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On the modulus of the Riemann zeta function in the critical strip. (English)

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http://www.mat.savba.sk/maslo/maslo.mat.savba.sk/

Summary: For the Riemann zeta-function $\zeta(s)$, defined for complex $s = \sigma + \mathrm{i}t$, we write $\sigma = \frac{1}{2} + \Delta$, and we study the horizontal behaviour of $|\zeta(s)|$ in the critical strip $|\Delta| \leq \frac{1}{2}$. We prove

$$\left|\zeta\left(\tfrac{1}{2} - \Delta + \mathrm{i}\,t\right)\right| \geq \left|\zeta\left(\tfrac{1}{2} + \Delta + \mathrm{i}\,t\right)\right|$$

for $0 \leq \Delta \leq \frac{1}{2}$, $2\pi + 1 \leq t$; and we give accurate but simple asymptotic estimates for the quotient $\alpha(\Delta, t)$ of these two quantities. Inequalities and numerical tables are presented which show just how accurate these estimates are. Several conjectures related to the Riemann Hypothesis are discussed as well.

Keywords : Riemann zeta function; modulus of $\zeta(s)$; functional equation; Stirling's series; horizontal behaviour of $\zeta(s)$

Classification :

∗11M06 Riemannian zeta-function and Dirichlet L-function

11M26 Nonreal zeros of zeta(s) and L(s,chi)