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On the number of prime divisors of the iterates of the Carmichael function. (English summary)

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Let $\omega(n)$ be the number of distinct prime factors of n, and let $\lambda_k(n)$ be the k-fold iterate of the well-known Carmichael λ function. With the help of the Brun-Titchmarsh theorem and two other prime related lemmas the author proves the following result:

Let $k \geq 1$ be a fixed integer; then the quantity $\mu_k(n)$, defined as

$$\mu_k(n) := \frac{\omega(\lambda_k(n)) - \frac{(\log\log n)^{k+1}}{(k+1)!}}{\frac{(\log\log n)^{k+1/2}}{k!\sqrt{2k+1}}},$$

is normally distributed, with the mean $\mu = 0$ and the variance $\sigma^2 = 1$. A similar theorem is then proved for the prime function $\mu_k(p+a)$.

Both these results are Carmichael λ function analogs of results proved by N. L. Bassily, the author and M. Wijsmuller [J. Number Theory **65** (1997), no. 2, 226–239; MR1462839 (2000c:11159)] for the k-fold iterations of Euler's φ function. In fact, as is the case in many related situations, the proof of the above theorem is obtained by showing that—"on average"—the difference between $\omega(\lambda_k(n))$ and $\omega(\varphi_k(n))$ is negligible.

Reviewed by *Filip Saidak*