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An elementary proof of a theorem of Delange. (English, French summaries)

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Let $\omega(n)$ be the number of distinct primes dividing the positive integer n and for real $x \geq 2$ let $\mathcal{G}(x) = \sum_{n \leq x} (\omega(n) - \log \log x)^2$. P. Turán (1934) proved that $\mathcal{G}(x) \ll x \log \log x$ and stated that $\mathcal{G}(x) = x \log \log x + o(x \log \log x)$. H. Delange (1971) refined the Selberg analytic method to obtain the asymptotic development

$$\sum_{n \leq x} \omega(n) = x \log \log x + Bx + \sum_{m=1}^k B_m \frac{x}{(\log x)^m} + O\left(\frac{x}{(\log x)^{k+1}}\right).$$

In this paper, by using the Mertens formula

$$\sum_{p \leq x} \frac{1}{p} = \log \log x + B + O\left(\frac{1}{\log x}\right),$$

and the Murty bound

$$\sum_{p \leq x/2} \frac{1}{p \log(x/p)} \ll \frac{\log \log x}{\log x},$$

the author proves that

$$\sum_{pq \leq x} \frac{1}{pq} = (\log \log x)^2 + 2B \log \log x + C + O\left(\frac{\log \log x}{\log x}\right),$$

from which, and by Abel partial summation exclusively, he derives the values of the first and second moment of $\omega(n)$ to obtain

$$\mathcal{G}(x) = x \log \log x + \theta x + O\left(\frac{x \log \log x}{\log x}\right),$$

where θ is an effective constant given in the paper.

Reviewed by *Fethi Ben Saïd*