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Berenhaut, Kenneth S. (1-WKFR); **O'Keefe, Augustine B.** (1-WKFR);
Saidak, Filip (1-WKFR)

Recursive sequences of the form $y_n = a_n y_{n-1} + y_{n-2}$ with integer coefficients. (English summary)

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Consider linear recurrences of the form

$$y_n = a_n y_{n-1} + y_{n-2}\ (n \geq 1),$$

with $a_i \in \mathbb{N}\ (i \geq 1)$, $y_1 = 0$ and $y_0 = 1$. Note that the first term in y_i is simply the product of $a_1, a_2 \ldots, a_i$, which renders it convenient to write $y_i(a_1, a_2, \ldots, a_i)$. Using an elementary, algebraic approach, the authors prove the following three main results about the values of the recurrence elements under fixed sum constraints.

I. If $\sum_{i=1}^{k} a_i = N$, then

$$y_k(a_1, a_2, \ldots, a_k) \leq y_k(w, x, x, \ldots, w, w, \ldots, w),$$

where $x = [N/k]$ and $w = x + 1$; the number of w's is $N - kx$, while the number of x's is $k(x + 1) - N$.

II. If $\sum_{i=1}^{k+1} a_i = N + M$ and $\max a_i = M$, then

$$y_{k+1}(a_1, a_2, \ldots, a_{k+1}) \leq y_{N+1}(M, 1, 1, \ldots, 1).$$

III. If $a_j \geq 1$ ($j = 1, 2, \ldots, k$) and $\sum_{j=1}^{k} a_j = N$, then there exist $b_1, b_2, \ldots, b_k$ with $\sum_{j=1}^{k} b_j = N$ and $b_j \in \{[N/k], [N/k] + 1\}$ ($j \in \{1, 2, \ldots, k\}$) such that

$$y_k(a_1, a_2, \ldots, a_k) \leq y_k(b_1, b_2, \ldots, b_k).$$

Reviewed by *Vichian Laohakosol*