ATTACTOR POTENTIAL FROM ELECTROENCEPHALOGRAM

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ABSTRACT

The digitized electroencephalogram output from 16 channels for (1) eyes closed (normal) (2) eyes open (normal) and (3) a subject under meditation are subjected to a singular value decomposition to obtain the eigen values and eigen function of the attractor distribution. We then evaluate effective interaction potential as a function of these eigen values and eigen functions at different skull points, using an inverse scattering technique. This gives a global potential distribution. We make a detailed comparison of these distinctly different dynamical cases. We further define a Coherence index at these 16 locations from the amplitude and phase synchronization indices. We have also defined a new parameter Learning index and these could be two additional invariant parameters besides the usually known ones. The merits and demerits are discussed in the concluding section.
1. **Introduction**

Brain as a dynamic system is distinctly different in nature from known physical systems, since they are highly complex and deal with billions of input-output signals. In developing the brain dynamics, one has to take into cognizance the complex, nonlinear, non Marcovian character (Sreenivasan et.al., 1999) together with the fact that the elemental constituent cell viz. neurons are constantly being generated and destroyed without striking an equilibrium brain state even in adult primates (Gould et.al., 1999). Further the new neurons generated at the basal ganglia migrate both tangentially and dorsally from a lateral origin (Kaas and Reiner, 1999). If we view this system as a collection of attractors then the complexities in the actual system would manifest itself into a multi species collection of attractors consisting of point attractors, limit cycles, regular attractors consisting of Torii as well as strange attractors. Various attempts have been made to understand the dynamics both classically [McCullough and Pitts, 1943; Anderson and Cooper, 1978, Parikh and Pratap, 1984 and Cowan, 1991] and quantum mechanically (Donald, 1990). The main analytical techniques used for those of neural networks, nonequilibrium statistical mechanics as also that of master equation. Pratap (2000) has given a recent review. In particular Donald (1990) approached the problem as a collection of quantum mechanical switches placed in a wet medium and operated by electrical signals travelling along axons. All these attempts were aimed at exploring the working of the threshold effect in neural dynamics. One should however realize the fact that while this threshold effect is indeed important, there are other equally significant features in brain dynamics. On a single neuronal level, the signals do not experience any attenuation as they travel along the system and that while their amplitude is independent of the stimulus strength the firing frequency is directly proportional to
the strength. These regular features are however lost in the signals observed in the actual cortex domain as observed in an EEG. The collective behaviour in the signals that is observed in the cortex domain is sensitive to both amplitude and frequency for different kind of stimuli that are generated in different pathological cases. Furthermore any model should account for the various higher functions of the brain such as recognition, memory, recollection, cognizance, association and generalization- to mention a few.

This paper is organized in the following manner. In the next section we introduce two invariant parameters called Coherence index, $C_i$ and Learning index, $L_i$. We have evaluated the Coherence index by two different methods for a coupled chaotic Rossler oscillator system. We have shown the inadequacy of the method adopted by Rosenblum et.al. (1996). In section 2.2, we have defined the Learning index and we are yet to develop an algorithm to estimate this. We are at present working on this problem. In section 3, we have determined the eigen values and eigen functions for an attractor gas using the singular value decomposition method and evaluated the effective interaction potential existing in the attractor gas at various points(Indic & Pratap). The last section enumerates the conclusions and future directions.

2. Invariant Parameters

The brain state is characterized by the invariant parameters. The known parameters are (1) embedding space dimension $\varepsilon$, (2) attractor basin dimension $B_i$ (3) attractor dimensions $D_{ij}$ (4) generalized entropy $I_{ij}$ and (5) Lyapunov exponents/functions $\lambda_{ij}/\alpha_i$. The definition of all these parameters are given in text books (Schuster 1995). This list is by no means complete. Furthermore while in the above list (1)-(3) are static geometric parameters; only (4) & (5) characterize the dynamic state of the system. We add here two new dynamic parameters as (6) coherence index $C_i$ (7) learning index $L_i$. 
Both these adumbrate the dynamic state of the brain. We shall define these parameters in the following.

