In 1959, W. Sierpiński used Dirichlet’s 1837 theorem on primes in arithmetic progressions to show that for any chosen base $g \geq 2$ there exist infinitely many primes that have the first digit $b$ and the last digit $b'$ when expressed in base $g$, where $b, b'$ are any fixed integers from $\{1, 2, 3, \ldots, g - 1\}$ and $(g, b) = 1$.

In this paper the author generalizes this theorem to “admissible” vectors $L$ of digits, of lengths 1 and 2. In fact, he estimates the number of such $k$-digit primes (in base $g$) as

$$C_L \left(1 - \frac{1}{g}\right) \frac{g^k}{\log g^k} + O\left(\frac{g^k}{k^2}\right) \text{ as } k \to \infty,$$

where $C_L$ is a constant depending only on the digit vector $L$ and the base $g$.

With the help of the Riemann Hypothesis for the $L$-functions with characters modulo $p^m$ ($m \in \mathbb{N}_0$), it is also shown that the length of the “admissible” digit vectors could be increased considerably—all the way to $(1 - \varepsilon)k^{1/2}$. The proofs of both of these theorems are nice and compact.

**Reviewed** by Filip Saidak

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