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Square-free values of the Carmichael function. (English summary)

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Let $\lambda(n)$ be the Carmichael function, defined as the exponent of the group $(\mathbb{Z}/n\mathbb{Z})^\times$. Then it is shown that

$$\#\{n \leq x: \lambda(n) \text{ square-free}\} \sim \kappa \frac{x}{(\log x)^{1-\alpha}},$$

where κ is a positive absolute constant, and

$$\alpha = \prod_p \left(1 - \frac{1}{p(p-1)}\right)$$

is Artin's constant. One can get similar results for k -free values of $\lambda(n)$. The proof uses a simple characterisation of those n for which $\lambda(n)$ is square-free, along with the observation that

$$\#\{p \leq x: p-1 \text{ square-free}\} \sim \alpha \frac{x}{\log x}.$$

Reviewed by *D. R. Heath-Brown*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.