2.1 Coherence index $C_i$

This is defined in terms of phase and amplitude synchronization. If two chaotic signals were coupled in a nonlinear manner, then the differences in phases of the individual constituents would be an arbitrary function of time. This can be written as

$$|\phi_1(t) - \phi_2(t)| = \Omega(t)$$  

If we expand the function $\Omega(t)$ in a Taylor series, then the constant (the first term) indicates synchronization. If the individual units are chaotic, then the system is considered synchronized if the average of $\Omega(t)$ is equal to a constant. This implies that the nonlinear coupling does not result on an average in creating instabilities. Physically this means that the oscillations dub their energy into a synergic single mode and create a resonance phenomenon. We shall denote this constant by $S_p$. In this evaluation one normally ignore the effect of amplitude (Rosenblum et al, 1996). If however we determine a similar constant for the random amplitude and define a constant as $S_a$ Then we can define a coherence index by taking the harmonic mean of the two viz.

$$C_i = \frac{2S_p S_a}{S_p + S_a}$$  

where $i$ denotes the position on the skull space.

Since $S_p$ and $S_a$ have values between 0 and 1, $C_i$ is in the interval

$$0 < C_i \leq 1$$  

A perfectly coherent system is such, if both amplitude and phases are perfectly synchronized. (i.e. $S_p = S_a = 1$) then $C_i = 1$

Rosenblum et.al. (1996) have shown that in the case of two chaotic coupled Rossler systems, the phases get locked for a range of coupling parameters. But in real
systems, such as the attractors in the human brain, where there are more than two interacting chaotic attractors such a calculation is of little use, since even the governing equations are not known. We therefore have to resort to Poincare maps to get the phases and amplitudes for the search of synchronization/coherence phenomena. It may however be mentioned that what we are measuring at a given point in the skull space is the resultant phases without stipulating the number of attractors or their coupling. It should further be realized that this is probably the first time when one is using Poincare maps for a quantitative estimation of the dynamic state of human brain.

Rosenblum et al (1996) in their calculations have assumed (a) that the signal is an analytic function of time, and (b) the imaginary part can be written as a Hilbert transform of a real part. While the assumption (b) implies that the imaginary part can be derived from the real part using a transform, the analytic function in reality depends only on a single variable viz. s (t). This needs a real justification. Secondly in general the assumption (a) cannot be justified in a real world process, more so when the process is highly nonlinear and chaotic. We tried to examine these points by evaluating the phase and amplitude synchronization indices as well as the Coherence index for the Rossler coupled nonlinear oscillators by using Rosenblum's assumption as well as using Poincare map. The results are presented in Figure 1 (a&b).

Figure 1 (a) gives the phase and amplitude indices as well as coherence index using Rosenblum's approach. The evaluation has been done for windows of 1024 data length. It may be observed that while the synchronization index does exhibit fluctuation in the range 0.1 to 0.5 the amplitude fluctuation is in the range 0.4 to 0.6.

In Figure 1(b) we evaluated the phase and amplitude using Poincare map. The independence between the phase and amplitude synchronization indices is sharply seen and that the phase synchronization is in the interval 0.7 to 0.9 while the same
quantity for the amplitude is in the interval 0.3 to 0.5. The coherence index is always less
than 0.5. This agrees with the inference of the authors that the phase is highly
synchronized while the amplitude is random. This also brings to light the fact the degree
of coherence is small when the two variables are widely separated. This incidentally
justifies the fact that one should look for coherence and not for the phase synchronization
and further that one should get the phase information from Poincare maps and not by
assuming Hilbert transform. It may be observed that this is probably the first time one is
using a Poincare map for a quantitative estimation of a dynamic parameter.

Search for synergic effects in chaotic systems have been done in earlier
times by Lanczos and Gellai (1975), Pratap (1977). In both cases, a random sequence of
numbers were subjected to a Fourier representation and they both observed that the
vectors showed bunching at more than one sectors in the unit circle.

