Due: Mon Mar. 29, 2010.

First, read Section 7.1, exercises for Section 7.1, and solutions to selected exercises in textbook. Then answer the following questions.

**Question 1** (10%)
Write down the proposition denoted by the following expression, where the variables take values in the domain \{0, 1\}.
\[
\forall y \exists x \ p(x, y)
\]

**Question 2** (10%)
Write down a quantified expression over some domain to denote each of the following propositions or predicates
\[q(0) \lor q(1)\]
\[p(x, 0) \land p(x, 1)\]

**Question 3** (10%)
Explain why the following expression is a wff.
\[
\exists x \forall y \ (p(y) \rightarrow q(f(x), y))
\]

**Question 4** (10%)
For the following wff, label each occurrence of the variables as either free or bound:
\[
\forall y \ q(y) \land \neg p(x, y)
\]

**Question 5** (20%)
Let \(B(x)\) mean \(x\) is a bird, let \(W(x)\) mean \(x\) is a worm, and let \(E(x, y)\) mean \(x\) eats \(y\). Find an English sentence to describe the following expression:
\[
\forall x \forall y \ (E(x, y) \rightarrow B(x) \land W(y))
\]
\[
\forall y \ (W(y) \land \exists x \ (B(x) \land E(x, y)))
\]

**Question 6** (10%)
Let \(e(x, y)\) mean that \(x = y\), let \(p(x, y)\) mean that \(x < y\), and let \(d(x, y)\) mean that \(x\) divides \(y\). For the following statement about the natural numbers, find a formal quantified expression.
Any two nonzero natural numbers have a common divisor.

**Question 7** (10%)
Given the wff \(W = \exists x \ p(x) \rightarrow \forall x \ p(x)\), find all possible interpretations of \(W\) over the domain \{a, b\}. Also, give the truth value of \(W\) over each of the interpretations.

**Question 8** (10%)
Find a model for each of the following wffs:
\[
\exists x \ p(x) \rightarrow \forall x \ p(x)
\]
\[
\forall x \ (p(x, f(x)) \rightarrow p(x, y))
\]

**Question 9** (10%)
Find a countermodel for each of the following wffs:
\[
\exists x \ p(x) \rightarrow \forall x \ p(x)
\]
\[
\forall x \ (p(x, f(x)) \rightarrow p(x, y))
\]