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Non-abelian generalizations of the Erdős-Kac theorem. (English)

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The well known Erdős-Kac theorem states that $(\omega(n)-\log\log n)/(\log\log n)^{\frac{1}{2}}$ is normally distributed, where $\omega(n)=\sum_{p|n}1$. The authors consider the analogue of this result when n is replaced by $f_a(n)$ defined for $(a,n)=1$ to be the order of $a \pmod n$, so $f_a(n)|\varphi(n)$. They prove a conjecture of *P. Erdős* and *C. Pomerance* [Rocky Mt. J. Math. 15, 343-352 (1985; Zbl 0617.10037)] under a hypothesis weaker than the generalized Riemann hypothesis. Write $\Phi(\alpha,\beta)=\frac{1}{\sqrt{2\pi}}\int_\alpha^\beta e^{-t^2/t}dt$. For q prime and ω_q a primitive qth root of unity, let $L_q=\mathbb{Q}(\omega_q,\sqrt[q]{a})$, a non-abelian extension of $\mathbb{Q}$, and $\zeta_q(s)$ be the corresponding Dedekind zeta function. Under the hypothesis that, for some $\theta<1$, $\zeta_q(s)$ has no zero in the region $\mathrm{Re}(s)>\theta$ for every prime q, the authors establish that for squarefree $a\geq 2$

$$\left|\left\{n\leq x:(a,n)=1,\alpha\leq\frac{\omega(f_a(n))-\frac{1}{2}(\log\log n)^2}{\frac{1}{\sqrt{3}}(\log\log n)^{3/2}}\leq\beta\right\}\right|\sim\Phi(\alpha,\beta)\frac{\varphi(a)}{a}x$$

as $x\to\infty$. This is proved by first obtaining the corresponding result for $\Omega(f_a(n))$, where $\Omega(n)=\sum_{p^\alpha\|n}\alpha$, and then exploiting the relationship between the two functions. The authors also establish a number of related theorems including the corresponding result for a prime variable: with the same hypothesis,

$$\left|\left\{p\leq x:(a,p)=1,\alpha\leq\frac{\omega(f_a(p))-\log\log p}{(\log\log p)^{1/2}}\leq\beta\right\}\right|\sim\Phi(\alpha,\beta)\pi(x)$$

as $x\to\infty$. This is deduced from the corresponding result for $\omega(p-1)$ after showing that $\omega(p-1)-\omega(f_a(p))$ is smaller than $\omega(p-1)$ for almost all primes p.
Two of the other theorems do not require any hypothesis in their proof, but one of them highlights where the quasi-GRH is invoked, namely in finding asymptotic formulae for the sums $\sum_{p\leq x}\Omega(f_a(p)p^{-1}$ and $\sum_{p\leq x}\Omega^2(f_a(p))p^{-1}$.

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