2.2 Learning Index

It is well known by now, that when the brain through its sensory channels
receives a signal, a certain neuronal firing pattern is established in the neocortex domain.
If similar signals are given repeatedly, then the resulting pattern attains a certain degree of
permanence. The pattern generated can be expressed by a distribution of attractors
characterized by the invariant parameters $\varepsilon_i$, $B_i$, $D_{iq}$, $I_{iq}$, $\lambda_{iq}$ and $C_i$.
Here $I$ denote the electrode location index. $\varepsilon_i$, $B_i$, $D_{iq}$ are characteristic parameters
depicting the geometry of the attractors while $I_{iq}$, $\lambda_{iq}$ and $C_i$ denote the dynamic state of
the attractor system. Since $D_{iq}$, $I_{iq}$, $\lambda_{iq}$ are all generalized parameters, the suffix $q$ take
values from 0 to infinity. The fact that new neurons are generated and destroyed regularly
even in adult primates (Gould et al, 1999) implies that at no stage is the neuronal number
conserved.
If we denote by $\Omega$, a pattern generated by a set of attractors specified by the invariant parameters, we can then construct a phase space spanned by these parameters and any pattern generated would be represented by a point in this space. The trajectory, which the point generates in time, would then represent the evolutionary path of the system. As in Statistical Mechanics, if we take all plausible patterns and their evolution in time, these trajectories form an incompressible flow and one can define a Liouville density

$$\rho = \rho(\Omega, t)$$  \hspace{1cm} (4)

satisfying the Liouville equation

$$\frac{d\rho}{dt} \equiv 0$$  \hspace{1cm} (5)

In (4)

$$\Omega = \left\{ \Omega_i \right\} = \left\{ \varepsilon_i, B_i, D_{iq}, I_{iq}, \lambda_{iq}, C_i \right\}$$  \hspace{1cm} (6)

We can now define the probability of one pattern being generated in the system (which is equivalent to one particle distribution function) as

$$f(\Omega, t) = \int \rho(\Omega, t) d\Omega'$$  \hspace{1cm} (7)

where the prime denotes that the integration is effected for all $\Omega$ s of (6) except for $i = k$, and

$$d\Omega = \left\{ d\Omega_i \right\} = \left\{ d\varepsilon_i, dB_i, \{ dD_{iq} \}, \{ dI_{iq} \}, \{ d\lambda_{iq} \}, dC_i \right\}$$  \hspace{1cm} (8)

The one particle distribution is the, most significant quantity, since we want to calculate the pattern, which gets established as a consequence of repeated similar input signals. Since the system is non-equilibrium in nature, there are two distinct methods of solving this problem. One can follow the method of BBGKY, where if we integrate the
equation (5) it results in a hierarchy of equations, and one has to truncate the hierarchy by imposing arbitrary conditions on the distribution function (Bogolubov, 1962). We can also define time scales following the method developed by Prigogine and co-workers (Balescu, 1975) and select terms pertaining to a specified time scale from the general solution (2). These methods are however feasible only if we know the interactions in the system precisely.

The general solution of (2) can be written as a non-Marcovian Integral equation as

\[ \rho(t) = \exp(-Lt)\rho(0) + \int_{0}^{t} d\tau G(t-\tau)\rho(\tau) \]  

where \( L \) is the Liouville operator. In this all interactions are included in the definition of the transition probability \( G \) or the Green's function as is known in field theory.

We shall follow the method adopted by Parikh and Pratap (1984) and obtain the "one particle (pattern) distribution function". We shall denote the afferent pattern by \( A \) and the efferent pattern by \( U \) and the background pattern as \( R \). Each of these are functions of attractor variables. We now define the probability of occurrence of \( U_k \) at time \( t \) as

\[ f(\{U_k\},t) = \int_{0}^{t} d\tau \int d\{A_i\} \int \int d\{U_i\} \int \int d\{R^0_j\} 
G(\{U_k\}\{A^\mu_i\}\{U^\nu_i\}\{R^{00}_j\},t-\tau)\rho(A_i,U_i,R_j,\tau) \]  

where \( G \) is the transition probability of taking \( \rho \) at \( \tau \) to \( \rho \) at \( t \), which is a function of \( \{U_k\} \). In (7) \( \mu, \nu \) are indices pertaining to the attractor, while i,j denote the pattern generated by the afferent and efferent processes. It may be noted that time occurs as \( t-\tau \) in \( G \) and \( \rho \) is at time \( \tau \). Hence the evolution depicted in (7) is path dependent or the equation is non-Marcovian implying thereby that the equation has a built in memory.
We shall use the above formulation in defining a learning index. We shall designate each pattern generated by the afferent-efferent pattern as a “concept”. If two successive afferent patterns are such that their difference is comparable to the standard deviation of the various patterns generated by the random spontaneous background neural firings, we then say that the two patterns are similar. Having defined similarity in patterns, we shall now define learning index. We say that a concept is learnt if the efferent patterns generated by a sequence of similar afferent patterns converge to a set of constant patterns. These statements can be quantified by defining proper transition probability.

