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Báez-Duarte, Luis (YV-IVIC)

A general strong Nyman-Beurling criterion for the Riemann hypothesis. (English summary)


Let $C_0$ be the space of continuous functions on $[0, \infty)$ vanishing at infinity, $C_{00}$ be its subspace of compactly supported functions, and $C_1^0$ be the subspace of continuously differentiable functions. Also, let $L_p = L_p(0, \infty)$, $\chi = \chi_{(0,1]}$ be the characteristic function of $(0, 1]$, and $\rho_1(x) = \{\frac{1}{x}\}$. In a recent paper [Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. 14 (2003), no. 1, 5–11; MR2057270 (2005b:11135)] the author proved a strengthened version of the Nyman-Beurling criterion [see B. Nyman, On the One-Dimensional Translation Group and Semi-Group in Certain Function Spaces, Thesis, University of Uppsala, 1950; MR0036444 (12,108g); A. Beurling, Proc. Nat. Acad. Sci. U.S.A. 41 (1955), 312–314; MR0070655 (17,15a)] for the Riemann Hypothesis. It had the form

$$\text{RH} \iff \chi \in B(\rho_1)^{L_2},$$

where $B(f)$ is the linear hull of the set $\{K_n f : n \in \mathbb{N}\}$ of positive integer dilations $K_n$ of $f$.

In the paper under review the author continues his research in this area. Let the class $G$ of “good kernels” be the family of functions $g \in C_1^0 \cap L_1$, for which $\int_0^\infty t |g'(t)| dt < \infty$. For functions $f \in G \cap C_{00}$ which also satisfy the condition $Z(\hat{f}) := \{s : \hat{f}(s) = 0, \ 1/2 < \text{Re}(s) < 1\} = \emptyset$, he proves

$$\text{RH} \iff f \in B(P f)^{L_2},$$

where $P$ is the so-called Müntz operator [see Ch. H. Müntz, Mat. Tidsskr. B 1922, 39–47; JFM 49.0124.01]. The main result is obtained by establishing a more general, “smooth” version of Müntz’s classical formula.

Reviewed by Filip Saidak

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