Let $\omega(n)$ denote the number of distinct prime factors of $n$ and put $L(n) = \log \log n$. A fundamental theorem of probabilistic number theory due to Erdős and Kac states that the quantity $Q(n) = (\omega(n) - L(n))/\sqrt{L(n)}$ is normally distributed. Here the normal order $L(n)$ of $\omega(n)$ serves as the mean, and $\sqrt{L(n)}$ as the standard deviation. Likewise, the quantity $Q(p-1)$ is normally distributed (with $p$ prime), as was established by H. Halberstam [J. London Math. Soc. 30, 43–53 (1955; Zbl 0064.04202); 31, 1–14, 14–27 (1956; Zbl 0071.04203)]. Let $a$ be a squarefree integer and $f_a(p)$ the order of $a$ modulo $p$. M. R. Murty and the author [Cand. J. Math. 56, No. 2, 356–372 (2004; Zbl 1061.11052)] established, under the assumption of a quasi Generalized Riemann Hypothesis (qGRH) for Dedekind zeta functions of Kummer fields of the type $\mathbb{Q}(\zeta_q, a^{1/q})$, that $(\omega(f_a(p)) - L(p))/\sqrt{L(p)}$ is normally distributed. They used that for almost all $p$, under qGRH, $\omega(f_a(p))$ is rather close to $\omega(p-1)$ and made use of Halberstam’s result. In the present well-written paper the result of Ram Murty and the author is rederived by computing all the higher moments corresponding to $\omega(f_a(p))$, and by comparing them, via the Fréchet-Shohat theorem, with estimates due to Halberstam concerning the moments of $\omega(p-1)$.

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