
MR2099755 (2005i:11138a)[Liu, Yu-Ru \(3-WTRL-PM\)](#)**A generalization of the Turán theorem and its applications. (English summary)**[Canad. Math. Bull.](#) **47** (2004), no. 4, 573–588.[11N37 \(11N80\)](#)[Journal](#)[Article](#)[Doc
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MR2099756 (2005i:11138b)[Liu, Yu-Ru \(3-WTRL-PM\)](#)**A generalization of the Erdős-Kac theorem and its applications. (English summary)**[Canad. Math. Bull.](#) **47** (2004), no. 4, 589–606.[11N37 \(11N80\)](#)[Journal](#)[Article](#)[Doc
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[References: 27](#)**[Reference Citations: 2](#)****[Review Citations: 0](#)**

Two related papers published by the author in this issue of the journal deal with two fundamental results of probabilistic number theory, namely theorems of Turán (1934) and Erdős-Kac (1939).

In the first paper the author establishes a generalization of the classical Turán theorem (concerning the normal order of $\omega(n)$ —the number of prime factors of n) for free abelian monoids, elements and primes of which satisfy certain simple density conditions (denoted by (A) and (B) in the text). Under the assumption of these two conditions the author proves analogues of Mertens' theorems, and then she uses them in combination with a method found in the reviewer's paper (which is cited incorrectly here) to prove the desired sharp asymptotic formula. Applications of the general theorem are numerous—to number fields, function fields, geometrically irreducible varieties, etc. One only needs to verify that (A) and (B) are satisfied.

In the second paper, in the same setting and under the same density conditions (A) and (B), the author proves a similar extension of the theorem of Erdős and Kac to free abelian monoids. This is done, once again, with the help of the Mertens-type estimates, this time combined with an advanced probabilistic machinery, a review of which is provided, but the depth of which is

clearly underestimated. The results proved have several interesting applications, but the final philosophical conclusion the author arrives at—that in this general context both theorems of Turán and Erdős-Kac “require only conditions (A) and (B)” and are therefore “of the same difficulty”—is incorrect. It is true that both results can be proved using (A) and (B), however the latter requires a considerably more “deep” theory than the former, including the central limit theorem, the Fréchet-Shohat moment theorem and a great deal of measure theory. In fact, it is safe to say that in any possible setting the Erdős-Kac theorem will always be “deeper” than Turán’s theorem.

Reviewed by [Filip Saidak](#)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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