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**Zbl 1085.11048**

Saidak, Filip

**An elementary proof of a theorem of Delange.** (English)

C. R. Math. Acad. Sci., Soc. R. Can. 24, No. 4, 144-151 (2002).
http://comptes.math.carleton.ca/

Let $\omega(n)$ denote the number of different primes dividing $n$. In 1916, Hardy and Ramanujan established that the normal order of $\omega(n)$ is $\log\log n$. P. Turán derived an innovative proof of this result in 1934 by first showing that

$$S(x) := \sum_{n\le x}(\omega(n) - \log\log x)^2 \ll x\log\log x.$$

The aim of the paper under review is to obtain by elementary methods the stronger result

$$S(x) = x\log\log x + \Theta x + O\left(\frac{x\log\log x}{\log x}\right),$$

where the constant $\Theta$ is given explicitly. The author points out in the acknowledgements that *P. Diaconis*, *F. Mosteller* and *H. Onishi* established the same result in [J. Number Theory 9, 187–202 (1977; Zbl 0355.10043)]. A key ingredient in both proofs is an asymptotic formula with an error term $o(1)$ for the sum $\sum_{pq\le x}\frac{1}{pq}$, where $p\,q$ are primes.

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*Keywords* : normal number of prime factors; theorems of Turán and Delange
*Classification* :
∗11N37 Asymptotic results on arithmetic functions
11K36 Well-distributed sequences and other variations
11K65 Arithmetic functions (probabilistic number theory)