
MR1958980 (2003k:11144)
Puchta, Jan-Christoph (4-OX)
Primes in short arithmetic progressions.
Acta Arith. **106** (2003), *no. 2,* 143–149.
11N13 (11N35)

References: 4 | **Reference Citations: 0** | **Review Citations: 0**

With the help of Selberg's sieve, Bombieri's version of the large sieve and some of Burgess' character sum estimates, the author proves the following theorem: Let $N, Q \in \mathbb{N}$, and $a_p \in \mathbb{C}$ for all primes $p \leq N$. Then

$$\sum_{q\leq Q}(q-1)\sum_{(a,q)=1}\Big|\sum_{\substack{p\leq N\\ p\equiv a \bmod q}} a_p - \frac{q}{\varphi(q)}\sum_{p\leq N} a_p\Big|^2 \ll_\varepsilon \left(\frac{N}{\log N}+Q^{2+\varepsilon}\right)\sum_{p\leq N}|a_p|^2.$$

Even under the assumption of the Generalized Riemann Hypothesis one would get only $Q^{4+\varepsilon}$ instead of $Q^{2+\varepsilon}$. As a corollary the author then deduces: Let $\pi(x,\chi)=\sum_{p\leq x}\chi(p)$. Then for any $x > Q^{2+\varepsilon}$ there exists a constant $C_\varepsilon > 0$, such that

$$\sum_{q\leq Q}\sum_{\chi \bmod p}{}^{*}|\pi(x,\chi)|^2 \leq C_\varepsilon\frac{x^2}{(\log x)^2}.$$

Weaker versions of these two theorems (i.e. with larger Q) were obtained in 1983 by P. D. T. A. Elliott [in *Studies in pure mathematics*, 157–164, Birkhäuser, Basel, 1983; MR0820219 (87c:11084)] and Y. Motohashi [in *Studies in pure mathematics*, 507–515, Birkhäuser, Basel, 1983; MR0820246 (87a:11088)], respectively.

Reviewed by *Filip Saidak*

[References]

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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2. H. Halberstam and H.-E. Richert, *Sieve Methods*, London Math. Soc. Monographs 4, Academic Press, 1974. MR0424730 (54 #12689)
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4. Y. Motohashi, *Large sieve extensions of the Brun-Titchmarsh theorem*, in: Studies in Pure Mathematics to the Memory of P. Turán, P. Erdős (ed.), Akadémiai Kiadó, Budapest, 1983, 507–515. MR0820246 (87a:11088)