached on the near side.

Non-vanishing $A$ is the hallmark of inclined flared disks. Its systematic variation with azimuthal angles cannot be replicated by any other means, easily distinguishing it from deviations from the perfect geometry of idealized models or noise in the data. Each flaring profile produces its own characteristic signature $A$. For example, it is easy to show that the constant grazing angle $\alpha$ used in figure 10 gives $\tan \beta \tan \alpha \simeq A/(2 + A)$ along the minor axis. Therefore, measuring $A$ determines the flaring profile once the inclination is determined from the eccentricity of the brightness contours.

5 DISCUSSION

Our results reveal numerous degeneracies that underscore the severe limitations of attempts to determine the dust morphology from SED analysis without imaging observations. The SED of a spherical shell with power-law density profile displays a dependence on the power index $p$ only when $p \gtrsim 2.6$, and then only in the spectral region $\lambda \lesssim \lambda_{\text{out}}$. All other regions of $p$ and $\lambda$ produce the same universal behavior $F_\nu \propto \nu^{p-1}$ (§2.4). The SED of a halo-imbedded disk remains the same when the viewing angle and the size of the disk central hole vary together as shown in figure 7. Above all, the expressions for the flux from a flared disk and its equivalent halo-imbedded disk are mathematically identical (§4.1). Any claim for preference of either geometry from a fit to the SED is meaningless when not backed up by imaging observations.

The dust optical properties introduce additional degeneracies. The results of §2.4 show that the frequency dependence of $\sigma_c$ and the radial dependence of the density profile can be interchanged on occasion without affecting the SED. The fundamental reason for all these degeneracies is that the flux from an optically thin source involves a volume integration (eq. 7) that tends to remove much of the dependence on the underlying morphology. Although the specific degeneracies we uncovered involve spherical geometry, the general analysis presented in §2 shows that the spherical idealization is not essential. The dust distribution can be flattened and even distorted into irregular shape before severely affecting the results. Only imaging can trace the actual geometry, and scattering provides a more faithful presentation because, unlike emission which involves also the dust temperature, it involves only the density distribution.

In spite of the attractiveness of the flared disk as a simple, physical model without additional components, imaging observations give irrefutable evidence for the existence of extended halos in many Haebes. Resolved images reveal sizes that greatly exceed any reasonable estimates for the dimensions of accretion disks. Combining space- and ground-based observations of HD 100546, the nearest Herbig Be star, Grady et al (2001) resolve both the disk, extending to $5''$ ($\sim 515$ AU), and a halo that extends out to $10''$ from the star. While the disk displays the elongation of inclined viewing with $i = 49^\circ$, the halo is roughly circular and background stars are visible through it. This halo cannot be possibly associated with the surface of the disk. The size decrease between 50 $\mu$m and 100 $\mu$m observed in MWC 137 by DiFrancesco et al (1994, 1998) cannot be explained by any single dust configuration, a conclusion reached in MIVE and further affirmed by the results of §2. Such behavior is possible only when the density distribution contains two distinct components, one optically thick, cool and compact, the other optically thin, warmer and more extended.

The origin of these halos has not been studied yet and could perhaps involve outflows such as disk winds (see, e.g., Königl & Pudritz 2000). Indeed, winds have been proposed on occasion to explain Haebes line observations (e.g., Bouret, Catala & Simon 1997). Accretion with the small rates of $\sim 10^{-8}$ $M_{\odot}$ yr$^{-1}$ has also been deduced from UV spectra of both Haebes (Grady et al. 1996) and TTS (Valenti, Basri & Johns 1993; Gullbring et al. 1998), and is consistent with halos with $\tau_{\nu} \sim 0.1$ (MIVE). These low rates cannot correspond to the main accretion buildup of the star but rather a much later phase, involving small, residual accretion from the environment. The corresponding accretion luminosities are only $\sim 0.1 L_{\odot}$, justifying their neglect in our calculations. It is also possible that in these halos we are getting a glimpse of what would eventually evolve into the equivalent of the solar system’s Oort cloud.

Whatever their origin, the halos seem prevalent in Haebes and would affect also the emission from flared disks. From the discussion of §4.1, the halo becomes the dominant component of the IR flux when it contains more dust than the halo-equivalent of the disk, i.e., whenever $\tau_{\nu_{\text{halo}}} > \frac{1}{2} H(R_d)/R_d$, where $\tau_{\nu_{\text{halo}}}$ is the optical depth across the halo at visual wavelengths ($2\tau_{\nu}$ for spherical halos). Even when not dominating the overall flux, the halo can still have a significant effect. The results of §3.1 show that even at much smaller $\tau_{\nu}$ the halo can dominate the disk heating and make a strong impact on its temperature profile. In TTS the halo inner cavity is much smaller than in Haebes, because of the lower $T_r$, and the importance of the halo contribution to disk heating is reduced relative to the star. Nevertheless, Kikuchi, Nakamoto & Ogoshi (2002) have recently concluded that halos are necessary supplements to flared disks also in flat-spectrum TTS. It appears that in most cases, flared disk in pre-main-sequence stars cannot be treated as if surrounded by vacuum.

ACKNOWLEDGMENTS

The partial support of NSF and NASA is gratefully acknowledged.

REFERENCES


© 2002 RAS, MNRAS 000, 1–11
Disks and Halos in PMS Stars

Königl, A. & Pudritz, R. E. 2000, Protostars and Planets IV, 759

© 2002 RAS, MNRAS 000, 1–11