Due: Tue. March 18, 2014.

**Question 1** (10%)
Write down a quantified expression over some domain to denote each of the following propositions or predicates. Specify the domain too.

$q(0) \lor q(1)$

$p(x, 0) \land p(x, 1)$

**Question 2** (10%)
Let $B(x)$ mean $x$ is a bird, let $W(x)$ mean $x$ is a worm, and let $E(x, y)$ mean $x$ eats $y$. Find an English sentence to describe the following expression:

$\forall x \forall y (E(x, y) \rightarrow B(x) \land W(y))$

$\forall y (W(y) \land \exists x (B(x) \land E(x, y)))$

**Question 3** (10%)
Let $e(x, y)$ mean that $x = y$, let $p(x, y)$ mean that $x < y$, and let $d(x, y)$ mean that $x$ divides $y$. For the following statement about the natural numbers, find a formal quantified expression.

Any two nonzero natural numbers have a common divisor.

**Question 4** (10%)
Given the wff $W = \exists x p(x) \rightarrow \forall x p(x)$, list ALL possible interpretations of $W$ over the domain \{a, b\}. Also, give the truth value of $W$ over each of the interpretations.

**Question 5** (10%)
Find a model for each of the following wffs:

$\exists x p(x) \rightarrow \forall x p(x)$

$\forall x (p(x, f(x)) \rightarrow p(x, y))$

**Question 6** (10%)
Find a countermodel for each of the following wffs:

$\exists x p(x) \rightarrow \forall x p(x)$

$\forall x (p(x, f(x)) \rightarrow p(x, y))$

**Question 7** (10%)
Use equivalences to construct a prenex normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

$\forall x \forall y (\exists z (p(x, z) \land p(z, y)) \rightarrow g(x, y))$

**Question 8** (10%)
Use equivalences to construct a prenex disjunctive normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

$\forall x \exists y p(x, y) \rightarrow \exists y \forall x p(x, y)$

**Question 9** (20%)
Use equivalences to construct a prenex conjunctive normal form for the following wff. Show your work. Write down the number of the equivalence used at each step.

$\forall x \forall y \forall z (p(x, y) \land p(y, z) \rightarrow p(x, z)) \land \forall x \neg p(x, x) \rightarrow \forall x \forall y (p(x, y) \rightarrow \neg p(y, x))$