Due: Tue. March 1, 2011.

First, read Sections 7.2.1. and 7.2.2. Then answer the following questions.

Question 1 (10%)
Consider the equivalence rule 7.12a of the textbook. Note that \( x \) does not occur free in \( C \) in this equivalence. Prove 7.12a using other equivalences. That is start with the left-hand-side of 7.12a, and use other equivalences to transform the lhs to equivalent formulas until you obtain the right-hand-side of 7.12a (or vice-versa). Write down your work step by step very clearly, stating which equivalence you are using for each step.

Question 2 (10%)
Repeat question 1 for equivalence 7.12b.

Question 3 (10%)
Repeat question 1 for equivalence 7.12c.

Question 4 (10%)
Repeat question 1 for equivalence 7.12d.

Question 5 (20%)
Similar to Questions 1-4, prove Equivalence 7.7a and 7.7b using other equivalences (in particular, you can use equivalence 7.6 in your proof)

Question 6 (10%)
Use equivalences to construct a prenex normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

\[
\forall x \forall y (\exists z (p(x, z) \land p(z, y)) \to g(x, y))
\]

Question 7 (10%)
Use equivalences to construct a prenex disjunctive normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

\[
\forall x \exists y p(x, y) \to \exists y \forall x p(x, y)
\]

Question 8 (20%)
Use equivalences to construct a prenex conjunctive normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

\[
\forall x \forall y \forall z (p(x, y) \land p(y, z) \to p(x, z)) \land \forall x \neg p(x, x) \to \forall x \forall y (p(x, y) \to \neg p(y, x))
\]