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**MR2131677 (2005k:11194)**
**Pappalardi, Francesco** (I-ROME3)
**A survey on $k$-freeness. (English summary)**
*Number theory*, 71–88, *Ramanujan Math. Soc. Lect. Notes Ser.*, 1, *Ramanujan Math. Soc.*, *Mysore*, 2005.
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In this paper the author gives a useful, nicely organized survey of results concerning $k$-th power free numbers, i.e. positive integers not divisible by a $k$-th power of a prime. The paper starts by stating a well-known theorem of Dirichlet:

$$S(x) \sim \frac{x}{\zeta(2)} + O(\sqrt{x}), \text{ as } x \to \infty,$$

where $S(x)$ is the number of squarefree natural numbers $\leq x$; and it finishes with some recent work of the author on the subject.

Both conditional (depending on the Riemann Hypothesis in the classical case and the $ABC$ conjecture in the polynomial case) and unconditional results concerning $k$-free numbers are discussed in this survey. One intriguing set of examples ([Axer (1911), Walfisz (1963), Montgomery & Vaughan (1981), and others]) gives the improvement of the error terms $R_k(x)$ in equations $S^k(x) = x\zeta(k)^{-1} + R_k(x)$, which generalize the above asymptotic formula of Dirichlet to $k$-free numbers. Other important results discussed in the paper include distribution of $k$-free numbers in:

(1) arithmetic progressions ([Prachar (1958), Hooley (1975), McCurley (1982), and others]);
(2) as values of polynomials in one variable ([Nagell (1922), Ricci (1933), Erdős (1953), Nair (1976), etc.]); and
(3) as values of various arithmetic functions ([Mirsky (1949), Pappalardi, Saidak and Shparlinski (2002), and Banks and Pappalardi (2003)]).

Five open problems are stated at the end of the paper. One should also note that the reference [35] on p. 83 should be changed to [32].

{For the entire collection see MR2131668 (2005j:11001)}

**Reviewed** by *Filip Saidak*