

Select alternative format: [BibTeX](#) | [ASCII](#)**MR1956284 (2003k:11131)**[Vu, V.H. \(1-UCSD\)](#)**Chernoff type bounds for sums of dependent random variables and applications in additive number theory.***Number theory for the millennium, III (Urbana, IL, 2000)*, 341–356, A K Peters, Natick, MA, 2002.[11K99 \(11P99 60E15\)](#)

Journal

Article

Doc  
Delivery**References: 0****Reference Citations: 0****Review Citations: 0**

Let  $t_1, \dots, t_n$  be independent binary (taking values 0 and 1 only) random variables. Define  $X = \sum_{i=1}^n t_i$  and let  $\mu$  denote the mean of  $X$ . Then the well-known Chernoff's inequality [H. Chernoff, *Ann. Math. Statistics* **23** (1952), 493–507; [MR0057518 \(15,241c\)](#)] states that there exists a constant  $c(\varepsilon)$  such that

$$\Pr(|X - \mu| > \varepsilon\mu) \leq 2e^{-c(\varepsilon)\mu}.$$

This result has many applications—the bound it proves is very desirable in number theory. However, in some problems concerning integer sequences, random variables used for modelling them are not completely independent of each other. Therefore, a different—dependent random variable—approach is needed. The paper under review discusses one method for getting such results.

A random variable depending on independent binary (atom) variables  $t_1, \dots, t_n$  can be viewed as a function from a product measure equipped  $n$ -dimensional unit hypercube to the real numbers. The following general principle is discussed: If all of the partial derivatives of a positive regular polynomial  $Y$  in  $t_1, \dots, t_n$  are relatively small, then  $Y$  is strongly concentrated around its mean. This, in turn, means that the tails of the distribution can be estimated effectively. In fact, as the author notes, one can obtain useful analogues of Chernoff's bound and inequalities of Azuma [see N. Alon and J. H. Spencer, *The probabilistic method*, Second edition, Wiley-Intersci., New York, 2000; [MR1885388 \(2003f:60003\)](#)] and of S. Janson [Random Structures Algorithms **1** (1990), no. 2, 221–229; [MR1138428 \(93a:60041\)](#)]. Applications of these results in combinatorics (the Erdős-Tetali theorem [P. Erdős and P. Tetali, Random Structures Algorithms **1** (1990), no. 3,

245–261; [MR1099791 \(92c:11012\)](#)]) and additive number theory (the Waring problem with thin subbases) are discussed.

No proofs are given, but some of them can be found in the author's earlier work [V. H. Vu, *Random Structures Algorithms* **16** (2000), no. 4, 344–363; [MR1761580 \(2001g:60060\)](#); J. H. Kim and V. H. Vu, *Combinatorica* **20** (2000), no. 3, 417–434; [MR1774845 \(2002b:05123\)](#); V. H. Vu, *Duke Math. J.* **105** (2000), no. 1, 107–134; [MR1788048 \(2002e:11134\)](#)].

{For the entire collection see [MR1956264 \(2003h:11003\)](#)}

**Reviewed** by *Filip Saidak*

© Copyright American Mathematical Society 2003, 2006