In constructing the model, one has to specify the transition probability. This operator transports the system state from \( t=0 \) to \( t=t \). Since we do not have any insight into the actual dynamics prevailing between the attractors our guide line would be (a) that the input patterns should be similar and (b) the emerging patterns in the asymptotic limit in time should be a set of constant patterns. In the final stage of the analysis we evaluate the statistical average of \( U_l \) using \( f(U_{\cdot k}, t) \) and take the asymptotic limit in time.

We shall define the transition probability in (10) as

\[
G\{\{U_a\}\{U^\alpha_i\}\{A^+_j\}\{R^0_j\}, t=\tau\} = \sum_k G_k\{\{U_a\}\{U^\alpha_i\}\{A^+_j\}\{R^0_j\}, t=\tau\} \tag{11}
\]

with the explicit form for \( G_k \) given by

\[
G_k = Sin(o(t-\tau))\prod_{\alpha} \delta\left(U^{\alpha}_a - \delta_{\alpha x}(U^{\alpha}_x - \sum U^\alpha_i \exp(-\sum_{\mu\nu} C_{\mu\nu} A^\gamma_j R^0_j))\right) \tag{12}
\]

and the distribution function as at \( \tau \) by

\[
\rho(\tau) = \prod_j \rho_j\{\{U^\alpha_i\}\{A^+_j\}\}\delta(R^0_j - A_j) \delta(\tau) \tag{13}
\]

In the above \( \delta_{\alpha x} \) is a Kronekar delta while \( \delta s \) are Dirac delta functions. Also \( U^\alpha_a \) is the asymptotic value of \( U_a \). It should be realized that the choice of time dependence through
a harmonic function in the transition probability and \( \delta \) function in the distribution amounts to taking the Markovian limit.

If we multiply the one particle distribution by \( U_i \) and perform all integration, we get

\[
U_i^0 = \left[ U_i^c - \sum_{\mu} U_i^{\mu} \exp\left( -\sum_{\mu\nu} C_{\mu\nu} A_i^\nu A_j^\mu \right) \right] \sin \omega t
\]

We shall take the asymptotic limit in time by first going to the Laplace space defined by

\[
f(z) = \int_0^\infty dt e^{zt} f(t)
\]

with the corresponding inverse transform. For \( \sin \omega t \) the transform becomes

\[
f(z) = \frac{\omega}{\omega^2 - z^2}
\]

where the Laplace parameter \( z \) is complex with the imaginary part being positive definite. The asymptotic limit amounts to the limit \( z \to 0 \). If we expand (16) binomially and take the limit \( z \to 0 \), the function reduces to \( \omega^{-1} \). We define \( \left( U_i^c / \omega \right) \) as the learning index since for repeated input of the afferent patterns, the efferent patterns tends to a constant learning index, as the exponential term vanishes since the exponent takes larger values.

3. Interaction Potential Distribution in the Skull Space

In this section, we evaluate the effective interaction potential at various points as inferred from electroencephalogram (EEG) (Indic & Pratap). The EEG data from each of 16 channels distributed in the skull space is subjected to singular value decomposition (Broomhead and King, 1986) obtaining the eigenvalues and the eigenfunctions. In these however the most dominant one is the first one, and the eigenvalues get diminished as one goes upto 5 (Parikh and Pratap, 1991). We evaluated these values for 15 windows for each eigen value, and took the average potential at each
channel for each eigen value. We then used these eigen values and eigen functions in a Hamiltonian and wrote the Schrodinger equation and evaluated the potential V at the various skull points. In the following, we have plotted the potential at the 16 skull points and drawn the isocontours for three different cases viz., (a) normal eyes closed (b) normal eyes open (c) meditation. We propose to continue this work for different pathological cases such as seizures, Alzheimer's disease as well as mental depression using a 64 channel data set, since we would get a closer grid distribution and thereby a more realistic isocontour. Hence the present work may be considered as a preliminary study.

