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De Koninck, Jean-Marie (3-LVL); **Luca, Florian**
Integers divisible by the sum of their prime factors. (English summary)
Mathematika **52** (2005), *no. 1-2,* 69–77 (2006).

Let $\beta(n)$ denote the sum of the distinct prime factors of n, and let

$$\mathfrak{B}(x) = \#\{n \le x\colon\ n \text{ is composite and } \beta(n) \mid n\}.$$

In this paper the authors prove that for all sufficiently large x we have

$$xe^{\left(-\frac{3}{\sqrt{2}}+o(1)\right)\sqrt{\log x \log\log x}} < \mathfrak{B}(x) < xe^{\left(-\frac{1}{\sqrt{2}}+o(1)\right)\sqrt{\log x \log\log x}}.$$

At the end they also conjecture that, as $x \to \infty$,

$$\mathfrak{B}(x) = xe^{\left(-\sqrt{2}+o(1)\right)\sqrt{\log x \log\log x}}.$$

Related questions were previously investigated by C. A. Spiro-Silverman [J. Number Theory **21** (1985), no. 1, 81–100; MR0804916 (87c:11086)], P. Erdős and C. Pomerance [Indian J. Math. **32** (1990), no. 3, 279–287; MR1088609 (92a:11091)] and most recently by R. C. Vaughan and K. L. Weis [Mathematika **48** (2001), no. 1-2, 169–189 (2003); MR1996369 (2004f:11099)].

Reviewed by *Filip Saidak*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.