Due: Thu Feb 17, 2011.

First, read Section 7.1, exercises for Section 7.1, and solutions to selected exercises in textbook. Then answer the following questions.

**Question 1** (10%)
Write down the proposition denoted by the following expression, where the variables take values in the domain \( \{0, 1\} \).
\[ \forall y \exists x \ p(x, y) \]

**Question 2** (10%)
Write down a quantified expression over some domain to denote each of the following propositions or predicates.
\[ q(0) \lor q(1) \]
\[ p(x, 0) \land p(x, 1) \]

**Question 3** (10%)
Explain why the following expression is a wff.
\[ \exists x \forall y \ (p(y) \rightarrow q(f(x), y)) \]

**Question 4** (10%)
For the following wff, label each occurrence of the variables as either free or bound:
\[ \forall y \ q(y) \land \neg p(x, y) \]

**Question 5** (20%)
Let \( B(x) \) mean \( x \) is a bird, let \( W(x) \) mean \( x \) is a worm, and let \( E(x, y) \) mean \( x \) eats \( y \). Find an English sentence to describe the following expression:
\[ \forall x \exists y \ (E(x, y) \rightarrow B(x) \land W(y)) \]
\[ \forall y \ (W(y) \land \exists x \ (B(x) \land E(x, y))) \]

**Question 6** (10%)
Let \( e(x, y) \) mean that \( x = y \), let \( p(x, y) \) mean that \( x < y \), and let \( d(x, y) \) mean that \( x \) divides \( y \). For the following statement about the natural numbers, find a formal quantified expression.

Any two nonzero natural numbers have a common divisor.

**Question 7** (10%)
Given the wff \( W = \exists x \ p(x) \rightarrow \forall x \ p(x) \), find all possible interpretations of \( W \) over the domain \( \{a, b\} \). Also, give the truth value of \( W \) over each of the interpretations.

**Question 8** (10%)
Find a model for each of the following wffs:
\[ \exists x \ p(x) \rightarrow \forall x \ p(x) \]
\[ \forall x \ (p(x, f(x)) \rightarrow p(x, y)) \]

**Question 9** (10%)
Find a countermodel for each of the following wffs:
\[ \exists x \ p(x) \rightarrow \forall x \ p(x) \]
\[ \forall x \ (p(x, f(x)) \rightarrow p(x, y)) \]