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Square-free values of the Carmichael function. (English)

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For a positive integer n the Carmichael function $\lambda(n)$ is the exponent of the multiplicative group modulo n. In this paper, the authors find an asymptotic formula for the number $L(x)$, which counts, for a given positive real number x, the number of positive integers $n \leq x$ having the property that $\lambda(n)$ is square-free. Their result is that

$$L(x) \approx \kappa \frac{x}{\log^{1-\alpha} x}, \qquad (1)$$

where $\alpha = \prod_{p\geq 2}(1 - 1/(p(p-1)))$ is the Artin constant and κ is a more complicated constant (explicitly determined). For the proof, the authors first use the Bombieri-Vinogradov theorem to show that if $\pi_{sf}(x)$ counts the number of primes $p \leq x$ with $p-1$ square-free, then $\pi_{sf}(x) = \alpha\pi(x) + O(x/\log^A x)$, where $A > 0$ is an arbitrary constant. Once equipped with this fact, the main result follows from the observation that the characteristic function of the set of those n such that $\lambda(n)$ is square-free is multiplicative and supported on π_{sf}, together with a well-known result of Wirsing relating the average value of a "small multiplicative function" $f(n)$ to $\sum_{p\leq x} f(p)$. The paper contains also some other interesting facts, such as a uniform upper bound for the function L in short intervals, a formula for the counting function of those positive integers n whose Carmichael function is k-free (for a fixed integer $k \geq 2$), as well as the results of some computations analyzing the speed of convergence to 1 of the ratio between $L(x)$ and the right hand side of (1).

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Classification :
∗11N25 Distribution of integers with specified multiplicative constraints
11N05 Distribution of primes
11N37 Asymptotic results on arithmetic functions