In dimensionless quantities, the Schrodinger equation may be written as

$$\frac{\partial^2 \psi}{\partial x^2} + [\lambda - V(\psi)] \psi = 0$$

(17)

where \(\lambda\) and \(\psi\) are the eigen values and eigen functions respectively and the potential V is a function of \(\psi\) since the process is nonlinear.

We solved this equation numerically for different \(\lambda\) and the corresponding \(\psi\). As has already been mentioned, since the first eigen value is dominant, we have plotted the resulting potential function at the 16 channel points and then drawn the isocontours. The results are revealing. We propose to continue this procedure for a 64-channel data set, since that would give a much closer grid system. We did this exercise for a normal case for eyes closed, eyes open and also for meditation. The isocontours are for different values of the potential and we thereby get a flow vector from the larger to a lower potential value.

In fig. 2 (a) for a normal person with eyes closed, we get a complex distribution with islands being formed both in the frontal and central-parietal regions. We have plotted the curves for \(V=3.3\) and 3.1. If we visualize an information flow vector
from a larger to a lower potential, the flow vectors are scattered in direction and that the vectors have a higher magnitude since the magnitude is \((\Delta V/\Delta r)\), \(\Delta r\) being the normal separation between the contours in the skull space. This could be understood as follows: Even though there are no ocular signals, the previous signals in the brain have registered patterns and since the subject is psychologically active, complex patterns are generated in the neocortex even when the eyes are closed, since the subject is in the conscious state.

However in the eyes open case, fig. 2 (b), ocular signals are received and hence the psychological complexity is reduced. Here however, the flow is confined mainly to the lower half of the brain. Nevertheless, it is significant that both lobes take active roles and hence the results indicate lack of localization in the brain function vis-à-vis right and left lobes. Here again the absolute value of the flow vector is comparable to the earlier one. This result however may be considered as tentative, since a 64/128-channel data set would give a better insight into the collective behaviour as against the localization phenomena.

The result is much more revealing in the case of meditation, fig.2 (c). The activities are much more spread in the lower half of the brain and again the gradients are much smaller, as compared to the previous one. This indicates that as one enters a meditation state, the activity is considerably reduced. Even though this was more or less an accepted situation, here we have shown that this is indeed true. This gives us a greater confidence in the veracity of this method, and the results this method would be generating.

4 Discussions and Conclusions

In this paper we have extended the domain of invariant parameters in the brain by defining (a) Coherence index and (b) Learning index. We have shown that phase and amplitude synchronization indices should be calculated to evaluate coherence index and
that these indices should be calculated from the Poincare map rather than in a Hilbert space. Again in defining the Learning index we have defined quantitatively a Concept and the Learning index. In investigating interactions in the system we considered a multispecies attractor gas in the brain and evaluated the effective potential by resorting to an inverse scattering mechanism in the frame work of nonequilibrium statistical mechanics as developed by Prigogine and coworkers at Brussel. We have yet to develop an algorithm to evaluate the Learning index. The global distribution of the effective potential is very sensitive to the brain state. We are exploring the possibility of using these parameters along with the already known ones to develop a diagnostic system by evaluating them on an online set up and this work is in progress.

Acknowledgement

One of us (PR) would like to thank the Council of Scientific and Industrial Research, Govt of India for awarding a fellowship during the period of working on this problem. We would like to express our deep sense of gratitude to Mr. P Indic for the help rendered during the period of working on this problem. We would like to thank Prof V P N Nampoori as well as Prof C P G Vallabhan and other members of the group for discussions as well as extending unstinted support during this period.
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Figure 1 (a), (b) The plot of phase (Sp) amplitude (Sa) and Coherence index (Ci) against equally spaced window interval. (a) is based on Rosenblum assumption and (b) obtained from Poincare’ map
Figure 2 (a) Eyes Closed

Figure 2 (b) Eyes open
Figure 2 (c) Under Meditation
The diagram of isocontours (of equipotential curves) in the skull space for three different cases obtained from normal subjects.