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**MR1954708 (2004b:11115)**

**Manstavičius, E.** (LI-VILN-DCS)

**Functional limit theorems in probabilistic number theory. (English summary)**

*Paul Erdős and his mathematics, I* (*Budapest*, 1999), 465–491, *Bolyai Soc. Math. Stud.*, 11, *János Bolyai Math. Soc.*, *Budapest*, 2002.

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This excellent, long overdue survey paper, concerning the theory of general functional limit theorems for partial sum processes, fills the gap left by all the existing textbooks and expository papers on the subject.

From the invariance principle of P. Erdős and M. Kac [Bull. Amer. Math. Soc. **52** (1946), 292–302; MR0015705 (7,459b); Bull. Amer. Math. Soc. **53** (1947), 1011–1020; MR0023011 (9,292g)], through important results of M. D. Donsker [Mem. Amer. Math. Soc., **1951** (1951), no. 6, 12 pp.; MR0040613 (12,723a)], Yu. V. Prokhorov [Teor. Veroyatnost. i Primenen. **1** (1956), 177–238; MR0084896 (18,943b)] and A. V. Skorokhod [Teor. Veroyatnost. i Primenen. **1** (1956), 289–319; MR0084897 (18,943c)] on properties of limit laws for the partial sum processes, to theorems of I. P. Kubilyus [Dokl. Akad. Nauk SSSR (N.S.) **103** (1955), 361–363; MR0072169 (17,239d)] and G. J. Babu [Sankhyā Ser. A **34** (1972), 323–334; MR0349616 (50 #2109)] concerning additive arithmetical functions, this paper gives a detailed and historically accurate description of the most basic facts and ideas of the theory.

Other noteworthy results discussed here include the one-dimensional limit theorem for sums of Legendre symbols with shifted arguments, due to H. Davenport and P. Erdős [Publ. Math. Debrecen **2** (1952), 252–265; MR0055368 (14,1063h)], Erdős' arcsine law and the author's recent extension of it [J. Théor. Nombres Bordeaux **8** (1996), no. 1, 159–171; MR1399952 (97k:11117)], and a well-known conjecture of Kubilyus and Ju. V. Linnik [Izv. Vysš. Učebn. Zaved. Matematika **1959**, no. 6 (13), 88–95; MR0136587 (25 #57)] concerning the Möbius function. Interesting work of P. Billingsley [Amer. Math. Monthly **80** (1973), 1099–1115; MR0345144 (49 #9883)], W. Philipp [in *Analytic number theory (Proc. Sympos. Pure Math., Vol. XXIV, St. Louis Univ., St. Louis, Mo., 1972)*, 233–246, Amer. Math. Soc., Providence, R.I., 1973; MR0354602 (50 #7080)], Babu

[Sankhyā Ser. A **35** (1973), no. 3, 307–310; MR0568272 (58 #27877)] and the author [Litovsk. Mat. Sb. **24** (1984), no. 3, 148–161; MR0779754 (86e:11065)] on the fundamental lemma of Kubilyus is also mentioned.

The paper lists 89 references, many of them from not very available Soviet journals. This adds to the fact that it is a must read for anyone who really wishes to understand the role of Brownian motion in the theory of prime numbers and arithmetical functions.

{For the entire collection see MR1954675 (2003h:00022)}

**Reviewed** by *Filip Saidak*